THE REVISED COURSE IN INDUSTRIAL INSTRUMENT TECHNOLOGY

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INDUSTRIAL INSTRUMENT TECHNOLOGY

by J. T. Miller, *B.Sc., F.Inst.P., M.S.I.T.*

PRICE THIRTY SHILLINGS

UNITED TRADE PRESS LTD

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The course is reprinted from INSTRUMENT PRACTICE, the journal that serves every industry using modern instrumentation ... ement of industrial processes.

...ugh Square, Fleet Street, London, E.C.4.

United Trade Press Ltd.,
Boswell House,
9 Gough Square,
Fleet Street,
London, E.C.4.

Fourth Impression

1970

Printed offset by Alden & Mowbray Ltd at the Alden Press, Oxford. 1969

Foreword

The original "Course in Industrial Instrument Technology" ran in *Instrument Practice* from February, 1951, to August, 1952. Twelve years ago there were no books whatever from British sources dealing with instrumentation and the courses in technical colleges were few and far between. Two works by British authors: *Instrument Technology* by E. B. Jones and *An Introduction to Process Control System Design* by A. J. Young, were being prepared but were not due to appear for two or three years. It was felt that a concise course dealing with the fundamentals of industrial instrumentation would at least bridge the gap until the books mentioned were published. More than that it was hoped the Course would stimulate interest in instrumentation amongst apprentices, students and the like for an industry which even then was developing at an extremely rapid rate. The main object was attained. The Course created great interest whilst running in *Instrument Practice*. After the last section had appeared, 500 sets of loose reprints were sold within a month It was then decided to issue the Course as a set of reprints bound in a paper cover. Many thousands of copies have been sold to date and there is an increasing demand for it.

A revision of the Course has long been projected but has for one reason or another been delayed until the present time. It has seemed logical to repeat the original procedure and issue the revised edition as monthly chapters in *Instrument Practice*. The present volume comprises these chapters in bound form.

In revising the material, one must remember that despite the considerable incursion of electronic techniques, many measuring and control functions are still carried out (and are likely to be carried out) by well tried non-electronic methods. Much, therefore, of the original Course remains, but opportunity has been taken to make some extensive rearrangement, particularly in the initial chapters, which now present some form of logical sequence. New material has been added on high vacuum measurement, on transducers and on feedback, whilst the chapters on automatic control have been largely re-written. To this must be added details of many electronic and nucleonic instruments where these have been applied to industrial processes.

Expansion of British industrial activities since the completion of the original Course in 1952 has been phenomenal. This, in turn, has meant a colossal increase in the number of measuring and controlling instrument systems used in industrial processes and it has brought into being many new techniques to implement existing designs.

It was tempting, therefore, to expand the Course to include such subjects as gas-liquid chromatography, industrial viscometry, absorptiometry, automatic titration and many other examples of continuous process analytical instrumentation. But to preserve a balance, this would have meant the inclusion of many other techniques with the consequent destruction of the whole nature of the Course. If one is aiming at a text book on instrumentation then one must be complete and this can mean a volume of 400—500 pages of normal dimensions. There is little need to add to the relatively large number of existing works and, indeed, I refer to these as sources of further reading throughout the Course. But one cannot run before one can walk, neither can one apply techniques before basic principles are understood and here, it is hoped, the present text will find its niche.

Hence, I would point out that the Course is intended to be a short concise introduction to industrial measurement and control and is not in consequence an exhaustive treatise on the subject. Not every instrument or system manufactured, therefore, is included, but the most important and widely used will be found in the following pages.

It is hoped that, above all, the Course will prove of some assistance and stimulation to those in the initial stages of education and training in measurement and control or to those who are considering entering what is now a key industry.

J. T. Miller

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Chapter 1

FLUID PRESSURE MEASUREMENT

WE are concerned with four main types of fluid pressure measurement:

1. Differential pressure, that is the difference between two pressures.
2. Gauge pressure, which is the pressure measured *above* the local atmospheric pressure.
3. Absolute pressure which is the *total* pressure measured from zero pressure as the datum point. When its value exceeds the local atmospheric pressure, it can be regarded as the sum of the local atmospheric pressure and the gauge pressure.
4. Vacuum, a condition in which the pressure is always less than that of the local atmosphere.

It is very convenient, if not orthodox, to consider the measurement of differential pressure first, since many of the common methods of measurement of gauge pressure and absolute pressure are but particular cases of differential pressure determination. For example, if the lower pressure of the two is made atmospheric, several of the instruments now to be described become gauge pressure devices. Similarly, if one pressure becomes zero, the instruments become suitable for absolute pressure measurement.

Static Pressure

The term "static pressure" is widely used in the measurement of pressure, liquid level and rate of flow. Where the fluid in a process is still or "static" the meaning of the phrase is reasonably clear. When a fluid in motion is encountered, however, the meaning may not be so obvious. An example may assist. Consider a pipe full of a fluid in motion. If a hole is drilled in the wall of the pipe and a pressure measuring instrument connected to it, the pressure indicated on the instrument is the static pressure at the tapping (see Chapter 4 on Flow Measurement).

Unless otherwise indicated, where reference is made to pressure in the text, static pressure is understood.

In differential pressure measurement, the difference involved is normally between two static pressures, and the difference may be relatively small compared with either of the two static pressures. These, as well as the differential pressure range, will determine the type of instrument to be used.

MEASUREMENT OF DIFFERENTIAL PRESSURE OR PRESSURE DIFFERENCE

Simple U-Tube Manometer

One of the most popular devices for measuring differential pressure is the U-tube manometer.

A column of liquid exerts a pressure proportional to its height, and is equal to $h\rho$ per unit area, h being the height and ρ the density of the liquid in appropriate units. Upon this fact is based the operation of the simple U-tube and its more advanced developments. *Fig.* 1 shows an elementary form, consisting of a glass tube bent into the shape of a U and partly filled with liquid

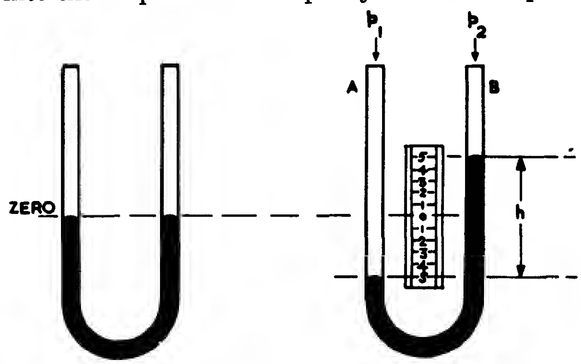


Fig. 1. *The principle of a simple* U-*tube manometer*

A scale is fixed between the limbs. It is a differential instrument in that it responds to a difference in the pressures exerted on the liquid in limbs *A* and *B*. The pressures should be regarded as absolute ones. If the higher pressure p_1 is applied to *A* and the lower pressure p_2 to *B*, the liquid in *A* will be forced down, that in *B* will rise, and the action will continue until pressure p_1 is balanced by the sum of pressure p_2 and that due to the column of liquid h between the two levels.

Then $$p_1 = p_2 + h\rho \qquad (1)$$

$$p_1 - p_2 = h\rho \qquad (2)$$

or $$h = \frac{p_1 - p_2}{\rho} \qquad (3)$$

ρ being the density of the liquid in the tube. The height h is therefore a measure of $(p_1 - p_2)$ the pressure differential.

Assigning some figures to equation (3),

if $p_1 = 8$ lb./sq. in. absolute

$p_2 = 6$ lb./sq. in. absolute

$\rho = 0{\cdot}03604$ lb./c. in. at 60°F. using water as the liquid

$p_1 - p_2 = 8 - 6 = 2$ lb./sq. in.

$$h = \frac{2}{0{\cdot}03604} = 55{\cdot}4 \text{ in. of water.}$$

It can be seen that for comparatively small differentials the size of instrument will be cumbersome if water is used as a liquid. For higher differentials, up to say 20 lb./sq. in., mercury is substituted for water.

If ρ for mercury at 60°F. is 0·488 lb./c. in. for $p_1 - p_2 = 20$ lb./sq. in.

$$h = \frac{20}{0{\cdot}488} = 41{\cdot}0 \text{ in. of mercury.}$$

Oils of various densities are used as alternative liquids to water for small differentials.

It will be noted that the densities given above are specified at 60°F. This is because the density of a liquid varies with temperature, and equation (3) shows that this can cause a variation in h for the same differential pressure. If large temperature changes occur, it may be necessary to correct all readings taken to a common basic temperature, e.g. 60°F.

Most U-tubes have scales starting at 0 for zero level and repeat the graduation numbers above and below. Each division is then half its normal value. A scale calibrated in inches would be numbered 1, 2, 3 at every ½ inch above and below the zero line, as in *Fig.* 1. It may be found that both limbs are not exactly the same as regards their bores. It is customary, therefore, to read each level and to take the average of the two readings, if these are not the same, for accurate work.

Range of Simple U-Tube

The simple U-tube is normally used between 0–4 in. water and 0–48 in. mercury (0–23·5 lb./sq. in. approx.) differential pressures.

High Pressure Industrial U-Tube Manometer

The next development in U-tube devices is indicated in *Fig.* 2. Here, one tube has become very much increased in diameter compared with the other.

It is still essentially a U-tube but the difference in diameters of the limbs brings about a modification of the basic equation. A differential pressure is applied between the limbs, the lower p_2 to the wide limb B, and the higher p_1 to the narrow limb A. The liquid falls in A and rises in B until the following pressure balance is realized:

$$p_1 = p_2 + (h + d)\rho \quad \ldots\ldots\ (4)$$

where h is the height the liquid in the narrow limb A has fallen from zero, and d is the distance the liquid in the wide limb B has risen from zero, and ρ is the liquid density.

Now, a volume of liquid has left the narrow limb, causing the level to fall, and has entered the wide limb, producing a rise of liquid there. The former is A_2h where A_2 is the area of the narrow limb, and the same volume in the wide limb is A_1d where A_1 is the area of the wide limb.

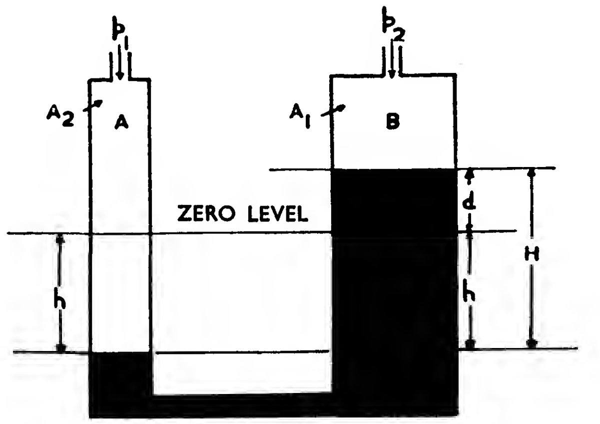


Fig. 2. *The principle of a* U*-tube manometer, one limb wider than the other*

Then $\quad A_1d = A_2h \quad \ldots\ldots\ (5)$

$$\text{Hence} \quad h = \frac{A_1}{A_2}d \quad \ldots\ldots\ (6)$$

Substituting in equation (4) for h,

$$p_1 = p_2 + d\rho\left(1 + \frac{A_1}{A_2}\right) \quad \ldots\ldots\ (7)$$

$$p_1 - p_2 = d\rho\left(1 + \frac{A_1}{A_2}\right) \quad \ldots\ldots\ (8)$$

$$\text{or} \quad d = \frac{p_1 - p_2}{\rho\left(1 + \frac{A_1}{A_2}\right)} \quad \ldots\ldots\ (9)$$

The distance d that the liquid rises in the wide tube is, therefore, a measure of the differential pressure $(p_1 - p_2)$.

To illustrate the working of equation (9) let us assign some figures to the factors.

Let $p_1 = 8$ lb./sq. in. absolute

$p_2 = 6$ lb./sq. in. absolute

$\rho = 0{\cdot}488$ lb./c. in.

$d_1 = 6$ in. $\quad d_1^2 = 36$ sq. in.

$d_2 = 3$ in. $\quad d_2^2 = 9$ sq. in.

$$\frac{A_1}{A_2} = \frac{\frac{\pi \times 36}{4}}{\frac{\pi \times 9}{4}} = 4$$

$$\text{Hence} \quad d = \frac{2}{0{\cdot}488 \times (1+4)}$$

$$d = 0{\cdot}82 \text{ in.}$$

Consider next that a pressure differential of 4 lb./sq. in. is to be measured, and that d is to be the same. To achieve this the only variables which can be adjusted are the density ρ and the diameter of the narrow tube. It is much more suitable to alter the latter, and substituting in equation (9)

$$0{\cdot}82 = \frac{4}{0{\cdot}488\left(1 + \frac{36}{d_2^2}\right)}$$

Solving this equation, it will be found that $d_2 = 2$ in. approx.

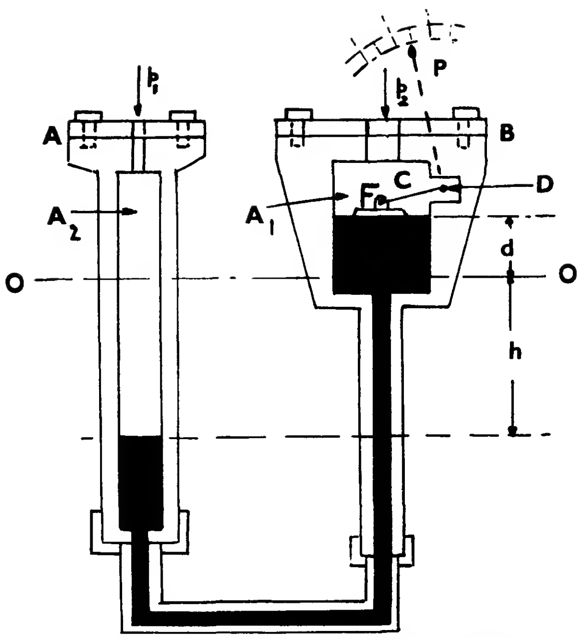


Fig. 3. *The basic construction of an industrial high pressure* U*-tube manometer with float operated mechanism*

The practical application of this is seen by referring to *Fig.* 3. It is a diagrammatic representation of a basic design used by many firms. It consists of two limbs comprising steel chambers with appropriate inlets for the applied pressures. One chamber is short and wide, corresponding to *B* in *Fig.* 2, and the other is taller and narrow, corresponding to *A* in *Fig.* 2. They are interconnected by steel tubing or castings so that the whole assembly may be robust and suitable for working at high pressures. With reference to the latter, although the differential $p_1 - p_2$ may be low, e.g. 2 lb./sq. in. or 4 lb./sq. in., the values of p_1 and p_2 may be very high: 1,000 lb./sq. in., 1,500 lb./sq. in., 5,000 lb./sq. in., etc. For example, p_1 may be 1,000 lb./sq. in., the differential 4 lb./sq. in., giving p_2 as 996 lb./sq. in. All parts of an instrument in contact with such pressures must be robustly built and capable of standing such values continuously without breakdown.

On the surface of the liquid in the wide chamber is a metal float connected by the arm *C* to a shaft *D*. The action of the float *F* in riding up and down on the surface of the liquid, as the level varies in response to changing differentials, rotates arm *C* and the shaft *D*. The latter is taken outside the chamber by a pressure-tight seal, and serves to operate a pen or pointer *P* by a linking mechanism.

If the instrument is so designed that the narrow tube, frequently termed the "range tube", may be changed for one of different diameter a means of altering the instrument range is obtained.

Cistern Manometer

It would make no difference to the working of the instrument if the narrow tube were inserted directly in the wide limb, or cistern as we shall now call it. *Fig.* 4 illustrates this. The advantage is that a very narrow column can be inserted in the cistern and a reasonably long single scale erected for measuring the rise in this column.

If a differential pressure $p_1 - p_2$ is applied to the cistern and column, A_1 is the area of the cistern, A_2 the area of the narrow tube, and h the rise of liquid from zero level in this tube, then

$$(p_1 - p_2) = h\left(1 + \frac{A_2}{A_1}\right)\rho \quad \ldots\ldots \text{(10)}$$

If the value of $\frac{A_2}{A_1}$ can be made so small that it may be neglected (equivalent to increasing the cistern diameter to such a value that the drop in level d from the zero level in the cistern is negligible), then

$$p_1 - p_2 = \rho h \quad \ldots\ldots\ldots\ldots \text{(11)}$$

When the ratio $\frac{A_2}{A_1}$ is not negligible, the scale may be calibrated in "contracted inches", that is, normal inches multiplied by the ratio $\frac{A_1}{A_1 + A_2}$.

0–12 ins. water to 0–48 ins. mercury would be typical extreme ranges.

Inclined Tube Manometer

A special development of the U-tube is the inclined tube manometer. It gives an increased length of scale compared with the simple U-tube for the same differential pressure. As its name implies, one limb consists of a glass tube inclined at an angle to the vertical or horizontal, and the other of a wide chamber or cistern. The sloping tube carries a scale adjacent to it, and the purpose of the inclination is to achieve a longer scale than the normal U-tube for the same differentials.

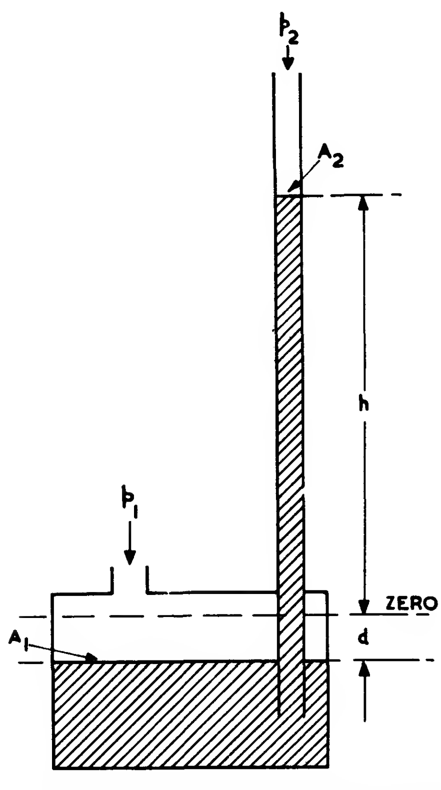


Fig. 4. *The principle of a cistern type manometer*

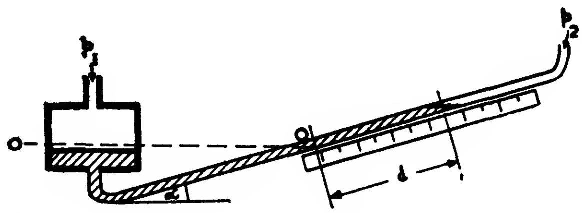


Fig. 5. The principle of an inclined tube manometer

Fig. 5 indicates the essentials and without much trouble the value of the differential pressure $(p_1 - p_2)$ can be derived as:

$$p_1 - p_2 = \rho d\left(\frac{A_2}{A_1} + \sin\alpha\right) \quad \ldots\ldots (12)$$

where ρ = the density of the liquid
A_1 = cistern area
A_2 = tube area
α = angle of slope of tube from horizontal
d = distance the liquid in the sloping tube has moved from zero.

Rewrite equation (12) as:

$$d = \frac{p_1 - p_2}{\rho\left(\frac{A_2}{A_1} + \sin\alpha\right)} \quad \ldots\ldots (13)$$

and assign values as follows:
$p_1 - p_2 = 2$ in. water
$\rho = 0{\cdot}036$ lb./c. in. (water)

$\alpha = 30°$ giving $\sin\alpha = \frac{1}{2}$

$\frac{A_2}{A_1} = \frac{1}{256}$ (diameter of cistern 4 in. and diameter of tube $\frac{1}{4}$ in.)

Then $d = 4$ in. approx.

On an ordinary U-tube, as in *Fig.* 1, the reading would have been 2 in., so that we have obtained a scale length of about double the value by using the inclined tube version.

A few inches of water is the normal range (0–0·5 to 0–12).

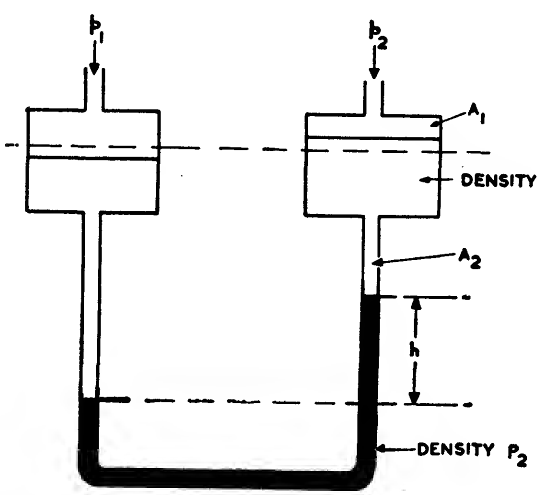


Fig. 6. The principle of a two liquid U-tube manometer

Two Liquid U-Tube Manometer

A variation of the normal design of U-tube is shown in *Fig.* 6. This contains two liquids, a lighter one situated above a heavier. At the top of each limb is a cistern, both being of the same area. If the pressure applied to one limb is p_1, and the other p_2,

$$p_1 - p_2 = h(\rho_2 - \rho_1) + \frac{A_2}{A_1}\cdot h\rho_1 \quad \ldots\ldots (14)$$

where
ρ_2 = the density of the heavier liquid.
ρ_1 = the density of the lighter liquid.
h = the difference in levels of the two limbs.
A_2 = the area of the tube.
A_1 = the area of the cistern.

By suitably choosing the two liquids, and the proportions of the tube and cisterns, the value of h may be made large for a comparatively small value of $(p_1 - p_2)$. This is brought out in the equation where the first factor is the most important. The smaller $(\rho_2 - \rho_1)$ becomes, the larger the value of h for the same differential pressure.

Diaphragm, Diaphragm Stack, or Bellows Pattern Differential Pressure Gauge

For many reasons, in industrial applications, the liquid manometer pattern of pressure measuring instrument is not suitable, and an alternative must be sought. One of these is the instrument which incorporates a diaphragm, capsule or bellows as its measuring device.

The simplest in conception is the single diaphragm instrument.

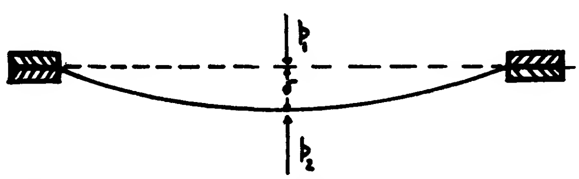


Fig. 7. Illustrating the deflection δ of a single diaphragm under the action of a differential pressure $(p_1 - p_2)$. p_1 is the higher pressure

Single Diaphragms

In its elementary form, a diaphragm is a thin flat plate of circular shape, fixed firmly round its edge. On applying a pressure to one side greater than that existing on the other, the diaphragm deflects in a direction away from the higher pressure, the movement being greatest at the centre (*Fig.* 7). At first sight, it would seem that by linking up the diaphragm centre with a pointer or pen a means of measurement is obtainable. Such a simple arrangement possesses limitations, particularly if the material is metal. The deflection is related to the radius of the diaphragm, its thickness, the modulus of elasticity of the material, and other physical factors. Only for relatively small movements is there a linear connection between pressure and deflection. When the latter approaches half the diaphragm thickness, divergence is noticeable, and above this value there can be considerable departure from a straight line relationship. Some modification of this simple case is desirable for practical instrumentation.

For very low pressures a diaphragm is required that is extremely flexible. A variety of materials have been tried such as colon leather, gold beater skin, nylon, rubberized fabric, and each manufacturer has his particular material. A means of connecting linkage to the diaphragm has to be introduced when using such fabrics, and it is customary to make the diaphragm assembly in the form of a ring of fabric with a disc of metal or other rigid material at the centre. Note that it is unnecessary for the diaphragm assembly to be circular. Quite often it is elongated. *Fig.* 8 indicates the principle. Gauges on these lines can be used up to 40 in. water, or just above 1·4 lb./sq. in.

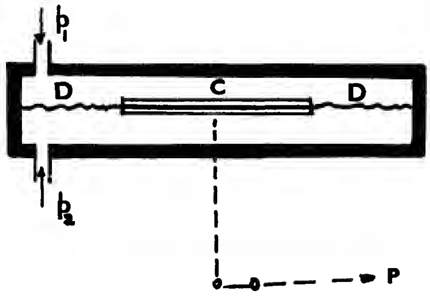


Fig. 8. *The principle of a fabric or "slack" diaphragm differential pressure gauge. D is the fabric and C the solid centre piece. To link C to pointer or pen P requires a sealing diaphragm in the wall of the chamber, magnetic coupling or similar devices*

Observe, in passing, that a problem exists in connecting a linkage system from the diaphragm *C* to pen or pointer *P*. This is solved, very often, by a flexible seal fitted in the wall of the instrument and used as a kind of pivot or displacement device. Solutions can be effectively made by devices of the type shown in *Fig.* 9*b* or by the construction of *Fig.* 13.

Fig. 9*a* shows the actual assembly of an industrial flexible diaphragm gauge manufactured by Bailey Meters & Controls Ltd.

As far back as 1844 Vidie first conceived the idea of introducing corrugations into circular metal

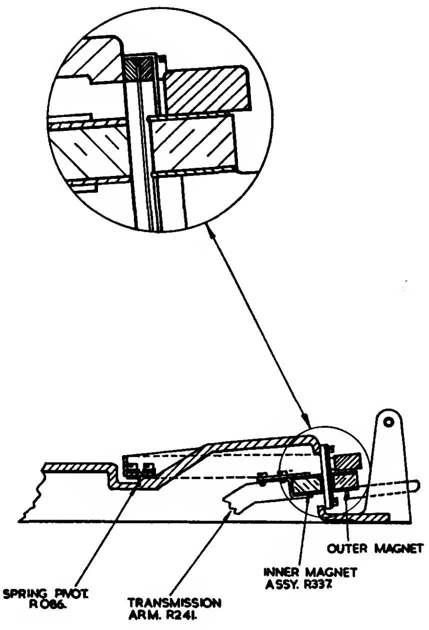


Fig. 9*b*. *Illustrating the method of coupling a pointer mechanism to a diaphragm, similar to Fig.* 9*a*, *by magnetic means. Reproduced by permission of Bailey Meters & Controls Ltd.*

diaphragms with the object of making them more flexible, so that they would give a bigger deflection than a plain diaphragm for the same applied pressure. It was found that the deflection bore a linear relation to the pressure provided the movement was not excessive. It will be apparent from *Fig.* 10 that several factors are involved.

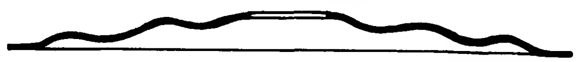


Fig. 10. *Section through a corrugated diaphragm*

These include the number of corrugations, the depth of each corrugation, the thickness of the diaphragm material, the radius of the diaphragm, the radius of each corrugation from the centre, the "set", and the physical properties of the

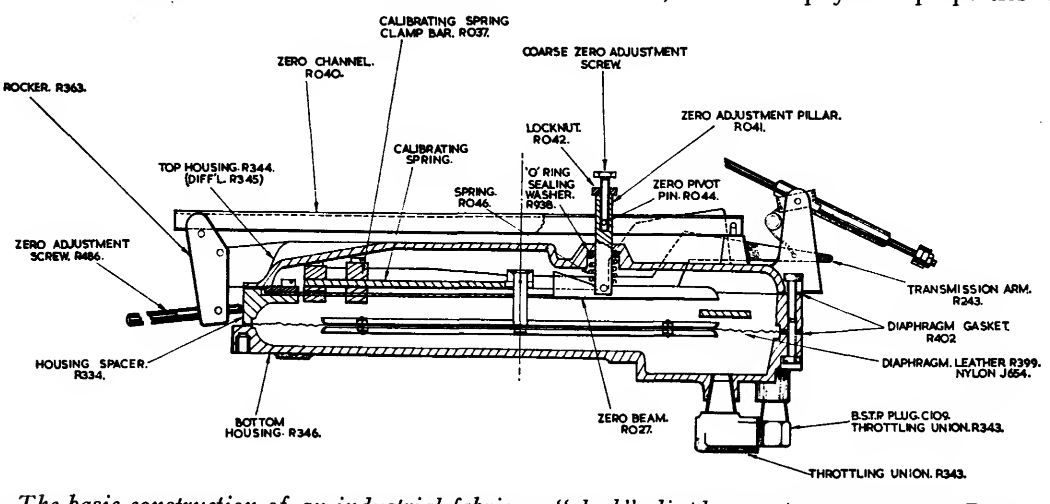


Fig. 9*a*. *The basic construction of an industrial fabric or "slack" diaphragm pressure gauge. Reproduced by permission of Bailey Meters & Controls Ltd.* (*This figure should be associated with* 9*b*)

material. The number of variables is so large as to render a general comprehensive mathematical formula covering the performance a matter of extreme difficulty.

The corrugated type of diaphragm is useful in itself but finds a wider application when it is assembled in stacks.

Diaphragm Stacks

For this purpose, diaphragms are pressed or punched out with a central hole. A number of these are taken, and soldered, welded or otherwise joined together in a leak-proof manner at the inner and outer edges. The whole assembly then forms a kind of metal concertina. If the interior and exterior are sealed off from one another (*Fig.* 11 shows this done in a simple manner) and a pressure difference is applied between inside and outside, a deflection of the stack is obtained. The value is many times that of a single diaphragm and is suitable to be used as the basis of a pressure gauge.

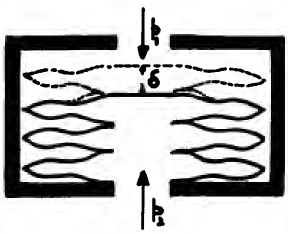


Fig. 11. *Illustrating the deflection δ of a diaphragm stack under the action of a differential pressure. p_1 is the higher value*

The number of diaphragms so used varies. It may be two, when the stack is known as a "capsule", or it may reach twenty or so.

Bellows

An alternative flexible device of somewhat more modern origin is the metallic bellows. Again it has general concertina shape, but its formation is totally different from the diaphragm stack. Initially, a thin walled tube is taken and formed (mainly by special hydraulic presses) into the corrugated shape shown in *Fig.* 12. To distinguish, it is better to call these convolutions. As in the previous case, sealing off interior and exterior and applying a pressure difference causes a deflection of the bellows of a sufficient value to be used in pressure measuring instruments.

Certain general rules have been laid down for the performance of metallic bellows. These are that the flexibility, defined as the movement in inches for each pound per square inch increase in applied pressure, is:

- (*a*) directly proportional to the number of convolutions,
- (*b*) directly proportional to the square of the outside diameter of the bellows,
- (*c*) inversely proportional to the cube of the wall thickness,
- (*d*) inversely proportional to the Young's Modulus of the bellows material.

Subject to the above conditions, bellows can be produced to quite large diameters, e.g. 12 in. The larger sizes, however, are not strictly speaking for inclusion in instruments, but are used on control valves and for special purposes.

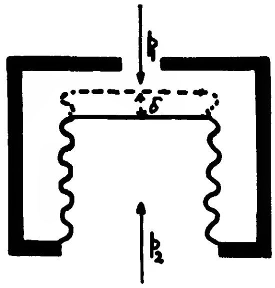


Fig. 12. *Illustrating the deflection δ of a bellows unit under the action of a differential pressure. p_1 is the higher value*

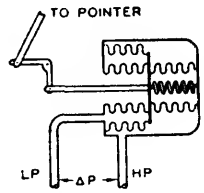


Fig. 13. *Illustrating the basic design of an industrial bellows differential pressure gauge. HP is the higher pressure, LP the lower. Reproduced by permission of Taylor Controls Ltd.*

Fig. 13 shows a bellows assembly used for differential pressure measurement. Note the double bellows arrangement to the left of the figure which solves effectively the pen or pointer linkage problem. The right-hand bellows acts as a seal for the range adjusting spring.

Rating

Bellows and stack assemblies have often been compared with a spring in that they are continuously extensible or compressible by applied force or pressure. Both, therefore, possess a *rate*, i.e. the force or pressure necessary to produce 1 in. of movement. The analogy with a spring is only a rough one, however, and should not be applied indiscriminately.

Materials

Diaphragm Materials

In this country, steel (of special composition), phosphor bronze, nickel silver and beryllium copper have been used.

Bellows Materials

Bellows materials have largely consisted of 80-20 brass (80 per cent copper and 20 per cent zinc), phosphor bronze, stainless steel and beryllium copper.

Ranges

The ranges of stack and bellows instruments can typically be between a few inches of water and two or three hundred lb/sq. in.

"Ring" Balance

The ring balance consists of a hollow or annular circular ring divided internally into two sections by a partition *C* in *Fig.* 14. On either side of the partition are flexible connections to the two pressure sources. The ring is pivoted at its centre on a knife-edge or has other convenient form of pivot, and when a pressure difference is applied to the partition a rotating moment is set up. The ring commences to rotate on its knife-edge in a direction away from the higher pressure. The action continues until the opposing moment, created by counter weight *M* at the foot of the ring, balances the rotating moment. The liquid in the ring acts as a seal.

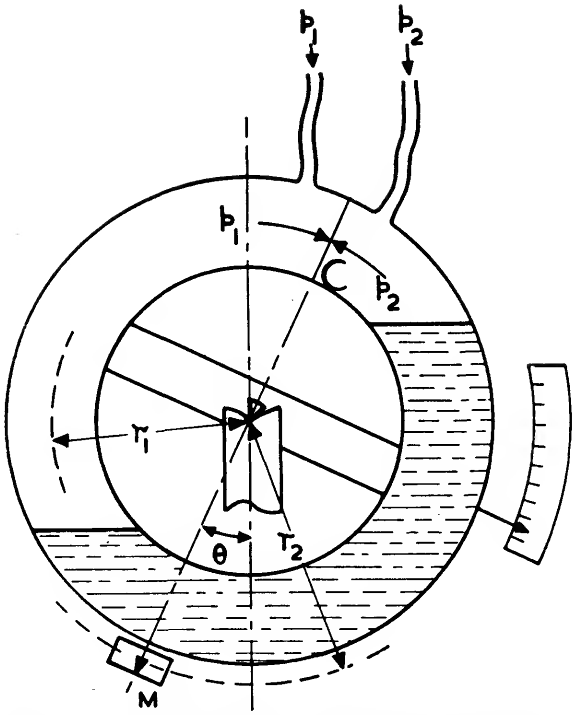


Fig. 14. *The principle of a ring-balance manometer*

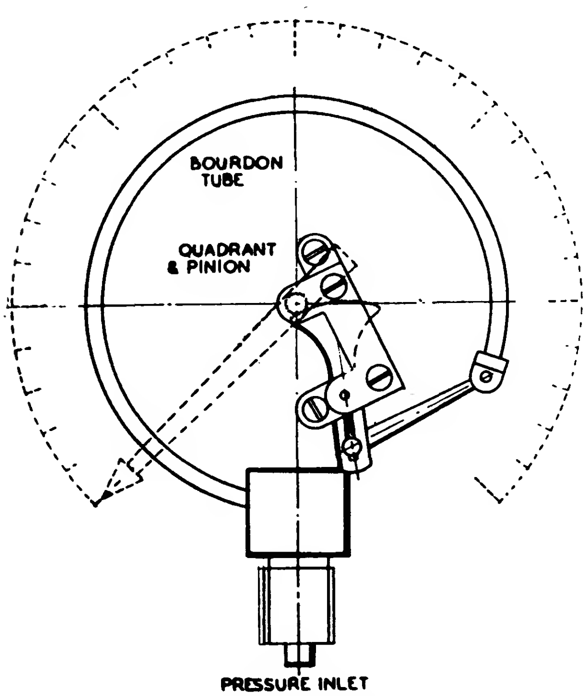
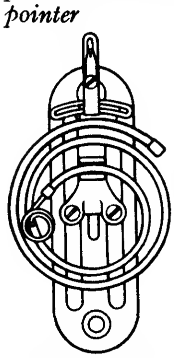
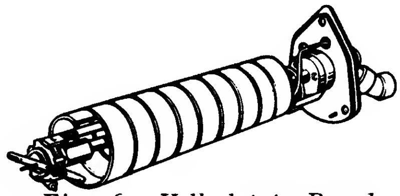


Fig. 15. *A C-type Bourdon tube with quadrant and pinion unit between the free end of tube and the pointer*





▲*Fig.* 16. *Helical type Bourdon tube*

◀*Fig.* 17. *Flat spiral Bourdon tube*

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The rotating moment $= (p_1 - p_2)Ar_1$ . . (15)
where $p_1 - p_2 =$ the differential pressure
$A =$ the tube cross-section area
$r_1 =$ the mean radius of the ring.
The restoring moment $= r_2M \sin\theta$ . . . (16)
where $M =$ the weight
$r_2 =$ the radius of its application
$\theta =$ the angle of rotation.
Equating,

$$(p_1 - p_2)Ar_1 = r_2M \sin\theta \quad \text{(17)}$$

$$p_1 - p_2 = \frac{r_2}{r_1} \cdot \frac{M}{A} \sin\theta \quad \text{(18)}$$

Thus the angle of rotation is a measure of the differential pressure, and a pointer attached to the ring may be used with a circular scale for measurement purposes. It will be noticed that there is no density factor involved in the equation so that the instrument cannot be affected by any temperature variations in the liquid density.

Ranges would vary typically between 0–1 in. and 0–12 in. water.

MEASUREMENT OF GAUGE PRESSURE

Bourdon Tube Pattern Instrument

This is one of the oldest pressure instruments, the tube having been patented by Bourdon in 1851. It has remained practically unchanged to this day and is one of the most common mechanisms in use.

It consists of a metal tube of approximately elliptical cross-section. The word approximately is used advisedly, since the normal method of manufacturing the bourdon is to take an ordinary tube of circular cross-section and flatten it by passing it through rollers. Having obtained the flattened tube, appropriate lengths are taken and are formed by special machines into a C shape as in *Fig.* 15, a long spiral (helical) form as in *Fig.* 16, or a flat spiral as in *Fig.* 17. In each case, one end of the tube so formed is closed and sealed, but left free to move. The opposite end is left open, but is mechanically fixed so that it is rigid. If now a pressure is applied to the inside of a tube greater than that existing outside, the elliptical or flat section changes its shape. Stresses are set up in the tube, and it begins to straighten out, with the result that the free end deflects, moving in an arc from its previous position. The amount of this deflection is proportional to the pressure difference between inside and outside. The latter is atmospheric for gauge pressure measurement.

It is unfortunate that, simple as the bourdon tube appears, the factors governing its operation are complex. Several papers have been written on the subject, some highly mathematical, but the chief stumbling blocks to any theoretical reasoning are the facts that the cross-section of the tube, when rolled flat, does not present any definite mathematical shape, i.e. it is not truly elliptical or rectangular, for instance, and the radius of the bend or turns alters during deflection. Apart from the dependence on the pressure difference existing between the outside and inside of the tube,

it is known that the deflection of the free end of the tube depends on the radius of the bend, the total tube length, the wall thickness of the tube, the major and minor axes (particularly the latter) of the cross-section, and the material of the bourdon tube. From data involving these factors in the possession of the manufacturers, it is possible to design any new tube required.

The usable part of the movement due to a **C** shape bourdon is never large, i.e. it may be of the order of $\frac{1}{8}$ in. or $\frac{1}{4}$ in. length of arc. With this pattern, therefore, it is necessary to introduce some form of multiplying mechanism between the tube and the pen or indicating pointer that it operates. Two of the most common mechanisms are the magnifying linkage, and the sector or quadrant and pinion movement.

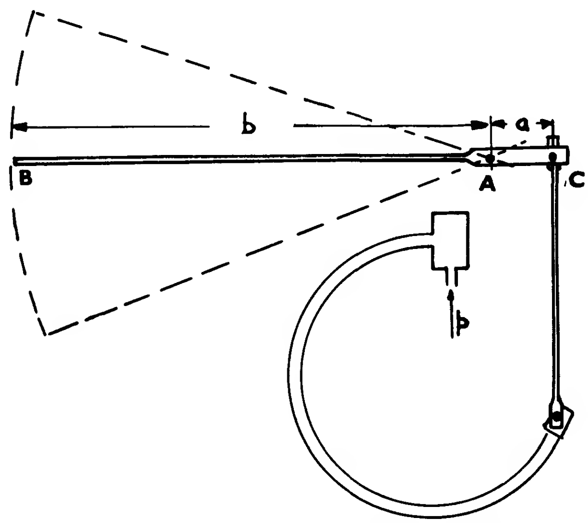


Fig. 18. *A simple diagram illustrating the effect of a magnifying linkage*

Fig. 18 shows, in a simple manner, the effect of a magnifying linkage on a pen arm. The arm is pivoted at *A*, the pen being at *B*. At end *C*, the pen arm is linked to the bourdon tube. Any deflection of the latter is magnified at *B* by the ratio $\frac{b}{a}$. To take some practical figures, if the bourdon deflection over the instrument range is $\frac{1}{4}$ in., *b* is 6 in., *a* is $\frac{3}{8}$ in., the movement of *B* will be

$$\frac{6}{\frac{3}{8}} \times \frac{1}{4} = 4 \text{ in.}$$

The angle the pen arm traces out in moving from one extreme of the chart to the other is about 38°. Whereas this angle is suitable for recording instrument pens, for indicating instruments of the circular scale type, the angle the pointer moves through can be as much as 300°. It is clear that ordinary linkage will become involved to produce this large angular movement and it is customary in such a case to use a sector gear and pinion movement as shown in *Fig.* 15. It should be noted that apart from the magnifying action of the gear and pinion, the sector itself, by virtue of its pivot position, can also magnify.

The **C** type bourdon possesses, as we have seen, limitations as to the amount of useful deflection it can produce, and these become pronounced at the lower pressure ranges, e.g. 0–5 lb./sq. in. Either the magnifying value would have to be so large as to produce errors, or the tube would have to be of such a thin wall that it would be unstable. If, however, a number of **C** bourdons could be so arranged that their individual deflections added up, larger deflections could be obtained. Practically, this is achieved by the spiral bourdons, either helical or flat, as indicated in *Figs.* 16 and 17 respectively. In such tubes, each turn may be roughly regarded as a **C** bourdon, with the movement due to each turn adding until the free end of the last turn has a deflection equal to the sum of these.

With these two types not only can we produce stable tubes at the lower pressures but we can very often dispense with the pen magnifying mechanism at different ranges.

Ranges of Pressures Measurable by Bourdon Tubes

Bourdon tubes are normally available for measuring up to 80 000 lb./sq. in., although higher ranges are occasionally used.

The lower range limit varies with different manufacturers, but 0–5 lb./sq. in. would seem to be a limiting working value.

No hard and fast rules can be laid down as to which type shall be used for various pressures, but past experience has shown that the helical bourdon is suitable from a range of 0–5 lb./sq. in. up to 0–80 000 lb./sq. in. The **C** type has been used up to 10 000 lb./sq. in. The flat spiral has been used for all the lower ranges, but not much above 0–100 lb./sq. in.

Materials for Pressure Bourdon Tubes

In this country, the most common materials are phosphor bronze or similar bronze alloys, steel, and beryllium copper. The use of a particular material may be governed by other factors than pressure values, e.g. corrosion, and there is no real fixed point at which to change over from one material to another. The higher pressures, however, demand a metal with a high elastic limit, which rules out phosphor bronze with a value of 15 tons per sq. in. Above, say, 2 000 lb./sq. in., therefore, a tough steel or beryllium copper is used. The latter alloy is being extensively used for the lower pressure ranges as well.

A low carbon molybdenum austenitic stainless steel is finding favour for bourdon tubes of all ranges and is suitable for the high pressure pattern of 80 000 lb./sq. in. The reader should consult "The Bourdon Pressure Gauge" by C. F. Budenberg for an excellent appraisal of bourdon tube practice. (*Trans. Soc. Inst. Tech.* Vol. 8, No. 2, June, 1956, page 75).

Diaphragm Stacks and Bellows

Both single diaphragm, diaphragm stacks and bellows may be used for gauge pressure measurement. The basic arrangements are those of *Figs.* 11 and 12. Here p_1 becomes $(p_g + p_a)$ where p_g is the gauge pressure to be measured and p_a the atmospheric pressure. p_2 is now p_a, the atmospheric

pressure. The deflection is proportional to the pressure difference $(p_g + p_a) - p_a$, i.e. proportional to p_g, the gauge pressure.

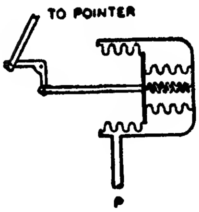


Fig. 19. Illustrating the basic design of an industrial gauge pressure instrument. Reproduced by permission of Taylor Controls Ltd.

U-Tube Manometer

For gauge pressure measurement, one limb, for example the right-hand one B of *Fig.* 1, is open to the atmosphere and p_2 becomes the atmospheric pressure p_a. p_1, again, can be considered as the sum of the gauge pressure p_g and the atmospheric pressure p_a.

Then,

$$h = \frac{p_1 - p_2}{\rho} \quad \ldots\ldots\ldots\ldots (20)$$

(from equation (3))

But now, $p_2 = p_a$

and $p_1 = p_g + p_a$

Substituting in (20)

$$h = \frac{p_g + p_a - p_a}{\rho} \quad \ldots\ldots (21)$$

$$h = \frac{p_g}{\rho} \quad \ldots\ldots\ldots\ldots (22)$$

So that h is now a measure of the gauge pressure p_g.

The same general conditions hold as for the measurement of differential pressure.

ABSOLUTE PRESSURE MEASUREMENT

Liquid Column Instruments

Consider the cistern pattern of *Fig.* 4, but with the tube now closed instead of open. As before, let the pressure acting on the column be p_2 absolute and on the cistern p_1 absolute. The basic equation still holds:

$$p_1 - p_2 = \rho h \left(1 + \frac{A_2}{A_1}\right) \quad \ldots\ldots (23)$$

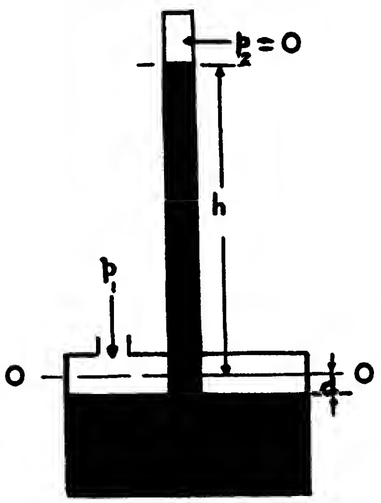


Fig. 20. An absolute pressure measuring instrument of the sealed column type

Now suppose that a perfect vacuum is produced in the space above the liquid, p_2 is now zero, as in *Fig.* 20, equation (23) becomes

$$p_1 = h\left(1 + \frac{A_2}{A_1}\right)\rho \quad \ldots\ldots (24)$$

The height h is now a measure of the absolute pressure p_1.

A special case is the barometer, where p_1 becomes p_a, the atmospheric pressure. Similar instruments are used, however, as testing devices for altimeters and other aircraft instruments.

Diaphragm Stacks or Bellows Instruments

It can be seen by referring to *Fig.* 21, if the pressure in A becomes zero, then the bellows *or* diaphragm stack unit becomes a possible absolute pressure device. In *Fig.* 21, by suitable matching, the effects on the two bellows due to atmospheric pressure and ambient temperature are equal over a reasonable range, and being in opposition are without influence on the instrument pen or pointer. Foxboro-Yoxall Ltd. who use this

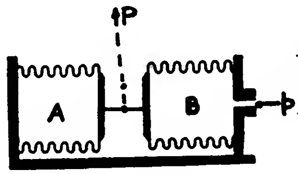


Fig. 21. The evacuated bellows type of absolute pressure measuring instrument

pattern of bellows unit evacuate bellows A down to $\frac{2}{10\,000}$ in. of mercury absolute pressure. This is equivalent to $\frac{5}{1\,000}$ mm. of mercury or 5 microns. A micron is a $\frac{1}{1\,000}$ part of a millimetre of mercury. This value is so low as to be insignificant for the ranges over which this unit is used and the vacuum bellows can be said to possess zero absolute pressure. With the external atmospheric effects balanced out, and negligible pressure in one bellows, the deflection of the pen or pointer must be proportional to the total pressure in the other, i.e. to the absolute pressure applied to this bellows.

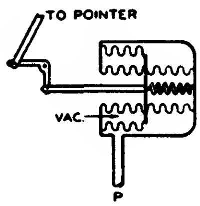


Fig. 22. An alternative design of evacuated bellows absolute pressure gauge. Reproduced by permission of Taylor Controls Ltd.

Stack and bellows absolute gauges range from a few inches of water to about 80 lb./sq. in.

STATIC HEAD CORRECTIONS

Consider first a pressure instrument with a single connection from the tapping point. The instrument can be situated above or below the tapping point. A head of fluid exists, as indicated in *Fig.* 23, which exerts a pressure equal to $h\rho$, where h is the vertical head and ρ the density of the fluid. When

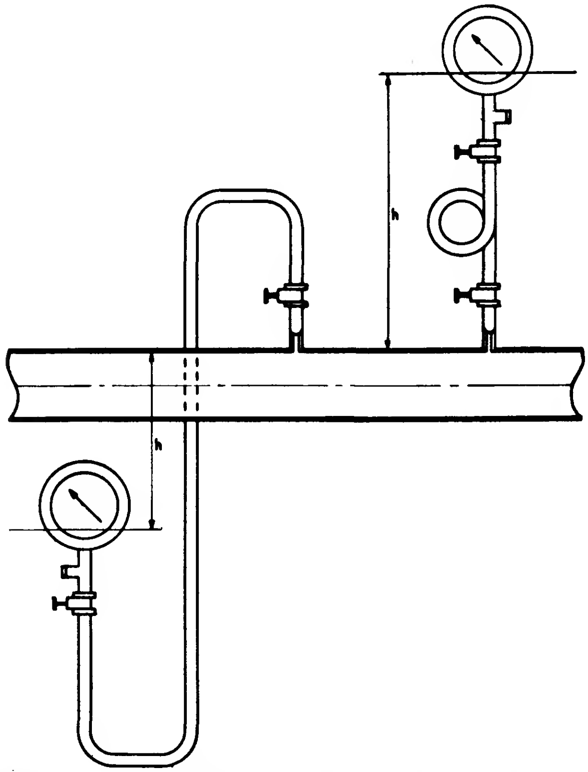


Fig. 23. *Illustrating the effect of static head on single connection pressure instruments*

the instrument is below the point of measurement, the pressure must be subtracted from the instrument reading, and when above the point of measurement, it must be added to the reading.

The trouble is only likely to be pronounced with fluids of comparatively large density, i.e. liquids. For example, a connecting pipe containing water exerts a pressure of 1 lb./sq. in. for each 2 ft. 3 in. approximately of its vertical height. Suppose an instrument is 4 ft. 6 in. above or below the point of measurement, then the error is 2 lb./sq. in. For an instrument measuring 5 000 lb./sq. in. the error is 0·04%. At 500 lb./sq. in. it is 0·4% and at 50 lb./sq. in. 4%. On low ranges, therefore, the error may be significant.

If the amount of correction is large, say 10% of the range, the maker should be advised when the gauge is ordered. Otherwise, it is usually practicable to reset the pointer to allow for the error.

In the case of differential pressures, the effect depends on the nature of the measuring instrument. For a "dry" type instrument using metallic bellows or diaphragms, with the two heads the same vertical height, the static pressure error due to the heads is neutralised. With a liquid manometer, however, an indirect correction is involved.

Let *Fig.* 24 be a simple U-tube installation. Each of the connecting tubes is filled with fluid.

If p_1 and p_2 are the two pressures,

$$p_1 + H_1\rho_1 = p_2 + H_2\rho_1 + h_m\rho_2 \quad \ldots\ldots (25)$$

Rearranging and remembering that $H_1 - H_2 = h_m$,

$$p_1 - p_2 = h_m\,(\rho_2 - \rho_1) \quad \ldots\ldots\ldots (26)$$

$(p_1 - p_2)$ may be replaced by $h_1\rho_1$ or hW where h_1 is the equivalent head of the fluid, density ρ_1, and h is the equivalent head of water, density W.

So that:

$$h_m = \frac{h_1.\rho_1}{(\rho_2 - \rho_1)} \quad \ldots\ldots\ldots (27)$$

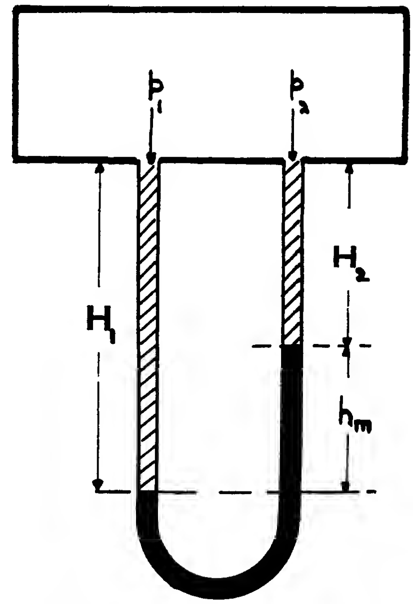


Fig. 24. *Illustrating the effect of static head on* U-*tube type instruments*

or

$$h_m = \frac{h.W}{(\rho_2 - \rho_1)} \quad \ldots\ldots\ldots\ldots (28)$$

The significance of (26), (27) or (28) is that the equivalent pressure of the instrument liquid column is given by $h_m(\rho_2 - \rho_1)$ and not by $h_m\rho_2$. In other words, the density has a reduced value equal to the difference between metered fluid and instrument liquid densities.

Similarly, equation (9) becomes:

$$d = \frac{p_1 - p_2}{(\rho_2 - \rho_1)\left(1 + \dfrac{A_1}{A_2}\right)} \quad \ldots (29)$$

giving a modified value of d, the rise of the float. If mercury is the instrument liquid, the value of ρ_1 for gases and vapours may be so small as to be neglected. For liquids, the values of ρ_1 are greater in comparison and must be taken into account.

Example:

Density ρ_2 of mercury at 60°F. = 0·488 lb./c. in.
Density ρ_1 of water at 60°F. = 0·036 lb./c. in.
Hence $(\rho_2 - \rho_1) = 0{\cdot}488 - 0{\cdot}036 = 0{\cdot}452$.
Thus the error in using 0·488 instead of 0·452 would be of the order of 7 per cent.

This subject will be found discussed with relation to particular applications in later Chapters (3 and 4).

Questions

1. *Define vacuum, gauge pressure, and absolute pressure. If I have a gauge pressure indicator giving a reading of 25lb./sq. in. and use the normal atmospheric pressure of 14·7lb./sq. in. instead of the actual value of 14·6lb./sq. in. to calculate the absolute pressure, what is the percentage error?* (*Ans.* 0·25% *or* ¼% *approx.*)

2. *In the meter shown in Fig. 3, the float chamber is 6in. in diameter, the range tube 3in., differential pressure 2lb./sq. in., float travel 0·82in., and liquid density 0·488lb./c. in. It is desired to increase the range to 4lb./sq. in., but only a 2½in. diameter range tube is available, what is the new value of float travel?* (*Ans.* 1·21*in.*)

3. *Using the equation* (14), *find the value of h for* $p_1 - p_2 =$ 0·1*lb./sq. in. if* A_1 = 4*sq. in.*, A_2 = 1/16*sq. in.*, ρ_1 = 0·030 *and* ρ_2 = 0·036*lb./c. in.* (*Ans.* 15·5*in. approx.* 16·6*in. if second factor is neglected*)

Chapter 2

HIGH VACUUM MEASUREMENT

Introduction

Some of the instruments described in Chapter 1 can be used for the measurement of vacuum conditions, particularly those in the section devoted to Absolute Pressure Measurement. How far these suit the particular requirements depends on how low a degree of vacuum one wishes to measure and the sensitivity desired.

For example, Bourdon tube instruments are marketed to measure from 760 to 20 mm. mercury to take a typical range, and sensitive diaphragm instruments can extend the range down to 1 mm mercury. The latter value should also be regarded as the limit of the U-tube manometer when used for vacuum measurement.

For the purposes of this chapter, however, the degree of vacuum with which we shall be concerned will be below 1 mm. mercury absolute pressure.

There are three arbitrary regions in which we are interested:

Medium high vacuum	1 to 10^{-3}mm. mercury
High vacuum	10^{-3} to 10^{-7}mm. mercury
Ultra high vacuum	10^{-7} mm. mercury and lower absolute pressures.

The term *Torr* is given to the unit 1 mm. of mercury.

It will be realised that the absolute pressure ranges specified above are vastly lower than any discussed in the previous chapter and require, therefore, very different techniques for measuring purposes. The following are descriptions of the instruments commonly used.

Mcleod Gauge

It is debatable whether the Mcleod gauge is really an industrial instrument in so far as plant operation is concerned, but models are manufactured for mounting on instrument panels and the instrument does serve as a calibration device for the other equipment described here. It is included for the sake of completeness.

The elements of the Mcleod gauge are shown in *Fig.* 25. It consists of the limb *A* sealed at the top, and the double limbs *B* and *C*. *A* and *B* are made the same diameter, and are virtually capillary tubes. *C* is wider in bore and helps to decrease errors due to capillarity. *C* and *B* are always connected to the system whose pressure is to be measured.

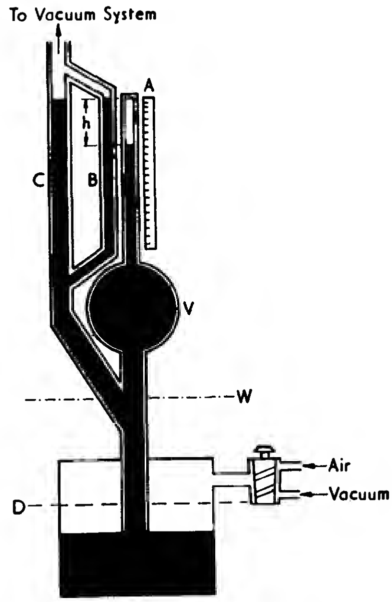


Fig. 25. *The basic features of a Mcleod gauge*

The principle of operation is simple. Mercury is initially at some level *D* below the bulb in the sealed limb. All three limbs are at this stage connected to the pressure to be measured. The mercury level is then raised so that it cuts off the gas in *V* and *A*, and compresses it. In actual use the mercury in *B* is brought to the same level as the top of the capillary in *A* which is then the zero of the scale. The gas in the limb *A* is compressed to a volume which depends on Boyle's law.

If *V* is the volume of the bulb and limb *A* above level *W*, p the pressure of the gas in the system connected to the limbs *B* and *C*, v the volume of gas in the sealed limb after compression, and p_1 the pressure of the gas in this limb after compression,

$$pV = p_1 v \text{ (p and p_1 in mm. Hg)} \quad . \ (30)$$

But $v = a\,h.$

where a = the cross-sectional area of the limb A.

h = the difference in levels between A and B.

Also, viewing the arrangement as a U-tube,

$$h = p_1 - p \quad \ldots\ldots\ldots \ (32)$$

Substituting and rearranging,

$$p = \frac{a\,h^2}{V - a\,h} \quad \ldots\ldots\ldots \ (33)$$

To measure a low pressure, the value of V must be large compared with the value of a. The ratio $\frac{V}{a}$ is termed the compression ratio. Actually, there are practical limits to its value. If a becomes too small, the mercury tends to stick in the capillary tube. If V is made large, an excessive weight of mercury may be involved, with consequent dangers to the apparatus. A typical industrial model has V equal to 400 cc and a equal to $0{\cdot}75 \times 10^{-2}$ cm^2, giving a compression ratio of 50 000 approximately.

The gauge has several disadvantages for general use. If the gas whose pressure is being determined contains any condensable vapours, Boyle's law is not obeyed and errors may occur. The measurement is, by the nature of the design, not continuous, and is not suitable for recording purposes. Precautions must be taken to prevent the mercury reaching the system under investigation, and cold traps are normally used.

The raising of the mercury to the required level is performed in several ways. For the laboratory, a reservoir of mercury connected by rubber tubing to the tube assembly or gauge head proper is often used. By raising the reservoir, the mercury is brought to the required level. In other models, a double passage cock enables air to be admitted into a mercury reservoir, in which a vacuum has been created. The air forces the mercury to the correct levels. For robust industrial applications, the Edwards "Vacustat", an all-glass design, has certain advantages. In this instrument, it is only necessary to rotate the gauge through a right angle for taking a reading. The gauge can be so constructed that it can be mounted on a panel.

The range of the Mcleod gauge is from 10 to 10^{-4} mm. mercury (10 to 10^{-4} Torr).

Pirani Gauge

Consider a heated wire element in a chamber of gas. The molecules of the gas strike the wire, and, in general, rebound with a greater energy than they had before impact. The energy is shared with other molecules by collision, until the walls of the vessel are reached. Reducing the pressure lessens the number of molecules present, and the path before collision becomes increased. It is possible to reach a stage where the molecules can travel from the filament to the walls of the vessel without suffering collision, i.e. the mean free path has become equal to or greater than the distance between walls and filament. Now, the thermal conductivity of the gas becomes pressure dependent, and the transfer of energy from wire to gas is a function of the gas pressure.

Suppose a substantially constant supply of heating energy to the wire is maintained. With varying pressure and related energy transfer, the temperature of the wire changes, and affords a method of pressure measurement. In addition, as a result of a temperature change, the resistance of the wire alters, providing a second method of pressure measurement.

To measure the temperature, a thermocouple is welded to the wire filament (*Fig.* 26) and a millivoltmeter measures the thermocouple e.m.f. (See chapter 9 for descriptions of thermocouples.)

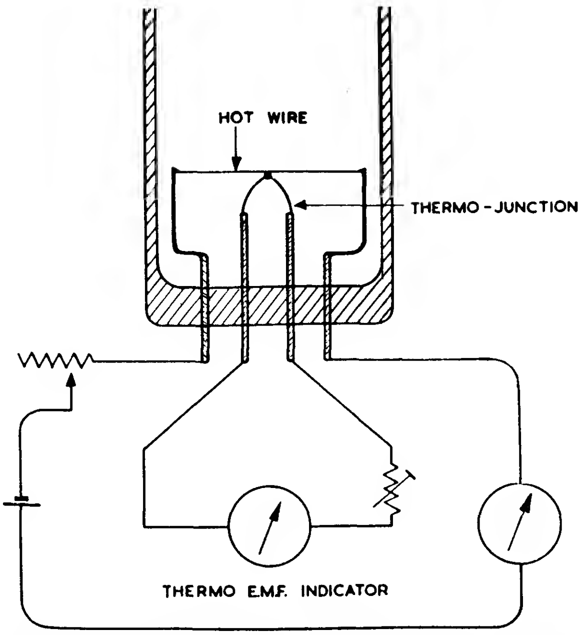


Fig. 26. *Thermocouple or thermo-junction gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

The meter may be calibrated directly in pressure values.

A resistance bridge is used when the resistance of the filament is to be measured. The normal method is to balance the bridge at some datum pressure and use the out-of-balance currents at all other pressures as measurements of the relative pressures. A simple bridge is indicated in ***Fig.* 27**.

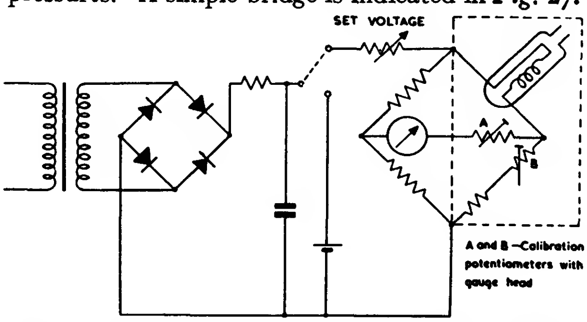


Fig. 27. *Simple circuit for Pirani gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

The Pirani gauge used in this fashion has the advantage that by making several improvements to the simple design the pressure range may be extended in both directions. A more sensitive meter may be used to measure the bridge out-of-balance, but to obtain the additional stability the bridge voltage has to be held constant. Low voltage transistor stabilized circuits using a Zener reference diode have been used with advantage. Considerable improvements in stability and, therefore, pressure range compared with the simple single element gauge are obtained by using a second gauge element either evacuated or sealed off at some fixed pressure as a compensating reference in the other bridge arm. Double range instruments with four element gauge heads in which

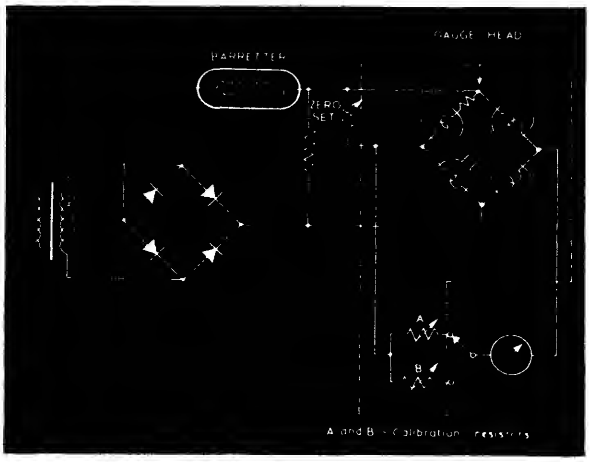


Fig. 28. *Simple circuit for double range Pirani gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

the bridge is balanced at low pressures (10^{-5} Torr) and at high pressures (atmospheric) respectively (*Fig.* 28) are possible.

The gauge is upset by contamination due to the gases, and "flashing" by passing a current through the filament is provided on most instruments to clear the trouble.

Alternatively, the filaments may be "blackened" so that under dirty conditions the calibration remains reasonably stable

The range of the Pirani gauge is from 10^{-4} mm. to 5 mm. of mercury approximately.

Thermionic or Hot Cathode Ionization Gauge

In its simplest form, the gauge consists of a glass envelope with outlet arrangements for connecting it to the system whose pressure is to be measured. Inside the envelope is a thermionic triode assembly comprising the normal filament (cathode), grid, and anode. We will suppose that the anode is at a negative potential and that the grid is at a positive potential with respect to the filament. The electrons emitted from the filament collide with the gas molecules present in the tube and ionize them. The positive ions travel to the negatively charged anode. An ionization current now travels round the anode circuit, and may be measured by a microammeter. This current is a measure of the absolute pressure of the gas in the gauge.

The electrons ultimately reach the positively charged grid and form an electron current round the grid circuit. The following approximate relation holds:

$$p = \frac{k\,c\,I_1}{I_2} \qquad \ldots\ldots\ldots\ldots (34)$$

where p = absolute pressure.
c = a probability factor.
k = a constant depending on the potentials used in the gauge, and the geometry of the electrodes.
I_1 = ionization current.
I_2 = electronic current.

Provided, therefore, that the potentials and the electron current remain unaltered, the pressure is proportional to the ionization current.

There is an alternative method of using the gauge by making the grid negative and the anode positive. The first method is claimed to be superior from a sensitivity point of view because the electrons in oscillating round the positive grid tend to have an increased path.

The importance of maintaining the electron current at a stable value during measurement has led to the introduction of stabilizing circuits.

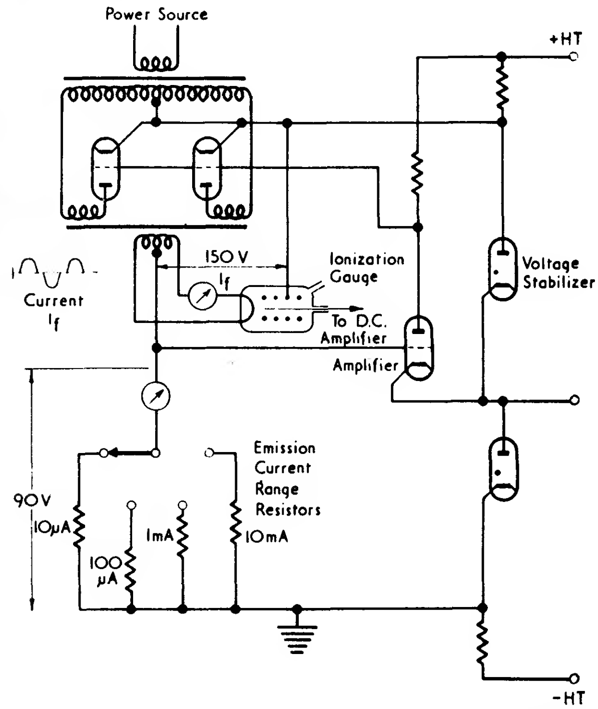


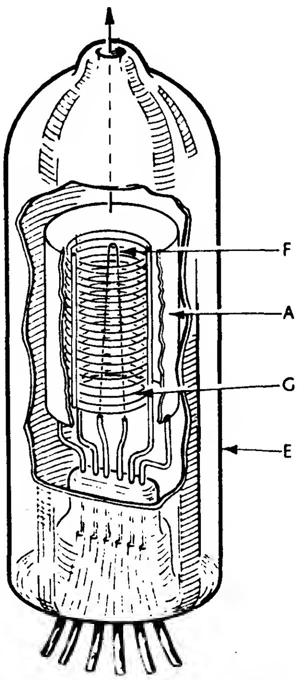
Fig. 29. *Supply circuit for hot cathode ionization gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

In *Fig.* 29 the emission current passes through the range resistors to obtain the negative bias voltage for the collector which can then be operated at earth potential. The voltage developed is compared with a reference voltage and the difference in levels serves to regulate the power valves supplying the filament voltage.

In the normal triode arrangement of *Fig.* 30*a*, the filament F is either pure tungsten, thoriated tungsten or a platinum alloy coated with barium and strontium oxides. The grid G typically takes the form of a cylindrical molybdenum wire helix surrounding the filament. The ion collector (anode) A is typically an open ended nickel cylinder surrounding the grid although in one design it comprises flat nickel plates parallel to the flattened sides of the helix.

One disadvantage of the hot ionization gauge is the tendency to produce X-rays when ionizing electrons strike the grid. The X-rays cause a secondary emission at the ion collector resulting in a current in the collector circuit in the same direction as that due to the ions. This has tended to restrict the lower limit of the gauge to 10^{-8} Torr.

In a desire to extend the range of the hot cathode ionization gauge, Bayard and Alpert made the ion collector in the form of a fine tungsten wire of about 0·1 mm. diameter and inverted the normal



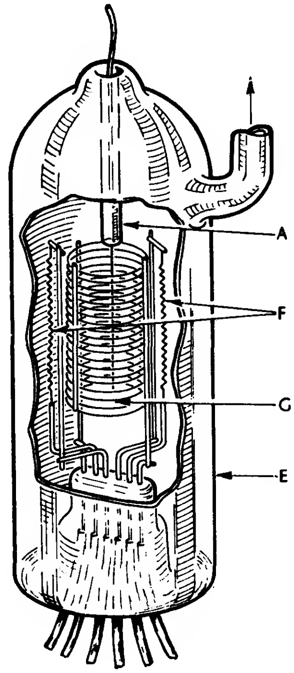


Fig. 30*a*. *Diagram showing essential features of an ordinary ionization gauge.*

Fig. 30*b*. *Diagram showing essential features of Bayard-Alpert ionization gauge.*

electrode arrangement. The wire ion collector is situated inside the helical grid *G*, and the filament *F* is outside the grid in the manner of Fig. 30b. The vastly smaller surface of the ion collector exposed to X-rays enables the lower limit to be extended to 10^{-11} Torr.

Cold Cathode Ionization Gauge

The simplest form of cold cathode ionization gauge is the discharge tube consisting of a glass tube, with two sealed-in electrodes joined to the vacuum system. When a high voltage is applied to the electrodes a glow discharge occurs at a low pressure, and as the pressure is reduced the colour and shape of the discharge can give a rough indication of the state of the vacuum and the gases that are present. At a pressure of about 10^{-2} mm. mercury the discharge blacks out. The

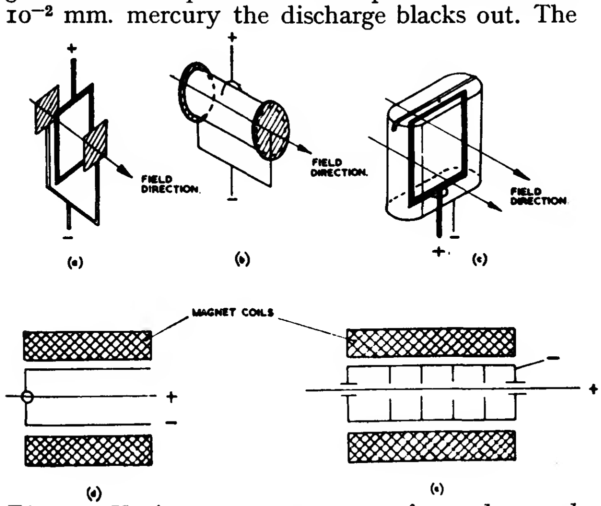


Fig. 31. *Various arrangements of anodes and cathodes for the Penning gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

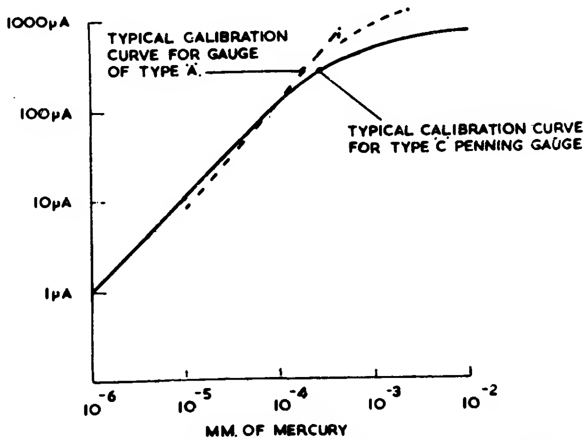


Fig. 32. *Calibration curves of two patterns of Penning gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

dimensions of the tube and the applied voltage determine the actual pressure at which blackout occurs and this condition is sometimes used to give an approximate indication of the degree of vacuum.

The Penning gauge is a more sophisticated development. In the Penning cold cathode ionization gauge two electrodes, an anode and a cathode are contained in a glass envelope. In the simplest design the cathode consists of two parallel plates and between them is a parallel loop of wire forming the anode (*Fig.* 31). The gauge is placed between the poles of an electromagnet in use so that the magnetic field is at right angles to the plates. The arrangement is that of a discharge tube, and emission of electrons from the cathode plates, takes place. The electrons move towards the loop anode in helical paths. The path length is increased enormously and the probability of collision of electrons with molecules to form ions is far larger than in the triode pattern of ionization gauge. The result is that a relatively big discharge current is obtained for a moderately high voltage.

The calibration curve of pressure plotted against meter reading is a straight line at the lower pressures. But above 5—10^{-4} mm. mercury nonlinearity begins. The slope of the line depends on the gas being measured (*Fig.* 32).

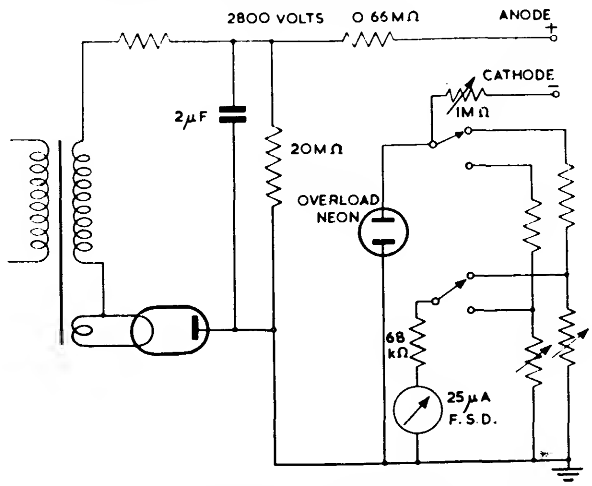


Fig. 33. *Supply circuit for Penning gauge. Reproduced with acknowledgments to Edwards High Vacuum Ltd.*

Cold Cathode Magnetron Gauge

A development of the cold cathode design is the magnetron gauge. A simplified diagram of an industrial mode is shown in *Fig.* 34 (Leybold-Elliott). The cathode K consists of two circular metal discs, the centres of which are joined together by a thin cylinder. This cylinder acts as an electron source. The anode A is a metal cylinder placed coaxially to the thin cylinder joining the discs. The anode-cathode assembly is enclosed in a glass envelope and placed between the poles of a magnet N-S so that the magnetic field acts along the direction of the cylinder axis. The strength of the field is normally about 1000 gauss. A potential difference of 5-6kV is applied between anode and cathode.

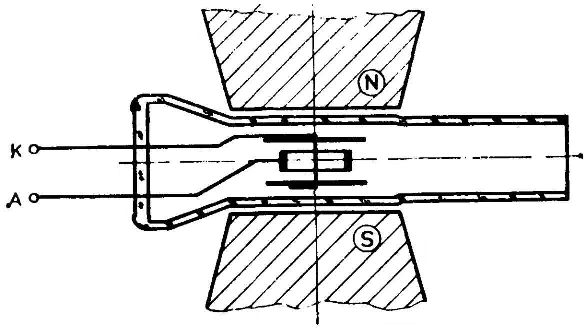


Fig. 34. *Diagram of cold cathode magnetron ionization gauge. Reproduced by permission of Leybold-Elliott Ltd.*

Electrons are emitted from the thin cylindrical cathode and, due to the disposition of the magnetic and electric fields, follow a cycloidal path. With such a path, they must collide many times with gas molecules, producing ions at each collision. The ions travel straight to the cathode. A current is set up and follows a linear relationship with the pressure in the range 10^{-4} to 5×10^{-10} Torr. Below 5×10^{-10} Torr the relationship is non-linear:

$$I = Kp^{1 \cdot 7}$$

where I = the ionization current
K = a constant
p = the absolute pressure

The overall range is 10^{-4} to 10^{-11} Torr.

The current in the linear portion of the range is about 45 times that of the Bayard-Alpert ionization pattern gauge.

In the particular pattern of gauge quoted, if it is used at very low pressures it must be baked out at 400°C or more for about 18 hours.

In some designs auxiliary cathodes consisting of annular ring pattern electrodes are placed between the disc cathodes and the anode.

Hot Cathode Magnetron Gauge

This is a variation in design which has for its object increasing of the path length of the electrons. It increases the probability of an electron striking a molecule of gas and producing ions. In the modification due to Lafferty, the magnetic field H is applied longitudinally instead of transversely. The tungsten filament F is surrounded by a

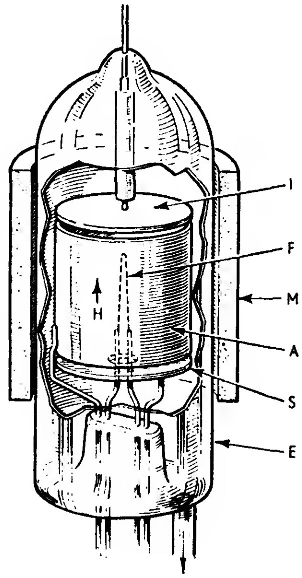


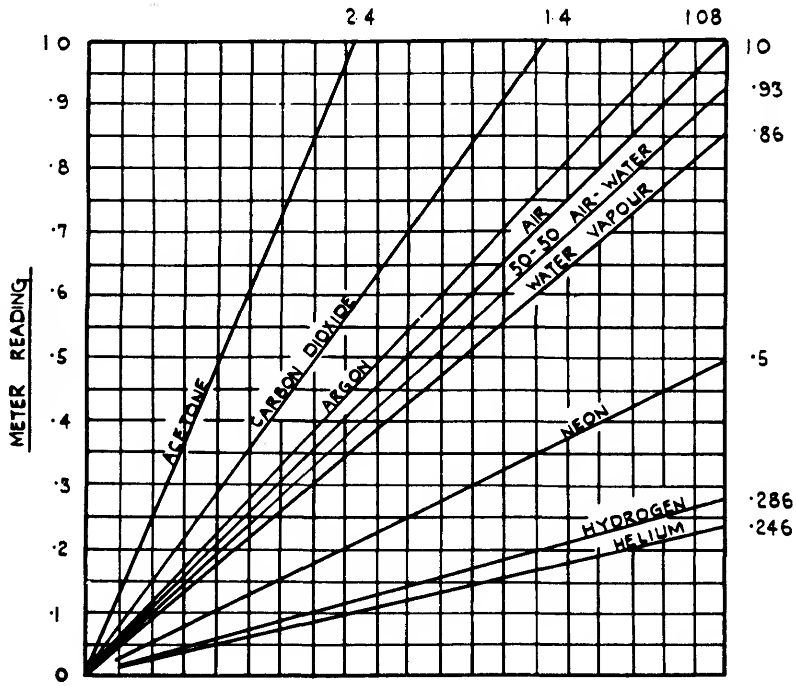
Fig. 35. *Diagram of hot cathode magnetron ionization gauge. Reproduced with acknowledgments to Journal of Applied Physics.*

cylindrical molybdenum anode A. Each end of the anode is nearly but not quite closed by two circular plates. One, I, acts as the ion collector and the other, S, as a shield.

The assembly is enclosed in a glass envelope, and a cylindrical magnet M slides over the glass envelope. The field strength is about 250 oersteds. Normal operating voltages are -45v on the ion collector and -10V on the shield. The gauge should be operated at low emission currents in the range 10^{-6}A to 10^{-9}A. The ion current has a linear relationship with absolute pressure down to 4×10^{-14} Torr. Above 5×10^{-8} Torr the relationship is non-linear and this represents an upper limit.

Alpha Particle Ionization Gauge

An alternative form of ionization gauge is the alpha particle pattern. The ionization action is caused by a weak radioactive source emitting alpha particles. The alpha particles colliding with the gas molecules cause ionization. Included in the gauge is a collector plate and grid assembly, and the positive ions are attracted towards the grid. An ionization current is established which is a measure of the pressure of the gas in the gauge, indeed, a linear relation exists between the two. The operation of the instrument depends on the probability of ionization which varies from one gas to another, hence the calibration curve of current versus pressure will be unique for each gas. The ionic current is of the order of 2×10^{-10} A/mm per mm. mercury absolute pressure. The currents will, therefore, be extremely small at the lower pressures, resulting in the need for complex d.c. amplification. This places a limit on the lower end of the range which typically is 10 mm. to 10^{-3} mm. mercury. *Fig.* 36 indicates the calibration curves for different gases of the Alpharay gauge.



RANGE 1 (X10)	0	100	200	300	400	500	600	700	800	900	1000	MILLIMETRES Hg
" 2 (X10)	0	10	20	30	40	50	60	70	80	90	100	MILLIMETRES Hg
" 3 (X10)	0	1	2	3	4	5	6	7	8	9	10	MILLIMETRES Hg
" 4 (X10)	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	MILLIMETRES Hg
" 5 (X10)	0	10	20	30	40	50	60	70	80	90	100	MICRONS Hg (·001 M.M)
" 6 (X10)	0	1	2	3	4	5	6	7	8	9	10	MICRONS Hg

Fig. 36. Calibration curves of the Alpharay ionization gauge. Reproduced by permission of Vacuum Industrial Applications Ltd.

Questions

1. In a Mcleod gauge the area of the measuring capillary tube is 1×10^{-2}sq.cm. The volume of the bulb, etc., is 300cc. By taking $p = ah^2/V$ instead of equation (33) find the value of h for $p = 2 \times 10^{-2}$mm. of mercury. Then calculate the error in using the above equation instead of equation (33). (Answer 7·74cm. Negligible.)
2. Draw the bridge circuit for the Pirani gauge when a second gauge is used for compensating purposes. How is contamination avoided or cured?
3. In the hot cathode pattern of ionization gauge the electron current increases from 10 to 11μA. If the absolute pressure being measured at 10μA is 1×10^{-6}mm. of mercury what is the apparent pressure at 11μA? All other factors remain the same. (Answer $0{\cdot}91 \times 10^{-6}$mm. mercury.)
4. What principal advantage does the Alpert-Bayard design have over the normal pattern of ionization gauge and what features of the design contribute to this advantage?
5. In the non-linear relationship for the cold cathode magnetron gauge if the current is doubled what is the ratio between the original and new pressure? (Answer 1·5 approx.)

Books etc., suggested for further reading

DUSHMAN, S. Chapter 5—Scientific Foundations of Vacuum Techniques. John Wiley, 1962.

PIRANI, M. and YARWOOD, J. Chapter 3—Principles of Vacuum Engineering. Chapman and Hall, 1961.

TURNBULL, A.; BARTON, R. S. and RIVIERE, J. C. Chapter 4—Vacuum Technique. George Newnes, 1962.

STECKELMACHER, W. The Measurement of Pressure in High Vacuum Systems. *Instrument Practice* Vol. 14, No. 5, May, 1960, p. 519.
Vacnique, Vol. 1, No. 3, July, 1961, p. 2.

Chapter 3

LIQUID AND SOLIDS LEVEL MEASUREMENT

INSTRUMENTS for the measurement of liquid level in a tank can be broadly classified under the following headings:

1. Mechanical and General.
2. Pneumatic.
3. Electrical.
4. Electronic.
5. Nucleonic.

The type of instrument to be employed is dependent on the liquid level range, the working pressures and the nature of the liquid.

In this chapter we are concerned with continuous measurement and devices solely for alarm or control purposes are not included.

I. MECHANICAL AND GENERAL INSTRUMENTS

Sight Glass

A simple and fairly common method of observing liquid level in a tank is to incorporate an external direct vision or glass gauge, a version of which is shown in *Fig.* 37. It comprises a vertical glass tube situated between two shut-off valves which enable the device to be isolated from the tank.

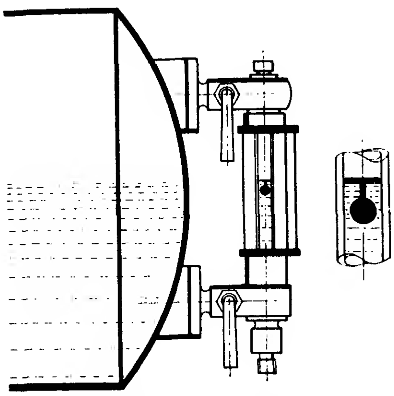


Fig. 37. *Sight glass liquid level gauge*

Liquid from the tank appears in the glass tube and indicates the level. The inclusion of a float marker improves the observation, particularly if the meniscus collects scum or scale. Toughened glass enables the gauge to be used up to 350 lb/in². If conditions due to the nature of the liquid or the working pressure appear hazardous then an alternative such as the Rotameter "Ekstrom" level gauge can be used.

"Ekstrom" Gauge

The "Ekstrom" gauge (*Fig.* 38) consists of a vertical metal branch connected to the tank, containing a float *M* incorporating a magnet. Adjacent to the branch is an entirely separate gauge glass *T* filled with a liquid such as butanol.

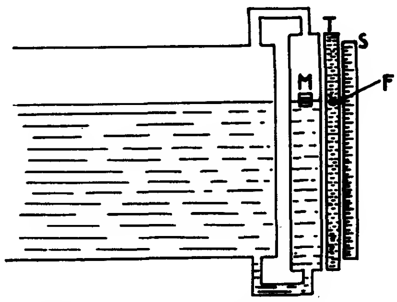


Fig. 38. *Principle of the "Ekstrom" gauge*

A hollow iron ball *F* is immersed in the butanol and follows the movement of the float magnet in the metal branch. A scale *S* may be added to the sight glass for measuring means. The gauge may be supplied to work at pressures up to 500 lb/in² and level ranges of the order of 18ft. Observe that breakage of the sight glass does not involve loss of process liquid as in the normal sight glass gauge described previously.

Float Gauges

Another simple means of measuring liquid level is the float operated instrument. Here, a float of some material not corroded by the liquid rests on the surface and a chain, tape or cable connected to it passes over a wheel or pulley to a counterweight. No slipping of the tape or cable on the drum must be permitted. As the liquid level varies, the float rises or falls and rotates the

pulley. Connection may then be made from the pulley to an indicating instrument either by gear train or linkage. This type is suitable for open pattern tanks only (*Fig.* 39).

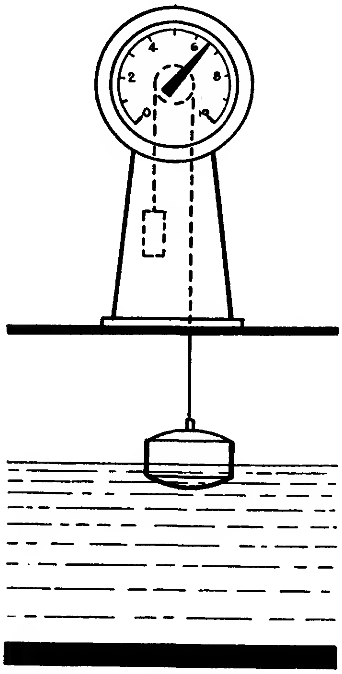


Fig. 39. *Float type gauge open tank*

A design for a closed tank is indicated in *Fig.* 40 Here a float is connected by a solid arm to a pivot. As the float rises and falls with level, the arm rotates and by suitable coupling operates the pointer of an indicating instrument. With a magnetic coupling no direct connection between inside and outside of tank is required. The general design is, of course, suitable for open tanks as well as closed tanks.

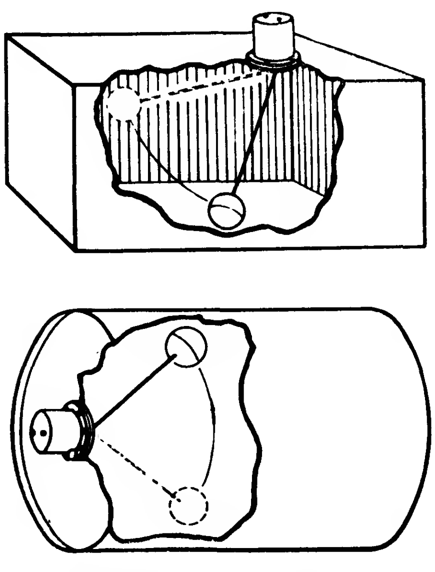


Fig. 40. *Float type gauge closed tank. Reproduced with acknowledgments to Bayham Ltd.*

Pressure Operated Instruments

In any tank of liquid, unless the tank is empty, there must always be a head of liquid relative to the bottom of the tank or some selected datum level near the bottom. This head of liquid exerts a pressure of $h\rho$, where h is the height of the liquid above the datum level and ρ the density of the liquid. As the tank empties or fills, there must be a varying head exerting a proportional varying pressure. Measurement of this pressure, then, can give information of the liquid level or, alternatively the volume content of the liquid in the tank. One precaution must be observed. The measuring instrument can be calibrated for one density only. Any departure from this value will cause incorrect readings.

Liquid Manometer Instruments

A very simple example of the use of a **U**-tube to measure the level of liquid in a tank is shown in *Fig.* 41. Note that this is a very ideal case where

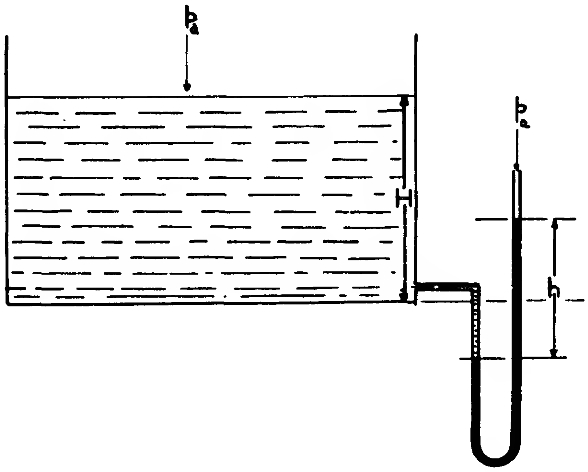


Fig. 41. *U-tube manometer ideal case*

the tank zero, used as the datum level, is on a line with the zero level of the **U**-tube and the tank is open to atmosphere. Using similar reasoning as in Chapter 1,

$$H = \frac{\rho_2}{\rho_1}. \quad - \frac{h}{2} \qquad \text{. . . . . (35)}$$

where H = height of liquid above the zero of the tank

ρ_1 = density of tank liquid

ρ_2 = density of manometer liquid

h = distance between levels of manometer liquid.

A rather more practical application, but still very simple, is indicated in *Fig.* 42. Here, the **U**-tube is below the tank, which is now closed, necessitating the connection to the limb B of the **U**-tube and the use of the manometer as a true differential pressure instrument. Observe two complications. There is now a head H_1 of liquid between the tank datum level and the level of liquid in the limb A of the manometer, for which allowance must be made, and there is another possible head H_2 acting on limb B due to the fluid above the liquid in the tank filling the downpipe

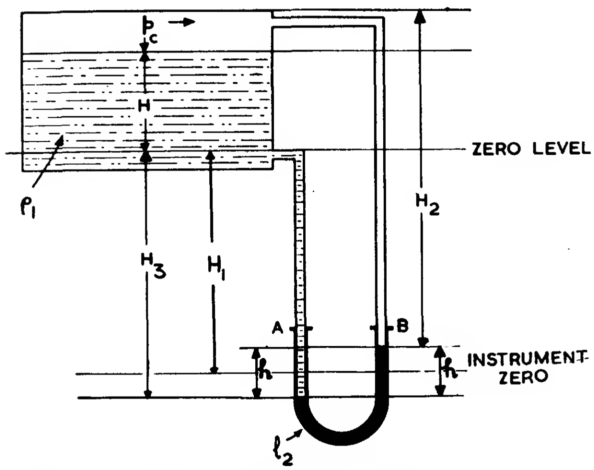


Fig. 42. *U-tube manometer below tank*

to limb B of the manometer. If the density of this fluid is very small compared with that of the liquid, the effect of this head may be neglected. Then,

$$H = \frac{\rho_2}{\rho_1}.h - \frac{h}{2} - H_1 \quad \ldots\ldots (36)$$

where H = height of liquid in tank above datum level

H_1 = height of datum level above manometer zero

ρ_1 = density of tank liquid

ρ_2 = density of manometer liquid

h = distance between levels of manometer liquid

With the ordinary glass U-tube manometer no very great operating pressures can be applied, and the all-metal version of *Fig.* 43 must be employed when these pressures are high. Equation then becomes

$$H = d\left[\frac{\rho_2}{\rho_1}\left(1 + \frac{A_1}{A_2}\right) - \frac{A_1}{A_2}\right] - H_1 \quad (37)$$

where ρ_2, ρ_1 and H are as before

A_1 = area of wide tube

A_2 = area of narrow tube

d = movement of float on liquid in wide tube.

One other application we must examine is when the level to be measured is virtually the interface between two fluids, the density of the upper one not being negligible.

With ρ_3 representing the density of the upper fluid in *Fig.*44 ,

$$H = \left[\left(\frac{\rho_2 - \rho_1}{\rho_1 - \rho_3}\right)\frac{A_1}{A_2} + \left(\frac{\rho_2 - \rho_3}{\rho_1 - \rho_3}\right)\right] d - H_1 . \quad (38)$$

A complication arises when a vapour condenses in the downpipe to the narrow limb, particularly when the pipe is partly full of condensed liquid and partly full of uncondensed vapour. Such conditions can lead to errors and it is useful to install a condenser near the upper outlet so that a column of liquid is obtained whose density can be estimated with reasonable accuracy.

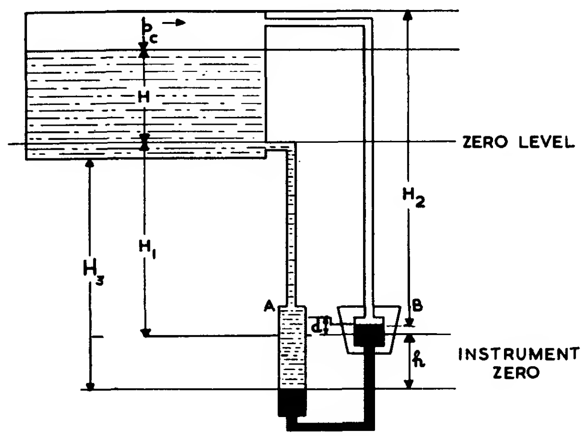


Fig. 43. *High pressure U-tube float pattern manometer below tank*

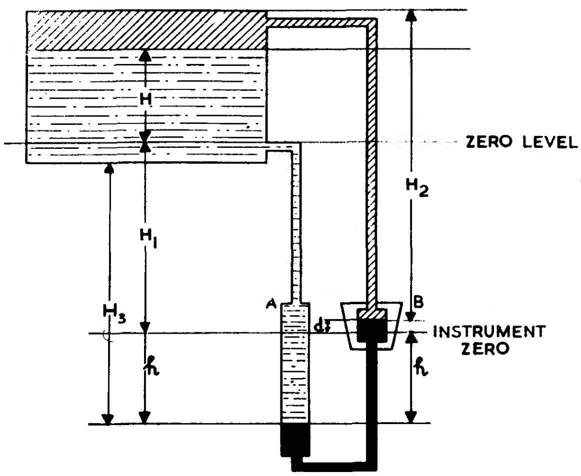


Fig. 44. *High pressure U-tube float pattern manometer below tank containing two fluids*

An interesting development of the U-tube pattern which renders it suitable for operation at high pressures and for distance is shown in *Fig.* 45. This is the Bailey-Jerguson instrument, and is especially applicable to the measurement of levels in a boiler drum. As will be seen, the gauge is basically a U-tube with a cistern or reservoir and a float chamber. The manometer liquid is mercury. The head due to the water in the drum is applied to the float chamber, where the mercury rises or falls in accordance with the level. The float is of stainless steel and rides on the surface of the mercury. Connected to the float is an armature of magnetic material enclosed in a stainless steel tube, and exterior to the tube is a pivoted magnetic yoke having fingers which form a magnetic coupling with the armature. To the yoke is attached the pointer which moves over a vertical scale. Thus, as the armature moves in proportion to the change in level, it is followed by the fingers of the yoke, which position the pointer on the scale.

In boiler installations it is necessary to ensure a constant head, and this is provided by the datum column comprising a condenser located at the top of the column. The gauge is suitable for working up to 1,500 lb/in². It may be located at a distance from the level to be measured, and can be

equipped with a high or low level alarm operated by electrical switches from the pointer mechanism.

An accuracy of ½ per cent of the scale reading is claimed with this instrument.

Where corrosive liquids are encountered it is necessary to install liquid seals at each outlet to prevent the possibility of the manometer liquid being attacked.

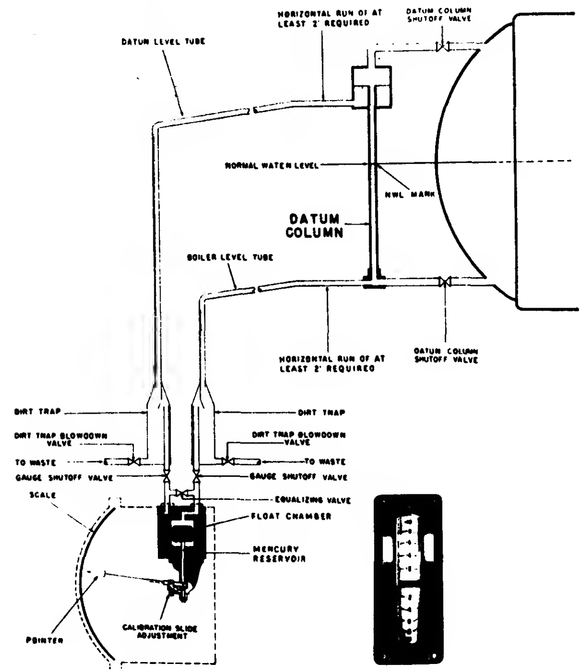


Fig. 45. *Bailey-Jerguson high pressure gauge*

Yet another variation in U-tube design is shown in *Fig.* 46.

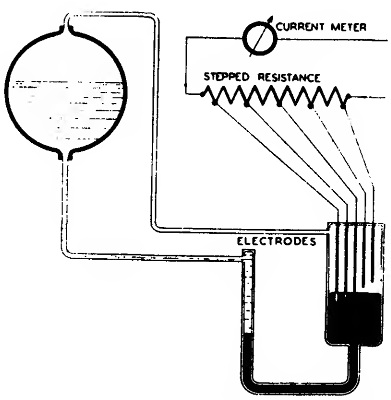


Fig. 46. *Electrode pattern high pressure manometer (Electroflo Meters)*

It consists of a U-tube constructed of steel to withstand high static pressures. In one limb is inserted a unit consisting of a large number (about 100) of vertical conducting rods arranged in spiral formation. These are connected to resistance elements in the head of the unit, in such a manner that they virtually form tappings from a continuous resistance. The rod lengths are graduated so that mercury rising or falling in the chamber, with changing level, makes or breaks contact with one rod after the other. If one circuit connection is made via the mercury, and another through the resistance assembly in the head, or as shown, the action of the mercury can be made to vary the amount of resistance in circuit.

The corresponding variation in current can be shown on an indicating or recording ammeter calibrated in terms of liquid level. This U-tube electrode unit is similar to that employed in flow measuring systems and will be discussed again when flow measurement is described.

Diaphragm, Diaphragm Stack or Bellows Instruments

Figs. 47 and 48 show how a single diaphragm unit may be used for liquid level measurement.

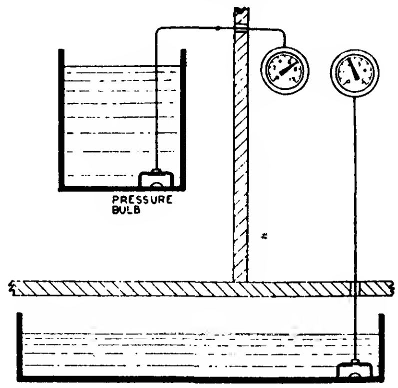


Fig. 47. *Pressure bulb inside tank*

The diaphragm is normally slack and is made of rubber, Neoprene or similar substances, and is secured in a housing. The chamber formed by the diaphragm and the housing is connected by capillary tubing to a pressure instrument of the bourdon, stack or bellows type. It can be seen that the liquid above the diaphragm must exert a pressure on it equal to $h\ \rho$ where h is the height and ρ the density. This pressure adjusts the position of the diaphragm and alters the pressure within the closed volume of air formed by the housing, capillary and bourdon, etc. The internal pressure is then observed on the pressure instrument which may be calibrated in terms of height of liquid above the diaphragm, or in terms of capacity, i.e. gallons of liquid in the container. This pattern of instrument is known as a pressure bulb.

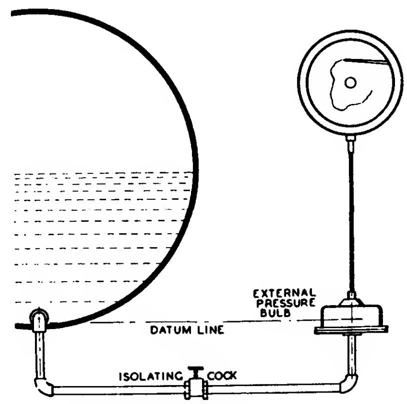


Fig. 48. *Pressure bulb outside tank*

Equally well, a bellows, diaphragm or stack unit may be inserted in the container base at a convenient spot, with the interior connected by

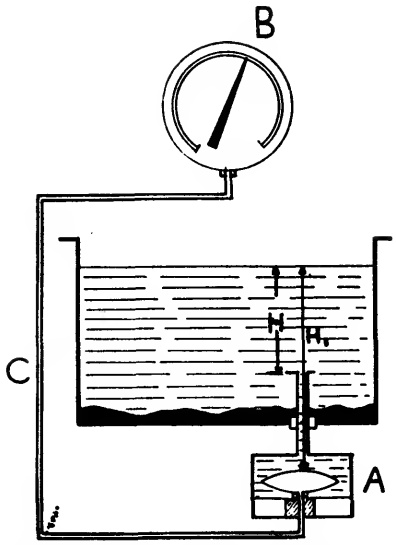


Fig. 49. *Diaphragm stack or bellows gauge. Observe the arrangement for obviating trouble due to sediment at the bottom of the tank*

capillary to the pressure instrument. The liquid above the unit will exert a pressure on the exterior of the stack, causing it to compress and thus adjust the pressure within the closed volume formed by the stack or bellows, capillary and bourdon in the pressure measuring instrument (*Fig.* 49).

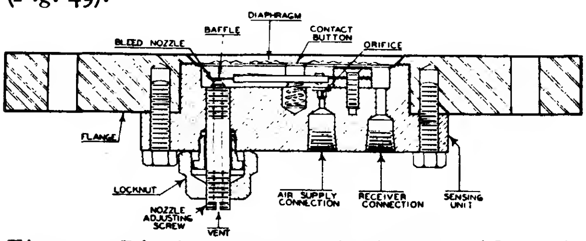


Fig. 50. *Diaphragm pattern level gauge with nozzle and flapper unit (Taylor Controls Ltd.)*

Another pattern of diaphragm instrument is that shown in *Fig.* 50. This is a force balance unit and is thus suitable for distance transmission. The pressure of the liquid acting on the diaphragm deflects it and moves the baffle nearer to or farther from the mouth of the nozzle. This adjusts the leak away to atmosphere and alters the back pressure in the chamber on the lower side of the diaphragm. Operation is continued until the back pressure equals the pressure due to the head of liquid on the opposite side of the diaphragm. The pressure in the chamber is then a measure of the liquid level and is transmitted to a suitable pneumatically operated receiving indicator or recorder, at the distance end, by the connection shown to the right of the figure.

This instrument has a minimum range of equivalent pressure 0-3lb/in^2., and maximum 0-45lb/in^2.

STANDPIPE SYSTEM OR FLUID PURGING SYSTEM

This is a simple means of obtaining a level or volume measurement with the indicating instrument above or below the tank.

The first arrangement shown in *Fig.* 51 involves a standpipe with a bell, a hand pump, an indicating instrument and a two-way cock or relief valve. The bell has a small aperture near its lower edge, and this aperture is placed so that it coincides with the zero level of the tank. To obtain a reading, the liquid is pumped clear of the standpipe and bell by a few strokes of the hand pump. If a two-way cock is fitted, it must be turned so that the indicator is isolated during the pumping operation.

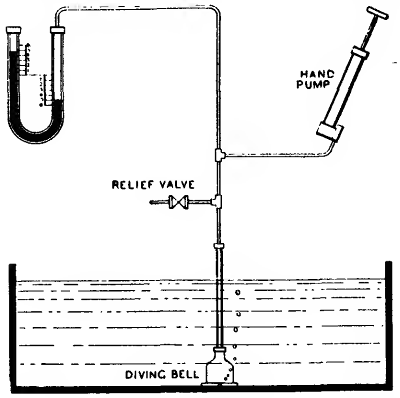


Fig. 51. *Standpipe installation with hand pump*

If a relief valve is fitted it should be adjusted to operate at a pressure a little above the maximum head likely to be experienced. When the pipe and bell are clear of liquid, the air pressure at the bell aperture balances that due to the head of liquid above it. This pressure is imposed on the indicator, whose reading then indicates the liquid level in the tank (or liquid volume). The capacity of the bell must be sufficiently large relative to that of the instrument, so that compression of the air in the system, caused by liquid rising in the bell, does not introduce errors which cannot be tolerated. From the nature of the equipment only periodic readings are possible, and for continuous indication or recording the arrangement shown in *Fig.* 52 must be used. The hand pump is replaced by a compressor and a standpipe without bell is installed. A small bubble indicator is included to

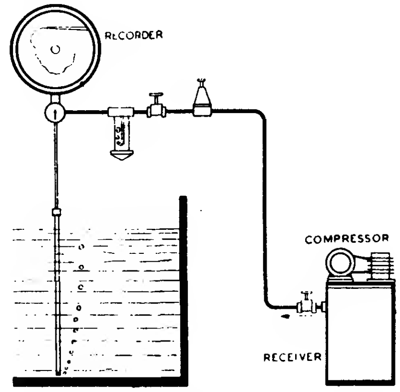


Fig. 52. *Standpipe installation with compressor*

give a visual indication of the air flow together with a pressure reducing valve. The rate of the flow must be as small as possible to obviate any significant pressure drop between the cock shown in *Fig.* 52 and the bottom of the standpipe, otherwise the pressure operating the recorder or indicator will be the one at the cock and not the one at the

bottom of the standpipe. A bubble rate of the order of 60 per minute is suggested.

Both of the foregoing examples deal with open tanks. With closed tanks it is customary to install a different system with two branches. One branch comprises a pressure reducing valve, a bubble indicator and a standpipe, as before. The other branch comprises a reducing valve, a bubble indicator but no standpipe and is taken to an inlet in the space above the liquid in the tank. The two branches are connected to a differential pressure indicator or recorder.

Air is commonly used for the purging fluid in standpipe installations, but where this would involve fire or explosion hazards, carbon dioxide, nitrogen or other inert gases may be used.

A liquid purge is sometimes necessary when dealing with asphalts, solids in suspension, corrosive liquids, etc. The small flow of liquid prevents the tank contents from entering the pipes of the level measuring equipment. The purging liquid depends on the nature of the tank liquid.

Buoyancy Type and Torque Tube

In the float-operated instruments developed from those of *Fig.* 40, the float has followed the change of level of the liquid, and the range has been limited to the length of the float arm, itself limited by the diameter of the vessel or the size of external float chambers. In a different design the change in buoyancy of the float is employed to detect change in level. The length of the float must be a little in excess of the change in level to be measured, but it can be of small diameter. For example, a float 50ft long can be used over the corresponding range in a column only 3ft wide, where the surface float type would be limited to something less than 6ft, i.e. the accurate movement of the maximum length of float arm that could be accommodated. The force generated by the buoyancy is partially balanced against a spring and the motion reduced considerably, so that it can be used to operate a nozzle and flapper device having air supplied through a restriction and at a constant pressure. Changes in level vary the flapper position and, hence, the air pressure in the system which is connected to instruments designed to measure pressure, but calibrated in terms of level.

The float can be located in the vessel itself, but preferably should be mounted in a separate chamber mounted externally and connected with the vessel at both the upper and lower ends. The reduced movement of the float is brought through the wall or casing by a torque tube which overcomes the sealing difficulties and prevents leakage of liquids or gases which might be injurious to health or add to the risk of fire or explosion. The construction of this unit is shown in *Fig.* 53. This figure is actually a control system but the principles involved are the same A typical range would be 50ft head of fluid with very high static pressures of the order of 5000lb/in².

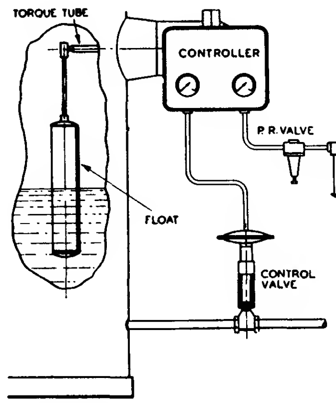


Fig. 53. *Buoyancy unit with torque tube transmission*

CAPACITOR TYPE INSTRUMENTS

This type of level instrument is suitable for liquids which can act as dielectrics. Consider a rod-like electrode, E in *Fig.* 54, inserted in a tank *T*, but insulated from the latter. The tank is earthed. The electrode and tank form a capacitor system whose dielectrics are the liquid itself and the air or gas in the space above the liquid. These form capacitances C_0 and C_1 as the principal components. Since the dielectric constants of liquids are normally much greater than those of gases, it can be seen that variations of the liquid dielectric by its rise and fall in the tank can alter the effective capacity of the system and so afford a measurement of the level of the liquid.

The measuring circuit is usually a bridge, of which the capacitor system due to the electrode, tank and its contents forms one arm. The opposite arm is also a capacitor, adjustable so that the bridge may be balanced at one datum value, e.g. zero level. The out-of-balance current at all other levels may then be used for measurement purposes and a suitable meter can be calibrated directly in level or contents units. The other two arms may be fixed resistors or fixed capacitors.

It is customary to use relatively high frequency supplies to the bridge, above 100 kc/s.

The range of the equipment may be a few inches or several hundred feet. The material of the electrode varies according to the application. For non-corrosive liquids most common metals are satisfactory. For corrosive liquids, stainless

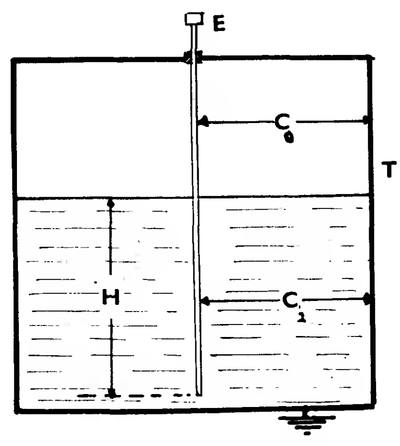


Fig. 54. *The principal capacitances in the capacitor probe system*

steel may be suitable, or it may be necessary to proceed to an electrode covered with p.v.c. or Fluon or similar substances. The former can be used up to 60°C and the latter to 320°C. In the case of very deep tanks the electrode has taken the form of a nylon rope weighted at the bottom, although, strictly speaking, this method is more popular in installations such as silos or grain elevators with solid contents.

NUCLEONIC TYPE INSTRUMENTS

The nucleonic type level instrument involves a radioactive source, a radiation detector and electronic measuring circuits including amplifier ratemeter and so on.

The source is placed externally on one side of the tank and the detector on the opposite external side of the tank. The liquid rising and falling inside the tank absorbs radiation and the change in intensity received by the detector is measured and gives an indication of the liquid level. In *Figs.* 55 and 56, two types of installation will be noted: one transmitting a relatively narrow beam of radiation and the other a deep beam of radiation extending over the range of liquid level it is desired to measure. The former is useful where the change in level is small, of the order of a few inches, and the other where changes in level up to 6ft are encountered. Sources normally used are Strontium 90 or Caesium 44 for beta radiation and Cobalt 60 and Caesium 137 for gamma radiation. A small radioactive source could be about 1in. long and $\frac{1}{8}$in. diameter. The deep source would be made of short rods placed end to end to make up the required length. All sources are housed in shielded containers with a shutter for transmission purposes.

In addition to the radioactive source, a typical equipment involves a single halogen quenched geiger counter as detector. The pulse signals so obtained are fed to a cathode follower circuit and thence to a backed-off ratemeter of high stability. The ratemeter output can be used to operate an indicator or recorder.

It is possible to use nucleonic liquid level instruments for tanks up to 15ft. to 20ft. in diameter.

As an indication of the effectiveness of the type of small liquid level changes, *Fig.* 57 shows the calibration for the molten glass feeder channel of *Fig.* 58 where it was necessary to measure accurately over a very small range.

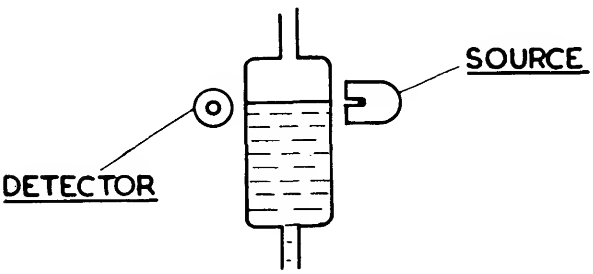


Fig. 55. *Diagram indicating the elements of a nucleonic liquid level gauge for small variations in level*

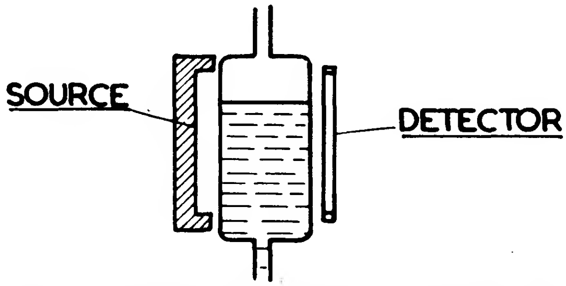


Fig. 56. *Diagram indicating the arrangement for a nucleon ic level gauge for larger variations than those of Fig.* 55

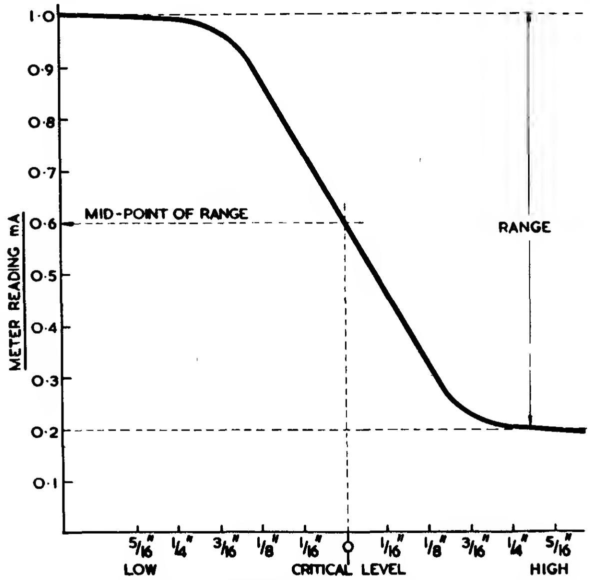


Fig. 57. *Calibration curve for nucleonic type level gauge for the installation of Fig.* 58

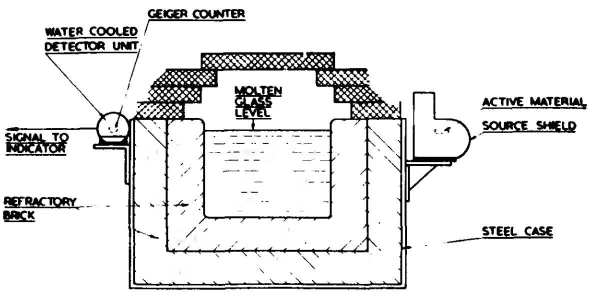


Fig. 58. *Arrangement for measuring the level of molten glass feeder channel*

MEASUREMENT OF THE LEVEL OF SOLID SUBSTANCES

There is a need in industry for the continuous indication of the levels of granular substances in containers between the full and empty conditions. One typical example is the measurement of the level of flour in a silo. It is relatively easy to provide an equipment which will indicate the upper and lower limits, but continuous measurement between these present difficulties. It can be seen that the majority of methods described for liquid level determination cannot be used in the case of solids. One exception is the capacitor probe method.

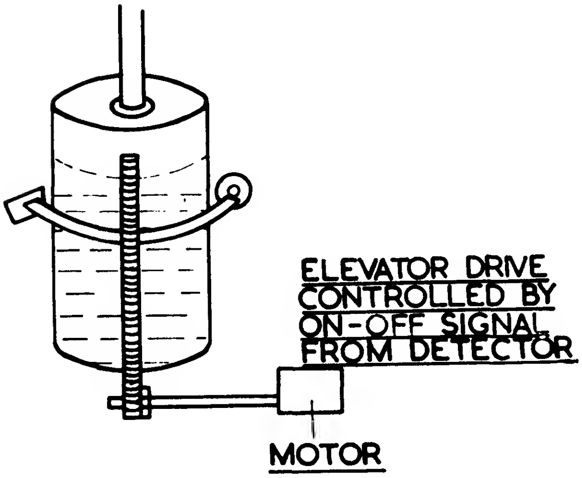


Fig. 59. *Possible arrangement of a nucleonic level measuring system for solids*

Nucleonic Methods

At the moment of writing this method has potential applications, particularly where the level has small variations. A suggested method is shown in *Fig.* 59. Here the transmitter and receiver are installed on a framework mounted in the rack member of a rack and pinion movement. A change in level produces a signal to operate the motor which drives the rack and pinion until the transmitter and receiver are again in line with the level of the solid in the container.

Capacitor Probe Method

The general arrangement is the same as that for liquid levels as indicated in *Fig.* 54. A variety of probe devices are available including the nylon rope previously mentioned which has been used for flour silos. One or two precautions must be observed. Filling and emptying a container with abrasive substances can damage the probe unless it is of a material, or is coated with a material, not affected by abrasive action. A change in moisture content of the substance can affect its dielectric constant and hence the capacity values. An error, therefore, may be introduced into the measurement. There is a tendency to introduce a device for compensating for moisture content changes into the capacitor probe equipment when it is used for hygroscopic substances.

Determination of Level by Weighing

Measuring the weight of material in a vessel is an indirect method of observing level since variations in level produce corresponding variations in weights. But the primary factor of importance in such cases would normally be weight and the method is not widely used for level determination only.

Books suggested for further reading

ECKMAN, D. P. Industrial Instrumentation. Chapter 9. Chapman and Hall, 1950.

JONES, E. B. Instrument Technology. Vol. 1. Chapter 2. Butterworths, 1953.

HOLZBOCK, W. G. Instruments For Measurement and Control. Chapter 5. Chapman and Hall, 1955.

—— Instrument Manual. Section X. United Trade Press, 1960.

KALLEN, H. P. (ED.). Handbook of Instrumentation and Controls. Section 6. McGraw Hill, 1961.

Questions

1. In the float operated gauge of *Fig.* 39, the float moves 20ft between maximum level and zero. If the pulley diameter is 3in. and the indicator pointer moves over 300° for full scale travel, what is the reduction needed between pulley and pointer? (Answer 31/1.)

2. Using formula (37) calculate the float travel d for a level range H of 40ft. Take ρ_2 as 0·488lb/in³ and ρ_1 as 0·036lb/in³. H_1 is 10ft. $d_1 = 8\frac{1}{4}$in. and $d_2 = 2$in. (Answer $2\frac{5}{8}$in.)

3. With the same figures as question 2 calculate d when the situation is as shown in *Fig.* 44. Use formula (38) and take ρ_3 as 0·016lb/in³ (Answer $1\frac{1}{2}$in.)

4. Assuming for calculation purposes that *Fig.* 54 can approximate to two coaxial cylinders, if the diameter d of the probe is 2cm, the diameter D of the tank is 100cm (1 metre), the dielectric constant K of the liquid in the tank is 2, find the capacitance involved if the length l of the tank is 200cm and is full of liquid.

$$\text{Capacitance C is } \frac{K0{\cdot}2416l}{\log_{10}\frac{D}{d}} \text{ microfarads}$$

(Answer 57 micromicrofarads.)

Chapter 4

DIFFERENTIAL PRESSURE FLUID FLOWMETERS

INTRODUCTION

FLUID flow in industrial undertakings occurs in two general forms: either as a flow in a pipe or conduit or, in the case of liquids only, as a flow in an open channel. In both cases, the rate of flow is of primary importance, and, in a large number of plants, the totalized flow over a specified period is required in addition. The rate of flow measuring instruments will be examined first.

RATE OF FLOW MEASURING INSTRUMENTS

This class may be broadly sub-divided into:

(a) Differential pressure flowmeters
  Orifice pattern
  Venturi and nozzle pattern
  Pitot tube pattern
  Dall tube pattern
  Miscellaneous tube patterns
(b) Variable area flowmeters
(c) Displacement and inferential flowmeters
(d) Electromagnetic flowmeters
(e) Ultrasonic flowmeters
(f) Anemometers.

DIFFERENTIAL PRESSURE FLOWMETERS

Before describing any particular models of differential pressure flowmeters, it will be necessary to consider some aspects of Bernouilli's Theorem.

Bernouilli's Theorem—Incompressible Fluids

Bernouilli's Theorem will be stated here, but not derived, since we are concerned only with the final result. *It must be pointed out that it applies to ideal fluids under streamlined flow conditions.* The full treatment may be found in standard textbooks dealing with fluid flow. For a stream of ideal fluid, in steady flow, with no frictional forces involved but acting under gravitational forces only, the sum of the pressure energy, kinetic energy and potential energy is a constant. Expressed mathematically,

$$pg + \frac{\rho v^2}{2} + g\rho Z = k \qquad \text{. . . . . (39)}$$

where
p = the pressure
ρ = the density
v = the velocity
g = gravitational acceleration
Z = height above an arbitrary datum plane
k = a constant.

The first term on the left hand side of equation (39) represents the pressure energy, the second the kinetic energy and the third the potential energy.

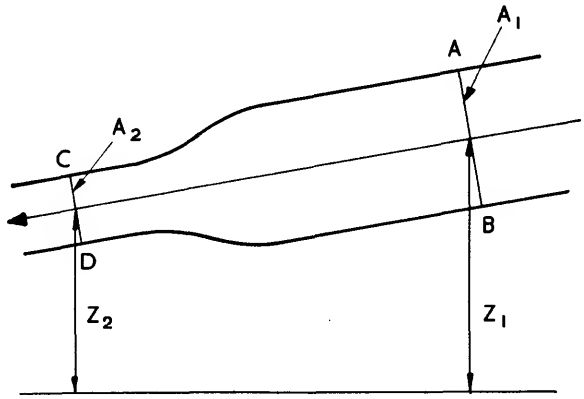


Fig. 60. A fluid stream constricted from area A_1 at AB to A_2 at CD

Now consider *Fig.* 60, a fluid stream constricted to flow from a section area A_1 at AB to a smaller area A_2 at CD. The fluid, at first, will be taken to be incompressible, so that the density is unchanged from AB to CD. Continuity of flow demands that the quantity of fluid Q flowing through A_2 per second is the same as that through A_1. Since Q is the product of the area and velocity at each section,

$$Q = A_1 v_1 = A_2 v_2 \qquad \text{. . . . . . (40)}$$

where
v_1 = the velocity at AB
v_2 = the velocity at CD.

This means that the kinetic energies obtaining at AB and CD are $\frac{\rho v_1^2}{2}$ and $\frac{\rho v_2^2}{2}$ respectively.

The potential energy at AB is $g\rho Z_1$, and that at CD, $g\rho Z_2$.

A pressure change is involved from AB to CD. The reason for this will be found in the various treatments of Bernouilli's Theorem. It can be deduced by considering the work done in transferring a small element of fluid from AB to CD. Let us designate the pressure energy at AB $p_1 g$ and that at CD $p_2 g$. The total energy at AB is the same as that at CD since no heat transfer, etc., is considered involved. Then, the following relation holds:

$$p_1 g + \frac{\rho v_1^2}{2} + g\rho Z_1 = p_2 g + \frac{\rho v_2^2}{2} + g\rho Z_2 = \text{k} \text{. (41)}$$

Rearranging (41),

$$\frac{\rho}{2}(v_2^2 - v_1^2) = g(p_1 - p_2) + g\rho(Z_1 - Z_2) \qquad \text{. . . (42)}$$

$$v_2^2 - v_1^2 = \frac{2g}{\rho}(p_1 - p_2) + 2g(Z_1 - Z_2) \qquad \text{. . (43)}$$

We have seen that

$$Q = A_1 v_1 = A_2 v_2$$

$$\text{If} \quad m = \frac{A_2}{A_1}$$

$$v_1 = m v_2$$

and (43) becomes

$$v_2^2(1-m^2) = \frac{2g}{\rho}(p_1-p_2) + 2g(Z_1-Z_2) \quad . (44)$$

giving

$$v_2 = \frac{1}{\sqrt{1-m^2}}\sqrt{\frac{2g[(p_1-p_2)+\rho(Z_1-Z_2)]}{\rho}} \quad . . (45)$$

But since $Q = A_2 v_2$ (45) can be written

$$Q = \frac{A_2}{\sqrt{1-m^2}}\sqrt{\frac{2g[(p_1-p_2)+\rho(Z_1-Z_2)]}{\rho}} \quad . (46)$$

A further step yields

$$Q = E.A_2\sqrt{\frac{2g[(p_1-p_2)+\rho(Z_1-Z_2)]}{\rho}} \quad . (47)$$

where $E = \dfrac{1}{\sqrt{1-m^2}}$ and is known as the *velocity of approach factor.*

Equation (47) is of the utmost importance as it infers that by constricting a fluid stream to a smaller area, a pressure differential may be set up which is a measure of the flow rate Q of the fluid.

The difference (Z_1-Z_2) can be written as H. Then

$$Q = E.A_2\sqrt{\frac{2g[(p_1-p_2)+\rho H]}{\rho}} \quad . . . . (48)$$

If the flow is through a horizontal pipe, $Z_1 = Z_2$ and (48) simplifies to

$$Q = E.A_2\sqrt{\frac{2g(p_1-p_2)}{\rho}} \quad . . . . (49)$$

$$\text{or } Q = E.A_2\sqrt{\frac{2gp_d}{\rho}} \quad . . . . (50)$$

where $p_d = (p_1-p_2)$

Compressible Fluids

With compressible fluids such as gases and vapours, the density does not remain constant when the pressure changes from p_1 to p_2. An adiabatic gas expansion is considered to take place, that is, no heat flows from or to the fluid, and no external work is done on or by the fluid. Applying certain thermodynamic laws, we find as a final result that the original equation (50) must be modified by a constant known as the *expansibility factor*, denoted by ε. The actual value of the factor is a function of the ratio of the upstream and downstream pressures, the gas specific heat ratio, and area ratios. Curves for deducing the appropriate value are available in B.S. 1042. Since the density alters with pressure, it must be specified at one particular pressure value. This is normally p_1.

Equation (50) now becomes

$$Q = E.A_2.\varepsilon\sqrt{\frac{2gp_d}{\rho_1}} \quad . . . . (51)$$

where ρ_1 is the density at pressure p_1.

Accepting equation (50) or (51) as a basis for measurement, the main problem to be solved is in what manner the alteration of area shall be made. The transition may take place abruptly or gradually. We shall consider the abrupt change first.

Concentric Orifice Plates

A universally used method of making an abrupt change in the cross-sectional area of a fluid stream flowing in a pipe is the concentric orifice plate. This involves a circular metal plate with a central hole or orifice concentric with the circumference of the plate. It is fixed between the pipe flanges and is located by the flange bolts. The orifice is then concentric with the internal bore of the pipe (see *Fig.* 61). Appendix 1 describes actual designs.

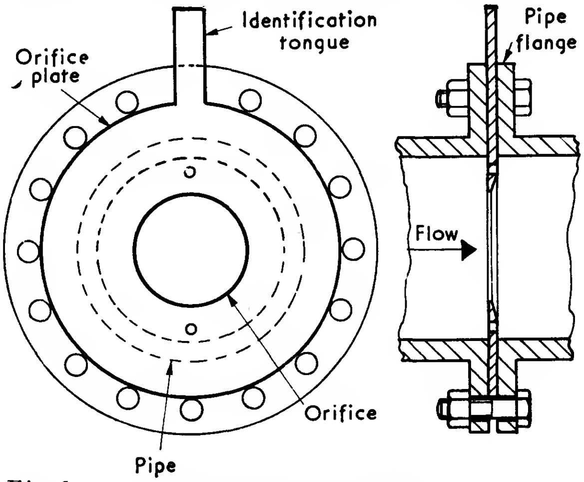


Fig. 61. *A concentric thin plate orifice*

The design and use of concentric orifice plates are covered in a British Standard Specification No. 1042, Part 1. It will be convenient before describing particulars to see what occurs when an orifice plate is inserted in a fluid stream in a pipe, and a liquid flow is considered. *Fig.* 62 illustrates the action in a simplified manner.

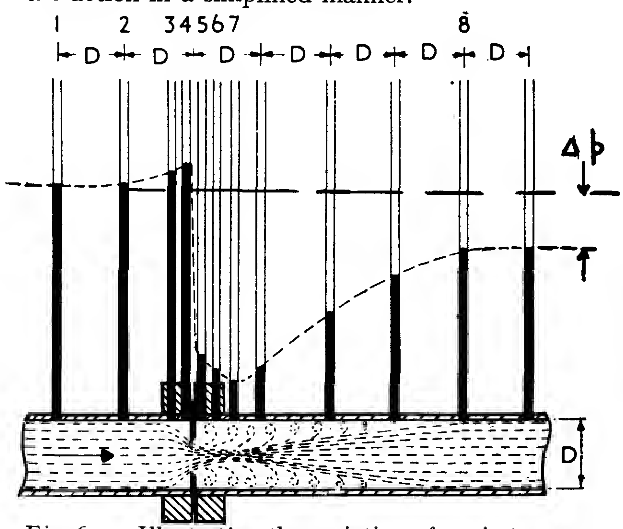


Fig. 62. *Illustrating the variation of static pressure upstream and downstream of the orifice*

Suppose that tubes are inserted through the pipe wall at the position shown in the diagram. The pipe liquid will rise in these until the pressure due to the column of liquid in each tube is equal to the static pressure at that position. The column heights are then a measure of the pressures, and from observing the different values we may trace the pattern of the pressure changes as we proceed

along the pipe. At positions 1 and 2 there is no pressure change worth specifying. At 3 and 4, just before the orifice, we find a slight increase in pressure. The stream is then constrained to flow through the smaller area of the orifice, from which it issues as a jet. At positions 5 and 6 there are lower pressures than at the upstream positions due to the change in the stream sectional area. Since this is smaller the velocity has increased, and the pressure has fallen. The stream or jet cross-section decreases in area after leaving the orifice until it reaches a point, indicated as 7 in the diagram, where it is a minimum and the velocity a maximum. This is mainly due to the liquid being directed inward as it approaches the orifice, and, through inertia effects, persisting in this direction for a distance after it leaves the orifice. The static pressure also reaches its minimum value at this position, which is known as the *vena contracta*. The distance from the orifice varies with the ratio of orifice diameter to pipe diameter, but an average value would be about one half the pipe diameter. From the vena contracta, the stream section expands until it reaches the pipe diameter at 8. Two facts emerge from a study of *Fig.* 62. One is that the downstream static pressure never recovers its upstream value. This would appear to be caused by the velocity changes being accompanied by considerable turbulence with resulting dissipation of energy involving a pressure loss. B.S. 1042 gives a curve of pressure losses in terms of percentage of the differential pressure across the orifice and the ratio of the orifice to pipe diameter. Taking a typical value of 0·6 for orifice to pipe diameter ratio, the percentage loss works out at 65 per cent of the differential pressure. Where pressure loss is important this factor should be borne in mind.

The second point which emerges is that there appears to be a variety of positions at which to take pressure tappings or connections for obtaining the differential pressure.

The following are the main tapping positions (shown diagrammatically in *Fig.* 63) and in appendix 1.

D and D/2 Taps (Radius or Throat Taps)

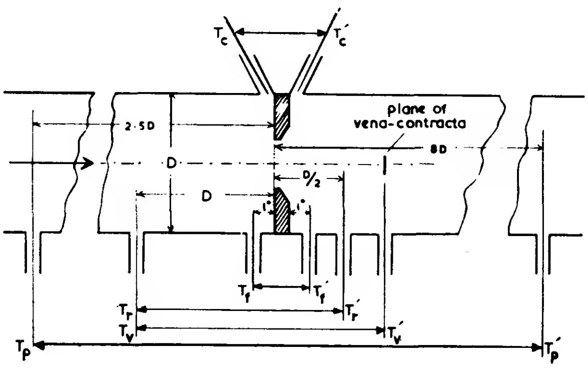
The upstream pressure tapping is taken at one pipe diameter, *D*, upstream from the face of the orifice, and the downstream pressure tapping is taken at one half pipe diameter, *D*/2, downstream from the orifice face, approximately the vena contracta position. The tappings correspond roughly to positions 2 and 7 in *Fig.* 62. (B.S. 1042, Part 1).

Corner Taps

Corner tappings are taken via holes cut obliquely through a flange or boss on the pipe, bringing the inside openings of the holes adjacent to the orifice Positions 4 and 5 in *Fig.* 62 correspond (B.S. 1042, Part 1).

Plate Taps

In this variety, connecting holes are actually bored in the orifice plate itself, each hole communicating with one face.



T_c T_c' = CORNER TAPS
T_f T_f' = FLANGE TAPS
T_r T_r' = RADIUS TAPS
T_v T_v' = VENA-CONTRACTA TAPS
T_p T_p' = PIPE TAPS

Fig. 63. *The position of various pressure tappings relative to an orifice*

Flange Taps

These are situated 1in. from the upstream and 1in. from the downstream face of the orifice plate, with the tappings bored through the flanges (B.S. 1042. Part 1).

Vena Contracta Taps

The upstream tapping is 1 pipe diameter from the upstream face, and the downstream tapping is determined from a curve relating the required dimension to the ratio of orifice to pipe diameter. These are very similar to the *D* and *D*/2 taps.

Pipe Taps

These may be 2½ pipe diameters upstream and 8 diameters downstream from the upstream face of the orifice plate.

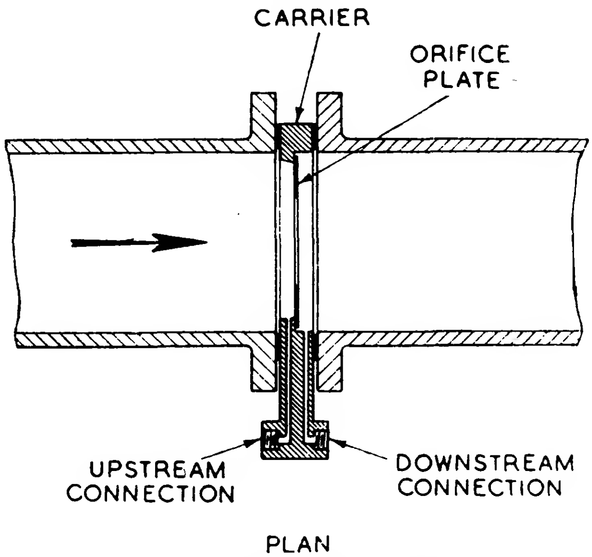


Fig. 64. *A carrier ring pattern orifice unit*

Carrier Ring

Where it is not desirable to drill or tap actual pipes, bosses, or flanges, a self-contained orifice assembly may be inserted between pipe flanges.

It consists of a metal ring holding the orifice plate, with tappings drilled through the ring to communicate with the upstream and downstream sides of the orifice. *Fig.* 64 shows diagrammatically a carrier ring assembly. One advantage of this type is that all drillings, etc., are carried out at the manufacturer's works, and errors due to site operations are eliminated.

Having established the possibility of a definite constrictive device for fluid flow measurement under ideal conditions, we must now examine what modifications are necessary in practice.

Turbulent Flow

In practically all cases of flow in pipes for industrial purposes the flow is turbulent, that is, the particles of the fluid do not follow paths parallel to the direction of flow. Some, if not all, of the particles have a transverse motion as well as a longitudinal one, and form little eddies or swirls giving rise to turbulence. Stream line or laminar flow formulæ will not apply here without modification and a new set of equations must be derived.

Discharge Coefficient

Due to friction and velocity distribution, the practical flow figures do not line up with the theoretical ones. Observe that the stream area contracts after leaving the orifice to the vena contracta position (*Fig.* 62). The cross-sectional area there may only be about 0·6 of that of the orifice, and since it is the stream area from which equations (50) and (51) have been evolved and the one, therefore, determining the measurement, the equation must include a correction factor for the friction effects, and another for the contraction effect. It is customary to incorporate these two into one constant, and denote it by name *Discharge Coefficient* and letter *C*.

Reynolds Number

All liquids are viscous in nature and the viscosity enters into the determination of flow constants, particularly the discharge coefficient. In fact, it may be shown that the latter is a function of $\frac{vd\rho}{\eta}$ where v is the mean velocity through the orifice, d is the orifice diameter, ρ the fluid density, and η its absolute viscosity. The term $\frac{vd\rho}{\eta}$ is known as the Reynolds number. This factor is dimensionless, and a useful criterion by which to compare flows in geometrically similar installations but with differing flow conditions. It also furnishes a means of indicating the conditions where stream line flow ceases and turbulent flow begins, and the Reynolds number for the transition region is about 2000-2200. In any of the official publications covering flow practice, e.g., "Fluid Meters" by the American Society of Mechanical Engineers, British Standard Specification No. 1042, and the German V.D.I. publication on "Rules for Normal Flow Nozzles and Orifices" will be found discharge coefficients prepared for varying Reynolds numbers and orifice/pipe diameter ratios so that calculation is rendered easier.

General Rules for Orifice Design

Some general rules relating to concentric orifices may now be quoted. In view of the comprehensive nature of B.S. 1042, Part 1, it is only possible to quote a few general provisions.

(1) The thickness of the orifice plate should be sufficient to prevent distortion by the differential pressure across it. This, however, does not normally call for excessive thickness and $\frac{1}{16}$ in. for pipes up to 6in. diameter (one or two firms go as low as $\frac{1}{32}$in.) and $\frac{1}{8}$in. above is quite normal. It may be necessary to proceed to $\frac{3}{16}$in. for some pipe and flow conditions. The maximum thickness allowed by B.S. 1042 is 0·1 or 0·05 of the pipe diameter, depending upon design conditions.

(2) The upstream edge of the orifice must be quite sharp except for conical entrance and quarter circle types and the bore should form a right angle with the face of the plate. Any alteration in this will affect the discharge coefficient.

(3) Coefficients are published for various types of orifices in pipe lines down to 1in. lated for m values between 0 and 0·7. The minimum orifice diameter varies between 0·2 and 0·6in.

(4) The orifice plate should possess an identification tongue as indicated in *Fig.* 61.

(5) The orifice plate must have a small hole drilled in it. In the case of liquid flow, the hole is situated above the orifice opening to allow passage of entrained air or gases and so prevent a gas or air pocket building up. In the case of air, gas, or vapour flow, the hole is placed below the orifice, and nearly flush with the pipe bottom, to allow condensed moisture to drain through. The hole must be at least 90° displaced from the pressure tapping.

(6) Orifice (either plate or carrier type) positioning is effected by the ring of bolts clamping the flanges together. The outside diameter of the orifice plate must be made so that it fits accurately within the bolt ring.

General Factors Affecting the Performance of Orifice Meters

1. In paragraph 2 of General Rules for Orifice Design reference was made to the sharp edge of an orifice. It may suffer from wear by abrasion, so modifying its effective diameter. At the same time, the internal surface of the pipe may undergo corrosion or erosion. From either aspect the ratio of the orifice diameter to pipe diameter can be affected seriously enough to introduce unacceptable errors. Calibration can also be affected by a difference between the actual diameter of a new pipe and its quoted value.

2. Certain precautions must be observed in the location of an orifice. A pipe bend or control valve if sufficiently close to the orifice can produce abnormal flow conditions, such as swirls or eddies,

influencing accuracy of measurement. It is usual to specify minimum straight lengths of unimpeded pipe upstream and downstream from the orifices.

In some cases, straightening devices are recommended upstream of an orifice to correct the distortion. Typical units are indicated in *Fig.* 65 as a set of vanes or a nest of pipes. The nature of the disturbance should be known, as the use of straighteners cannot be made indiscriminately. B.S. 1042, Part 1 makes recommendations regarding the installation of vanes, location of bends, etc., relative to the orifice, and is inclined to favour perforated plate services.

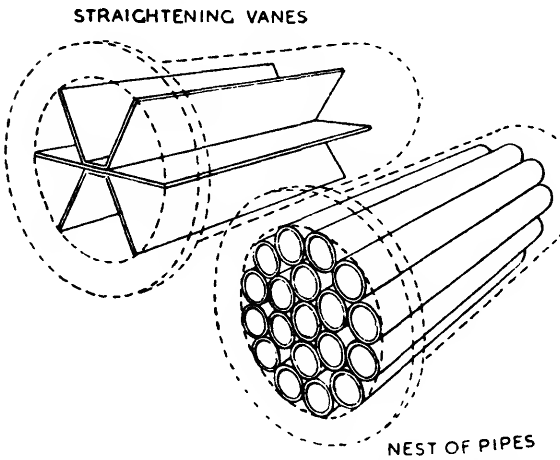


Fig. 65. *Flow straightening devices in a pipe*

3. The location of the measuring instrument relative to the orifice (or indeed other differential pressure elements) together with the nature of the fluid being metered give rise to certain requirements regarding piping and the use of seals and other devices. These will be discussed in Chapter 5 dealing with differential pressure flow measuring instruments.

Practical Forms of Equations

Equation (50) now takes the following practical forms:

$$W = 359{\cdot}2CZ\varepsilon Ed^2\sqrt{h\rho} \quad \ldots\ldots\ (52)$$

$$Qg = 2238CZEd^2\sqrt{\frac{h}{\rho}} \quad \ldots\ldots\ (53)$$

where W = the flow rate in lb/hour

Qg = the flow rate in gallons/hour

C = the discharge coefficient

d = the orifice diameter in inches

E = the velocity of approach factor

h = the pressure differential across the orifice in inches of water

ρ = the liquid density in lb/ft^3

Z = a compound factor. In the case of fluid flow it is the product of a correction for Reynolds number and a pipe size constant.

ε = expansibility factor. For liquids this may be taken as 1.

Equation (51) becomes:

$$Q = 359{\cdot}2CZ\varepsilon Ed^2\sqrt{\frac{h}{\rho}} \quad \ldots\ldots\ (54)$$

where Q = the flow rate of a fluid having a density ρ at the upstream pressure value. It may be necessary to correct to standard conditions.

The flow rate is normally in c.ft/hour.

ε, C, d, E, Z as in equation (52).

Segment (or Chord) and Eccentric Orifices

Whilst the concentric orifice is satisfactory for most fluids, where suspended solid is encountered it is preferable to use a segment type orifice or an eccentric one (*Figs.* 66 and 67). In the case of the segment or chord orifice, the solid segment is at the top part of the orifice plate, and the open part has its circumference coincident with the pipe so that passage of solid material is not interfered with, and there is no building up of solid matter against the upstream face of the orifice. The eccentric orifice follows a similar course with the lower part of its orifice opening flush with the lower part of the pipe.

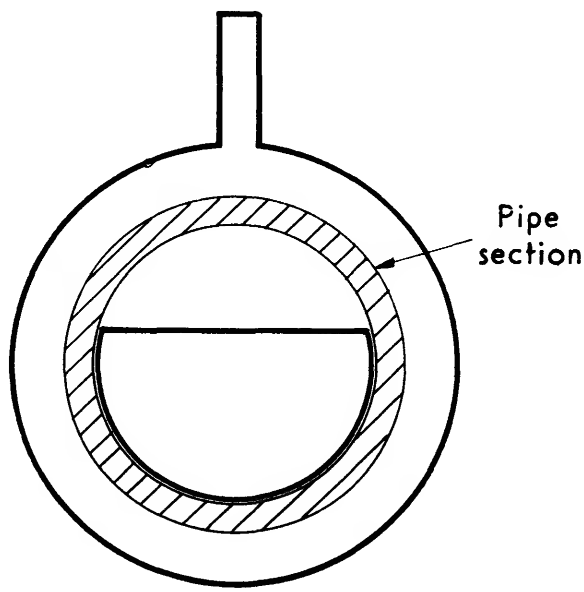


Fig. 66. *The segment pattern orifice*

There are one or two other aspects of these types of orifices which may favour their use in place of the concentric type. With a large *m* ratio, e.g., 0·7 or above, arranging the concentricity of the orifice opening relative to the pipe bore may be a matter of difficulty. With either of the above versions possible errors due to misalignment are eliminated.

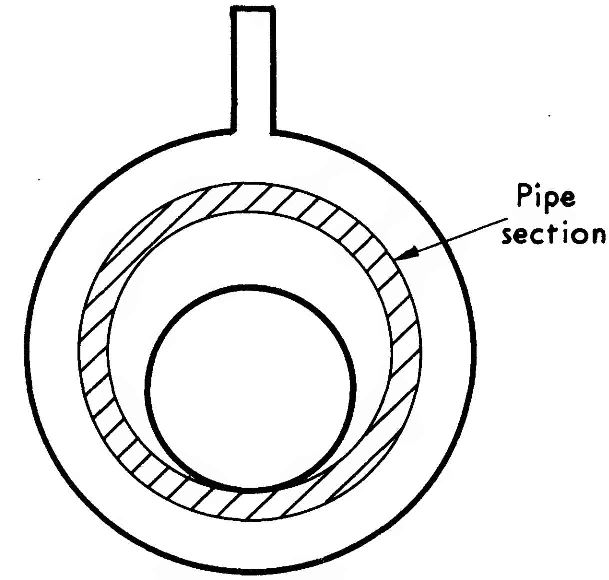


Fig. 67. *The eccentric pattern orifice*

Again, some gas flow mains can be of extremely large diameter, perhaps 3 or 4ft. The use of a segment orifice with a turned flange on it enables the orifice plate to be bolted to the pipe surface and dispenses with the necessity of having a pipe flange for installation.

Orifice Materials

Materials used for orifice plates include mild steel, stainless steel, Monel, phosphor bronze, gunmetal, depending on the application. A rough classification would be:

Water Metering: Gunmetal, bronze, stainless steel
Air Metering: Gunmetal, Monel, mild steel
Steam Metering: Stainless steel, Monel
Sewage, Fuel Oils, Coal Gas: Stainless steel.

Venturi Tube

We have seen the effect of inserting an orifice plate in a fluid stream, causing an abrupt change in stream area to produce a differential pressure. The operation can be accompanied by a fairly high permanent pressure loss, and, where pressure loss is important, it is necessary to turn to other methods of producing differential pressures. Let us consider devices with a gradual change in area. The first of these is the Venturi tube.

The principle was first explained by Venturi, the Italian scientist, as far back as 1797, but practical form was really given to it by Herschel in 1887. The basic design is relatively simple. There are virtually three sections to the tube: the inlet or upstream cone, the throat, and the outlet or downstream cone. *Fig.* 68 indicates the general lay-out of the long or standard pattern. The inlet cone tapers down from the pipe area to the throat section of smaller area to produce the necessary velocity and pressure change. The outlet cone expands from the throat to the pipe area. Pressure tappings are taken at the inlet entrance to the cone, and at the throat. The tappings take the shape of annular chambers, the inside surfaces assuming the form, as a rule, of smoothly machined liners with holes pierced at regular intervals round the circumference. This enables the pressure to be averaged before transmission to the measuring instruments. B.S. 1042 specifies conical angles between 5° and 15° for the outlet cone, the design of which has an influence on efficiency in terms of pressure loss. As an example, take a throat/pipe diameter ratio of 0·060 giving $m = 0{\cdot}36$. Expressed in the conventional manner, with a 5°–7° expansion, the net pressure loss is about 9 per cent of the differential pressure between inlet and throat, and with a 14°–15° (14 per cent. Compare these figures with the 65 per cent loss for an orifice/pipe diameter ratio of 0·60.

The effect of the inlet cone is not so important, and if a little greater loss of pressure may be tolerated, another pattern of Venturi tube may be used with considerable saving in weight and space. This type is known as the short pattern, and the inlet area change is made in several ways. In one form it takes the profile of a nozzle; in another it is almost as abrupt as an orifice.

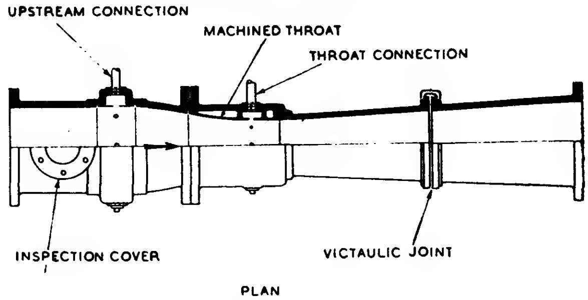


Fig. 68. *The elements of a Venturi tube*

Discharge Coefficients

The discharge coefficient of a standard type of Venturi tube is about 0·99 and remains substantially constant for all values of throat/pipe diameter ratios between 0·25 and 0·75 ($m = 0{\cdot}05$ to 0·55). At low values of Reynolds numbers, the Rd coefficient may fall to 0·95 and so modify the flow. In practice, the approach to the throat is often given a curved profile by means of its liner, to maintain a constant discharge coefficient.

For the short Venturi tube, the discharge coefficient ranges between 0·98 and 0·92.

BS. 1042 Part 1 covers pipe diameters down to 2in. but rules that the throat diameter must not be less than 0·8in.

The effect of a high discharge coefficient is apparent if any of the equations (52), (53) and (54) are examined (these apply equally to orifice plates, Venturi tubes or nozzles). For the same flow in a given pipe, and with the throat diameter the same as that of an orifice, a much smaller differential is needed, resulting in further decrease in pressure loss. Alternatively, with the same differential and throat diameter a considerably larger flow is achieved than with an orifice. For a simple and rough comparison let us consider an example.

Suppose the m value of a Venturi tube be 0·40, its discharge coefficient 0·99, and differential h_1. For an orifice of the same m ratio the discharge co-

efficient will be approximately 0·60. Let the differential be h_2. With the flow remaining the same in both cases:

$$0{\cdot}99\ \sqrt{h_1} = 0{\cdot}60\ \sqrt{h_2} \quad \ldots\ldots (56)$$

$$\sqrt{h_1} = \frac{0{\cdot}60}{0{\cdot}99}\sqrt{h_2} \quad \ldots\ldots (57)$$

$$h_1 = \left(\frac{0{\cdot}66}{0{\cdot}99}\right)^2 h_2 \quad \ldots (58)$$

$$h_1 = 0{\cdot}36h_2 \quad \ldots\ldots (59)$$

The differential required for the Venturi case is less than half that required for the orifice.

If the differential remains the same and the throat and orifice diameters are equal, and if Q_1 and Q_2 are the respective flows:

$$\frac{Q_1}{0{\cdot}99} = \frac{Q_2}{0{\cdot}60} \quad \ldots\ldots (60)$$

$$Q_1 = \frac{0{\cdot}99}{0{\cdot}60}Q_2 \quad \ldots\ldots (61)$$

$$Q_1 = 1{\cdot}66Q_2 \quad \ldots\ldots (62)$$

In other words, the flow has increased by more than half in the case of the Venturi tube for the same conditions, and the pressure loss, as we have seen above, is considerably less.

At first sight, the tube would appear to have immense advantages over the orifice plate. From a purely measurement point of view this is probably correct, but its use is only justified when the orifice plate cannot be used, e.g. where the orifice/pipe diameter ratio is in the region where the coefficients are uncertain, and where the pressure recovery is most important. Its cost compared with the orifice plate is extremely high, and the latter may be manufactured relatively quickly and easily in a normal workshop. Again, the dimensions of a Venturi can be very large. In a 10-in. diameter pipe, for a throat diameter of 7in., the overall length for a standard Venturi tube would be about 7ft 6in. Compare this with an orifice plate of thickness $\frac{1}{16}$in., which may even be installed on an existing plant by springing the pipe flanges apart a sufficient distance to allow for insertion. Careful consideration is always given to a flow measurement problem, therefore, before a Venturi tube is specified.

Constructional Features

To some extent, the construction of the Venturi-tube depends on the application. For normal uses, the sections would be of gun-metal, cast iron, or Meehanite, and smoothly machined liners of gun-metal or stainless steel inserted at the inlet and throat pressure tappings. The use of gun-metal or stainless steel reduces the risk of corrosion. To facilitate construction work a victaulic joint is sometimes inserted in the downstream cone. The extreme ends of the cast sections are flanged to mate with the pipe flanges, and with the adjacent section, and pressure tappings are arranged for screw-in or flanged connections depending upon the particular installation conditions.

For high pressure hot water flow, as in boiler feed water in a power station, the design may be modified. A fabricated cylindrical casing is used, and a gun-metal lining is inserted. The lining is made in three sections: inlet cone, throat, and outlet cone, profiled as for a standard Venturi. This design is suitable for pressures up to 1400lb/in². Another pattern has a maximum working pressure of 2000lb/in².

The Venturi tube possesses a big advantage over the orifice in that its section need not be circular. Square or rectangular shapes have been used for measuring large volumes of fluid flow. The non-circular section lends itself to constructional materials other than metal, and concrete has even been used for one or two very large flows. Note that the design renders the tube useful for fluids containing suspended matter because of its gradual area changes.

Formulae

Equations (52), (53) and (54) are suitable for Venturi tube calculations, but the orifice diameter now becomes the throat diameter and m is the throat/pipe area ratio for determining E.

Nozzles

The nozzle falls between the Venturi tube and the orifice plate as a means of flow measurement. It approximates to a Venturi tube with the curved form of approach, giving a gradual change of sectional area and has the same order of discharge coefficient. But the absence of a downstream expansion cone brings the pressure loss into the same region as that for an orifice plate. It is cheaper than a Venturi tube, and at high velocity flows its use in place of an orifice plate may be necessary. It is perhaps unfortunate that there is some divergence on nozzle design. B.S. 1042 specifies the I.S.A. nozzle and gives details of its design and coefficients. On the other hand, manufacturers have their own models, for which the I.S.A. nozzle coefficients do not apply. *Fig.* 69 indicates a nozzle with corner pressure tappings.

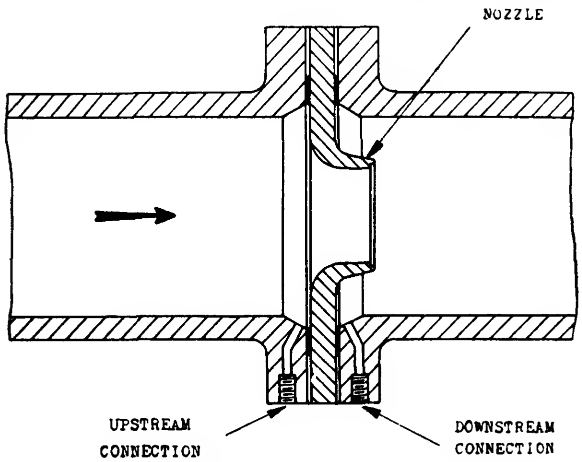


Fig. 69. *A typical nozzle unit*

A feature of the nozzle is that its graduated profile renders it useful where fluids with suspended matter are encountered. Again, as in the case of the Venturi tube, the nozzle possesses some advantages over the orifice plate, but its cost, whilst less than that of a Venturi, is still more than an orifice, so that its use requires consideration if cost is paramount.

Pitot Tube

Let us study the effect of placing a blunt object in a fluid stream as an obstruction to the flow (*Fig.* 70). As the fluid approaches the object, the velocity will decrease until it reaches zero at the point where it impinges on it. From the previous, a deceleration should mean an increase in pressure.

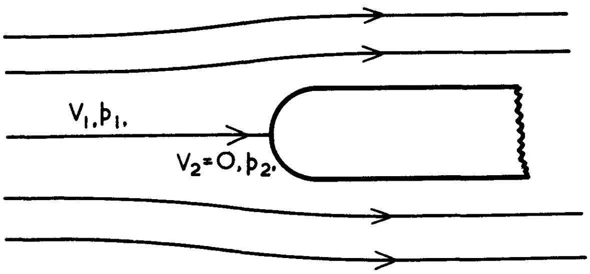


Fig. 70. *The effect of imposing a blunt object in the flow stream*

This would follow from Bernouilli's Theorem. From equation (41) this is:

$$\frac{p_1}{\rho} + \frac{v_1^2}{2g} = \frac{p_2}{\rho} + \frac{v_2^2}{2g} \quad \ldots\ldots \quad (63)$$

If p_1 and v_1 are respectively pressure and velocity upstream from the object, and p_2 and v_2 the respective pressure and velocity in the neighbourhood of the object, at the point of impact v_2 is zero. In other words, the kinetic energy has become converted to potential energy and the result is reflected in the value of p_2 at the impact point. This now consists of the normal static pressure plus another pressure produced as a result of the energy conversion. There is another way of explaining the same operation. It will be noticed that $\frac{p_1}{\rho}, \frac{p_2}{\rho}, \frac{v_1^2}{2g}, \frac{v_2^2}{2g}$ have the dimensions of a length or a head. $\frac{p_1}{\rho}$ and $\frac{p_2}{\rho}$ are, therefore, denoted by the term "pressure head" and $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$ by the term "velocity head". We can say that the velocity head $\frac{v_2^2}{2g}$ has become converted to a pressure head $\frac{p_x}{\rho}$. This will be additional to the normal pressure head $\frac{p_s}{\rho}$ and the sum of these two then gives us $\frac{p_2}{\rho}$:

$$\frac{p_x}{\rho} + \frac{p_s}{\rho} = \frac{p_2}{\rho} \quad \ldots\ldots\ldots\ldots \quad (64)$$

If $v_2 = 0$ equation (63) can be written:

$$\frac{p_1}{\rho} + \frac{v_1^2}{2g} = \frac{p_2}{\rho} \quad \ldots\ldots\ldots\ldots \quad (65)$$

Rewriting:

$$v_1^2 = 2g\left(\frac{p_2 - p_1}{\rho}\right)$$

$$v_1 = \sqrt{\frac{2g\,(p_2 - p_1)}{\rho}} \quad \ldots\ldots \quad (66)$$

We now replace the blunt object with a tube having a small opening facing the direction of fluid flow. Next consider that the tube is joined to one connection of a pressure measuring instrument of the diaphragm type suitable for measuring relatively small differential pressures (see Chapter 1). There is no flow through the tube and the point of impact or zero velocity can be considered to be at the impact hole. This produces p_2 in equation (66), and if a static pressure tapping is taken upstream, a little way from the tube, this gives us p_1. Both pressures are applied to the differential pressure instrument, and a means of measuring the velocity of the fluid is obtained since both g and ρ will be known. We have, then, the Pitot tube in its simplest form (*Fig.* 71).

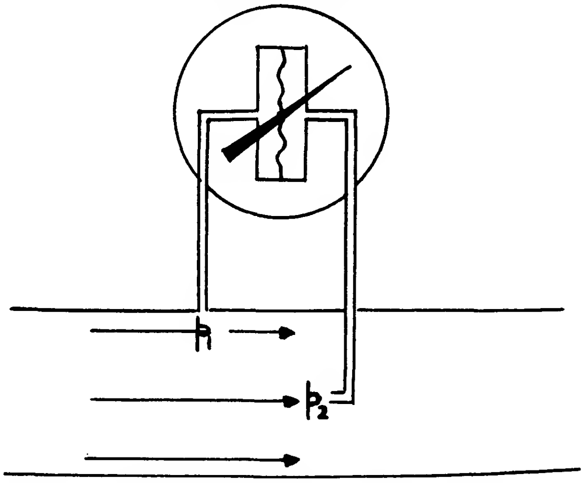


Fig. 71. *Separate static and impact pressure tubes*

It is very convenient to be able to measure the static pressure in the close neighbourhood of the tube and standard Pitot tubes are specified in B.S. 1042, based on N.P.L. research work. Both designs consist basically of inner and outer tubes. The inner one leads from the impact hole to one connection of a differential measuring instrument. The outer tube, referred to, sometimes, as the static tube, has a series of holes bored into it so that its interior connects to the outside surface to be in contact with the static pressure. This tube is joined to the second connection of the measuring instrument (*Fig.* 72).

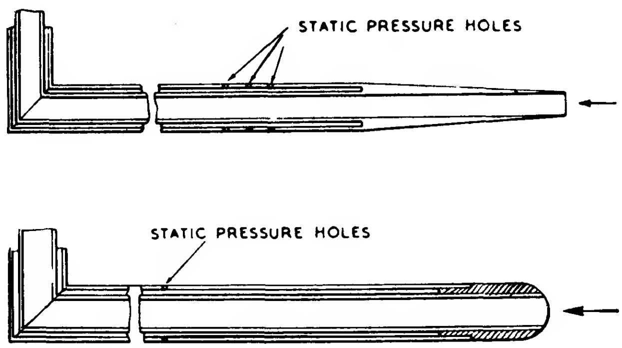


Fig. 72. *Standard pattern Pitot tubes*

The Pitot tube can only measure velocity at one position in the cross-section of a pipe. Now the velocity of a fluid in a pipe, taken across

the section, is not uniform, varying from zero at the pipe surface to a maximum at some point (not necessarily the centre) along a diameter. To find the mean velocity it is necessary to make a traverse of the pipe with the tube, taking the differential pressure at certain specified positions (B.S. 1042). An ideal distribution curve is shown in *Fig.* 73. For Reynolds numbers above 100 000,

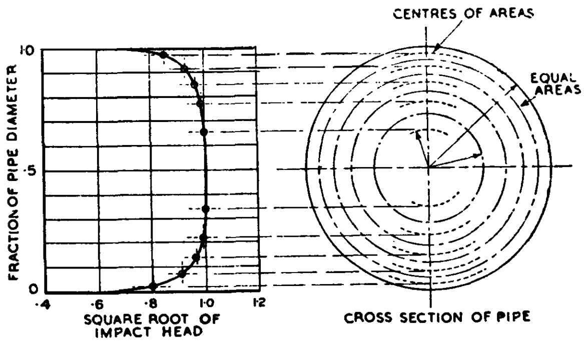


Fig. 73. *An ideal traverse of a pipe*

the ratio of average velocity to velocity at the centre of the pipe is frequently specified as 0·82 or 0·83. Whereas this value would apply for ideal cases for a curve of the type in *Fig.* 73, the actual curve may be different. The desirability of carrying out a traverse, therefore, is obvious. Once having determined the ratio value, the Pitot tube may be placed at the pipe centre and the instrument calibrated in terms of average velocity.

Another theoretically possible means of determining the average velocity is to select a position where the velocity corresponds to the average value. This has some practical drawbacks. The location may be near the wall of the pipe—a very approximate value being 0·25 of the radius in from the wall. It could be at a point where the velocity curve slope is fairly steep and any misplacement could lead to significant errors in velocity determination. At the centre of the pipe, by comparison, the curve is normally flatter and errors in location are not so serious.

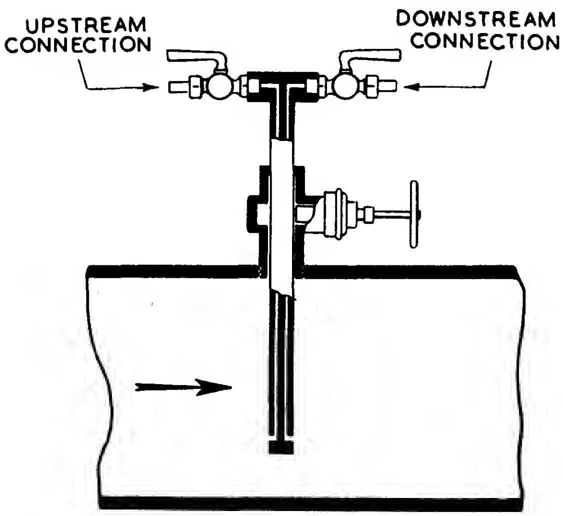


Fig. 74. *Double tip pattern Pitot tube*

Formulae

Equation (66) is not quite in a practical form. It is necessary to modify this by a velocity coefficient C in some types. For the B.S.-N.P.L. patterns, however, the value of C is taken as 1. Those for the single tube type may be lower depending on the design.

A formula commonly used for calculation purposes is:

$$v = 18{\cdot}29\sqrt{\frac{h}{\rho}} \quad \ldots\ldots\ldots \quad (67)$$

where v = the velocity of the fluid in ft/sec.
h = the differential produced in inches of water at 60°F.
ρ = the density of the fluid in lb/ft³.

Compressibility for industrial purposes does not introduce any serious errors by being neglected. Where extreme accuracy is required, however, the following basic formula must be used:

$$h = 0{\cdot}00299v^2\rho(1 + 0{\cdot}2 \times 10^{-6}\, v^2) \quad \ldots \quad (68)$$

where h = the pressure differential produced in inches of water at 60°F.
ρ = the fluid density in lb/ft³.
v = the velocity of the fluid in ft/sec.

To compare the two formulae, let v = 100ft/second, and ρ = 0·0764lb/ft³. From equation (67) h is 2·283, and from equation (68) h is 2·289. The difference is 0·006 or about $\frac{1}{3}$ per cent.

Two other types of Pitot tube deserve mention. One is the double tip pattern shown in *Fig.* 74, in which there are two holes, one facing upstream and the other downstream, the former measuring the impact head and the latter the suction head. The differential pressure obtained is greater than with the standard types, but is not double the value. Actually, the increase is between 35 per cent and 40 per cent depending upon the position of the tube in the pipe. The other type is the Pitot-Venturi. It is a combination of two concentric Venturi tubes, the outlet cone of the inner one terminating in the throat of the outer. The throat pressure of the inner tube and the impact pressure on an impact hole in the supporting tube give 7–10 times the differential produced with the normal types under the same conditions.

"Yaw"

It can be seen with all types of tubes there is a possibility of the axis of the head not being in alignment with the direction of flow, "Yaw" having been said to have taken place. The effect on the standard B.S. patterns is very small for any normal misalignment. At about 20° yaw, the error in velocity determination reaches 2 per cent. For the double tip type the error is about 1 per cent for a 5° yaw.

Advantages

1. The pressure loss caused by the insertion of a Pitot tube in a pipe or duct is very small unless its dimensions are large compared with the pipe diameter.
2. It is extremely useful for determining actual velocity profiles.
3. Its cost is low compared with that of a nozzle or Venturi tube. It is also cheaper than an

orifice, although this may depend upon the pipe diameter.

Disadvantages

1. The fluid must normally be moving at a relatively high velocity to produce a measurable differential pressure. About 50ft/sec of air produces a differential of 1in. w.g. and about 5ft/sec of water produces 10in. w.g. differential.

2. The small openings may become blocked if used with a fluid carrying solid particles.

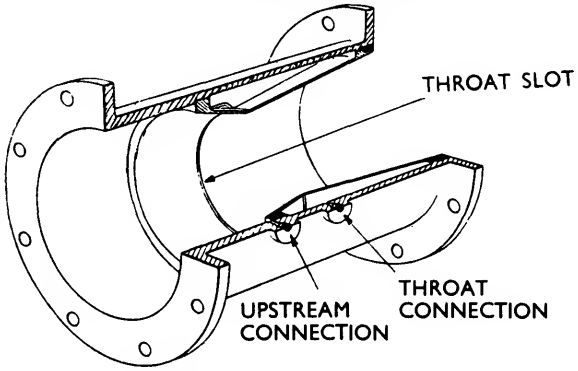


Fig. 75. Dall tube

Dall Tube

The principal features of the Dall tube are indicated in *Fig. 75*. It involves two truncated cones separated by a narrow throat. The throat length is between $0{\cdot}03d$ and $0{\cdot}1d$ where d is the throat diameter. The inlet cone has an included angle between 40° and 50°, the outlet cone between 12° and 17°. The mouth diameter D_m, the inlet pipe diameter D and the throat diameter d are connected by the following relation

$$D_m^4 - d^4 = k\,(D^4 - d^4)$$

where $k = 0{\cdot}5$ to $0{\cdot}75$.

Observe the diameter of the inlet cone is less than that of the pipe, resulting in a sharp step. This creates an impact pressure which is additional to the static pressure existing at the step. The high or upstream pressure connection is made just in front of the step. The other connection is made at the throat where the relatively abrupt change in area results in a marked static pressure depression. The original patent specification No. 689,474 claims a pressure loss expressed as 5% or 6% of the differential pressure. This compares with a loss of between 2 and 3 times this value with a normal Venturi tube. In addition, the Dall tube has the advantage of being considerably shorter than the normal Venturi.

Other Flow Tube Elements

Two other designs generally using truncated cones have been patented in this country. One is the Nathan tube patent No. 473,562 and the Bopp and Reuther tube, patent No. 764,889. A short description of these appears in an article by R. G. West in *Instrument Practice*, April, 1961, page 444.

Books and Literature suggested for further reading

WEST, R. G. *Instrument Practice*, April-July inclusive, 1961 (Nos. 4-7, Vol. 15). In this series of articles Mr. West investigates the empirical nature of flow formulae. His own research and that of others in the field indicates some fresh thought is necessary to place the fundamental laws of fluid flow on a sounder basis.

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British Standard Specifications

B.S. 1042 Methods for the Measurement of Fluid Flow in Pipes

Part 1 Orifice Plates, Nozzles and Venturi Tubes.

Part 2 Pitot Tubes

The original B.S. 1042 was published in 1943. There are many differences between the original version and Part 1 which appeared in 1964 and this part should be well studied in relation to this chapter.

Chapter 5

MEASURING INSTRUMENTS FOR DIFFERENTIAL PRESSURE FLOW DEVICES

SQUARE LAW EFFECT

The nature of the relation between rate of flow and differential pressure influences the design of measuring instruments for orifices, nozzles, Venturi and Dall tubes.

Taking one of the flow equations (52), at maximum flow L_1,

$$L_1 = 359{\cdot}1 C.Z.d^2.E\sqrt{h_1 W} \quad \ldots\ldots (68)$$

At any other flow L_2,

$$L_2 = 359{\cdot}1 C.Z.d^2.E\sqrt{h_2 W} \quad \ldots\ldots (69)$$

Dividing (69) by (68),

$$\frac{L_2}{L_1} = \sqrt{\frac{h_2}{h_1}} \quad \ldots\ldots\ldots\ldots (70)$$

$$\frac{L_2^2}{L_1^2} = \frac{h_2}{h_1} \quad \ldots\ldots\ldots\ldots (71)$$

$$h_2 = \frac{L_2^2}{L_1^2}.h_1 \quad \ldots\ldots\ldots\ldots (72)$$

Assign some figures to equations (68) and (69).

If $h_1 = 50$in. of water for full range,

and $L_1 = 200\,000$lb per hour,

at half the maximum flow,

$L_2 = 100\,000$lb per hour,

but $h = \frac{L_2^2}{L_1^2}.h_1$

$$h = \frac{(100\,000)^2}{(200\,000)^2}.50$$

$h = 12{\cdot}5$in. of water.

The square root effect is now apparent since for half the maximum flow we have only one-quarter the maximum differential. At still lower flows, e.g., $\frac{1}{5}$ maximum, or 40 000lb per hour, the differential is only 2in.

Suppose we wish to use any of the normal differential pressure instruments. Here, the movement or deflection of the measuring element has a substantially linear relation to the applied differential pressure. If a normal linear magnifying mechanism is used, the pen or pointer movement will be directly proportional to the differential pressure. In other words, if θ_1 is the maximum pointer travel in degrees over an indicator scale, θ_2 at any other value is

$$\theta_2 = \frac{L_2^2}{L_1^2}.\theta_1 \quad \ldots\ldots\ldots\ldots (73)$$

Taking a typical circular indicator scale span as 300° and keeping to the previous figures,

$$\theta_2 = \frac{L_2^2}{L_1^2}.\theta_1$$

$$\theta_2 = \frac{(100\,000)^2}{(200\,000)^2}.300$$

$$\theta_2 = 75^\circ$$

The pointer, therefore, only moves a quarter of the way round the dial for half maximum flow value. Most companies introduce a square law compensating device into their flow indicating or recording meters. The methods of so doing are numerous, and only one or two typical types will be described here.

METHODS OF SQUARE LAW CORRECTION WITH U-TUBE TYPE INSTRUMENTS

Simple Glass U-Tube

One method of obtaining a linear scale is actually a development of the inclined tube manometer shown in *Fig.* 5 of Chapter 1. The glass tube is curved to a parabolic law so that the rise of liquid in it bears a substantially linear relation to the rate of flow being measured.

This instrument is suitable for low static pressures. For the higher values, a float operated pattern must be used.

High Pressure U-Tubes

Ledoux Bell Pattern

In this method used by Bailey Meters & Controls Ltd., the basic U-tube equipment takes a special form. Generally, there is one chamber, as shown in *Fig.* 76, containing mercury. In this chamber is a hollow float of the shape indicated. The internal surface of the float is contoured to a given curve of the parabolic or square law family. The mercury rises or falls in the interior of the float as well as outside. The high pressure connection is taken to the inside of the float, and the low pressure to the outside. The application of the differential pressure tends to cause the float to rise in the mercury because of the higher internal pressure. At the same time, however, mercury is driven out of the interior of the float to the outside because of this higher internal pressure, raising the

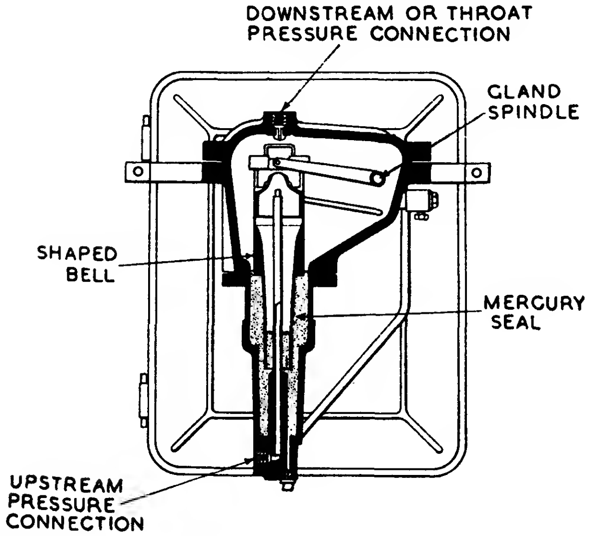


Fig. 76. Basic elements of the Ledoux bell pattern flowmeter

level there. This causes the float to rise to a higher level before equilibrium between the effective weight or buoyancy of the bell and the differential pressure takes place. The float travel will obviously depend on the amount of mercury leaving the interior of the float and entering the outside, and this again depends on the contour of the inside of the float. If this is shaped as already indicated to a generally parabolic curve, the float travel can be made to be substantially linear for all parts of the instrument range.

It may be of interest to note that this type of flowmeter can be used to a maximum of 6000lb/in² static pressure with a maximum differential pressure of 120in. of water (approximately 4½lb/in²). At lower static pressures, e.g., 2500lb/in² the maximum differential is 331in. of water (approximately 12½lb/in²).

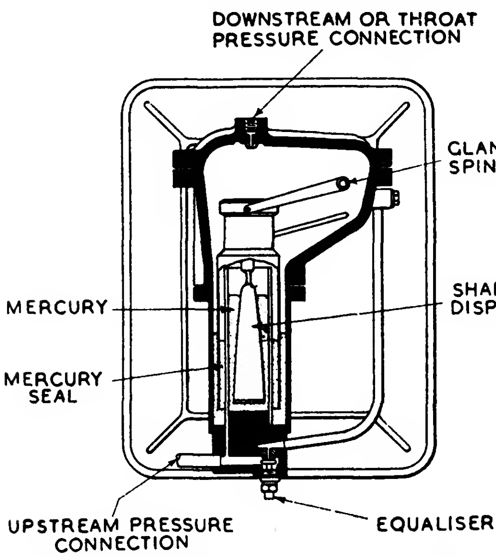


Fig. 77. Basic elements of the displacer pattern flowmeter

An alternative design of the same company used for lower static pressures and differential pressures is shown in *Fig.* 77. This involves a parabolic displacer. The displacer is immersed in mercury, being suspended from a bell which has a mercury seal. The upstream or higher pressure is applied to the inside of the bell, and the lower pressure to the outside. An increase in differential pressure results in an upward movement of the displacer and bell, but, owing to the change of buoyancy of the shaped displacer as it rises and falls, this movement is directly proportional to the flow rate.

Shaped Chamber

The design of *Fig.* 78 has been used to provide square law correction, particularly by Foxboro-Yoxall Ltd. It comprises a parabolic shaped chamber and a cylindrical one. The rise and fall of mercury and, hence, the movement of the float in the cylindrical chamber is substantially linear to the rate of flow. The theory of this design is not a simple one, and a full treatment will be found in Chapter 3 of reference 1. Broadly, the following relation holds:

$$A = \frac{A_1}{2\frac{C}{K}\sqrt{h}-1} \qquad \ldots\ldots (74)$$

where A = the area of the shaped chamber at differential pressure h

A_1 = the area of the cylindrical chamber

C and K = constants.

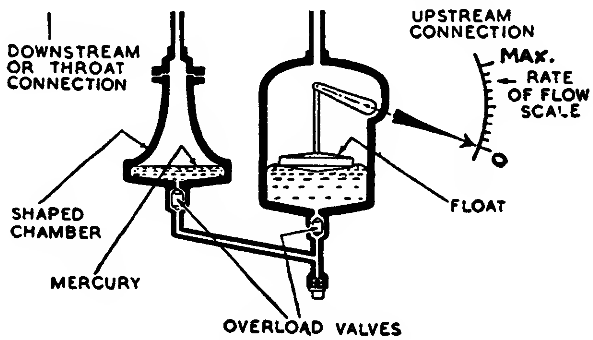


Fig. 78. Basic elements of the shaped chamber flowmeter

Electrical U-Tube Flowmeters

The electrical U-tube flowmeter of Electroflo Meters Co. Ltd. follows the general lines of *Fig.* 46 of Chapter 3. Again we have the limb in which is inserted a unit consisting of a large number (about 100) of vertical conducting rods arranged in spiral formation. These are connected to resistance elements in the head of the unit, in such a manner that they virtually form tappings from a continuous resistance. The rod lengths are graduated so that mercury rising or falling in the chamber, with changing flow, makes or breaks contact with one rod after the other. If a circuit connection is made via the mercury, and another through the resistance assembly in the head, the action of the mercury can be made to vary the amount of resistance in circuit. But now, in contrast to the linear arrangement for liquid level measurement, if the contour of the contacting ends of the rods is arranged according to a parabolic law, a means of correcting the square law effect is obtained.

If Q be the rate of flow, and (p_1-p_2) the pressure differential produced across the orifice, etc., $\sqrt{p_1-p_2}$ is proportional to Q,

i.e. $$\sqrt{p_1-p_2} = k_1 Q \qquad \ldots\ldots\ldots\ldots (75)$$

If h is the difference in levels of the mercury in the U-tube, h is proportional to (p_1-p_2), so that

$$\sqrt{h} = k_2 Q \qquad \ldots\ldots\ldots\ldots (76)$$

The conductance K, defined as $\frac{1}{R}$ where R is the circuit resistance, is made proportional to $\sqrt{h}$ by the parabolic contour of the rods.

Hence $\sqrt{h} = k_3K$ . . . . (77)

From equations (76) and (77), therefore,

$$k_3K = k_2Q \qquad (78)$$

i.e., the conductance varies directly as the flow rate.

The flow meters used in this scheme are electrical instruments with scales calibrated directly in flow units. It is necessary, of course, to ensure that variations in circuit supply voltage do not cause changes in instrument readings. Each indicator or recorder is of a special moving iron type with two operating coils. One coil is connected directly to the voltage supply and forms a voltage coil. The other is in series with the resistance unit and acts as a current coil (see *Fig.* 90). Any change in supply volts affects the current in both coils to the same degree, and since the instrument deflection is dependent on the ratio of these two currents, this ratio remains unaltered over a wide range of supply voltage variation. No errors occur, therefore, in the meter readings.

The space above the mercury is filled with oil in both limbs. This results in the whole resistance unit being oil enclosed and thus protected from the metered fluids.

Methods of Square Law Compensation with Ring Type Balances

The ring balance, as a differential pressure instrument, has an application as a flow meter for gas flow measurement. In so using it, there are various methods of square law compensation. Three typical ones follow.

Electroflo Meters Co. Ltd. suspend from one side of the ring one of their resistance units. As the ring rotates, the unit rises and falls in a mercury chamber, the action altering the circuit resistance or conductance as in their normal flow meter. By suitably contouring the contact ends of the rods, linear scales may again be obtained on the recording or indicating instruments.

Bailey Meters & Controls Ltd. utilize a special shaped displacer moving up and down in a cylindrical mercury chamber. Here the rotating force due to the differential is balanced by the buoyant force due to the displacer in the mercury. In addition, a shaped cam on the perimeter of the ring operates a follower on the pen arm assembly. By so contouring the cam, the combination of this and the displacer may be made to give a straight line scale.

In the George Kent version of the ring balance, the restoring weight M is suspended over a shaped cam surface.

Provided the angle of rotation is relatively small, e.g., less than 30°, the introduction of the cam results in a linear flow rate scale.

POINTS REGARDING THE INSTALLATION OF MEASURING INSTRUMENTS

For satisfactory measurement certain precautions must be taken in the installation of measuring instruments for differential pressure flow devices. The general aim must be to avoid conditions which would set up false differential pressures. Except in the case where sealing chambers are used, the fluid in the connecting pipes between the orifice etc. and the measuring instrument must be homogenous and any unwanted gas, liquid or solid, as the case may be, must be removed from the system by air vents, moisture sumps or similar devices. The two pipes involved must be at the same temperature, and it may be necessary to resort to lagging to avoid influences of external radiation and cooling.

It should be noted, in addition, that the conditions of *Fig.* 24 of Chapter I apply to U-tube pattern instruments when used with flow devices. As indicated in *Fig.* 79 in whatever manner the pipes are run there will always be a vertical head of liquid—above the instrument zero to influence the measurement. Formulae (26) and (29) of Chapter I apply. Similar formulae are involved when the instrument is mounted above the differential pressure device.

When metering liquids and steam, air and gas locks must not be formed, and when metering air and gases, water or liquid locks must be avoided. This normally means that piping must be run at a minimum slope and may have to include certain other arrangements. A brief description of normal practice follows.

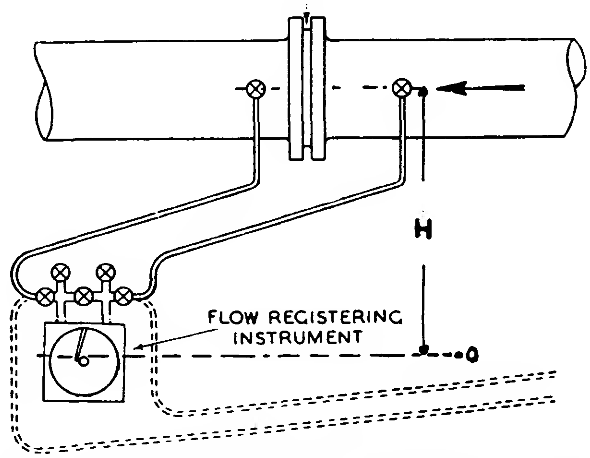
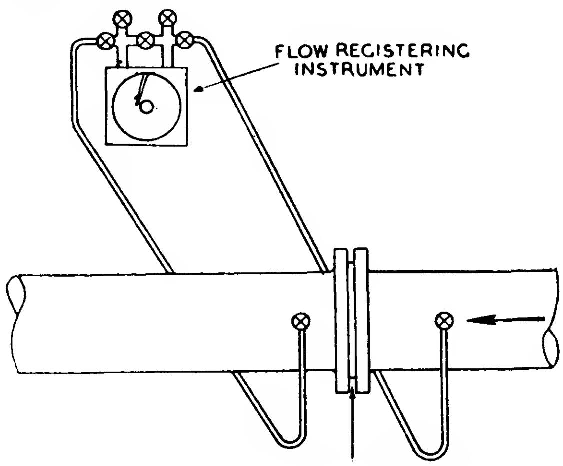


Fig. 79. *Measuring instrument mounted below flow element (liquids and steam)*

Fig. 80. *Measuring instrument mounted above flow element (liquids and steam)*



Liquids

The measuring instrument should preferably be below the flow detecting element, and the pipes should fall gradually to the instrument at a slope of not less than 1 in 20 (*Fig.* 79).

When the horizontal distance between the flow element and measuring instrument precludes this minimum slope, or where existing work obstructs this pipe run, the pipes should fall to a point below the instrument, and then rise up to it (*Fig.* 79, dotted).

When the measuring element is situated above the flow element, the pipes should dip down from the element and then rise gradually to the instrument at a slope of not less than 1 in 20 (*Fig.* 80). When the horizontal distance between the flow element and measuring instrument precludes this minimum slope, or where existing work obstructs the pressure transmission pipe run, these should be run to a point above the instrument and then fall to it, air collecting vessels with release valves being fitted at the highest point (*Fig.* 81).

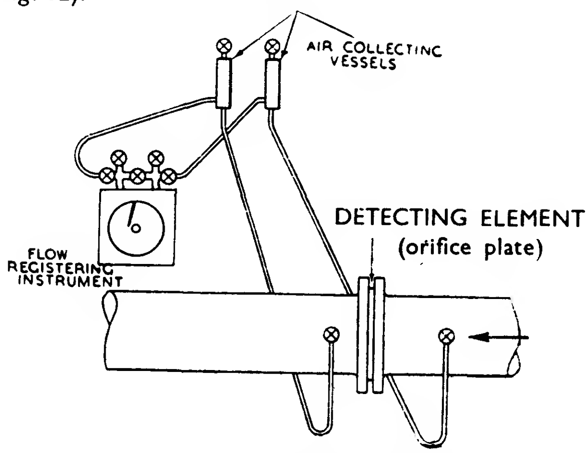


Fig. 81. *Measuring instrument mounted above flow element. Piping slope less than 1 in 20 (liquids and steam)*

Air and Gases

The measuring instrument is preferably mounted above the flow element, and the pipes should rise gradually to the instrument at a slope of not less than 1 in 20. When the horizontal distance between the two elements precludes this minimum slope, or existing work obstructs the pipe run, the pipes should be run to a point above the instrument and then fall to it (*Fig.* 82).

If the instrument is mounted below the flow element, the pipes should first rise and then fall gradually to the instrument at a slope of not less than 1 in 20. When the horizontal distance between the two elements precludes this minimum slope, or where existing work obstructs this pipe run, the pipes should fall to a point below the instrument and then rise to it, moisture sumps with drain valves being fitted at the lowest points (*Fig.* 83)

Steam

The pipe runs are the same as for liquid meters. Condensation or cooling chambers should be as close to the primary element as possible. When the main is vertical, or inclined at an angle to the horizontal, both cooling chambers must be fixed at the same level, e.g., if the flow is vertically downwards the lower pressure cooling chamber is brought up to the level of the higher pressure vessel. A large-bore pressure transmission pipe must be used for this purpose, and it must

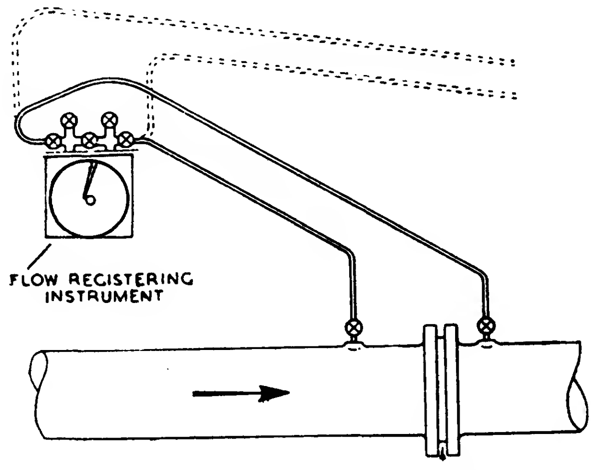


Fig. 82. *Measuring instrument mounted above flow element (air and gases)*

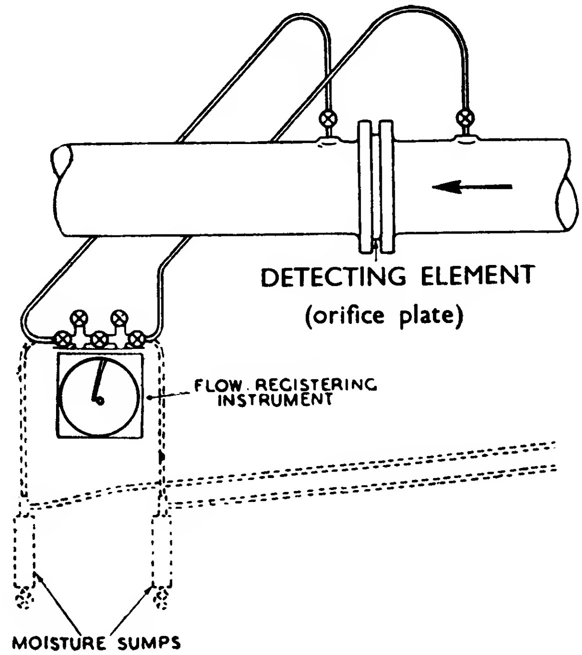


Fig. 83. *Measuring instrument mounted below flow element (air and gases)*

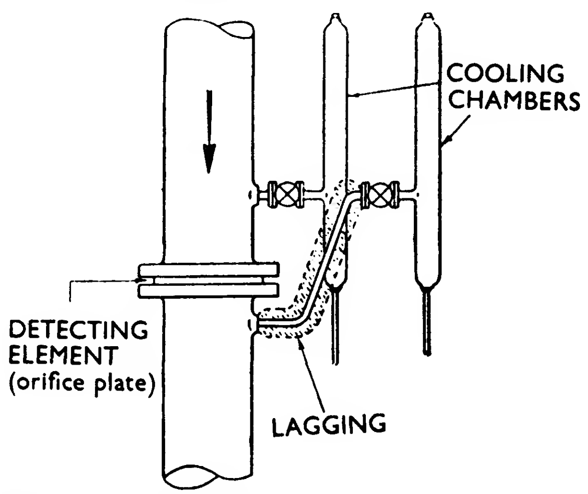


Fig. 84. *Non-horizontal flow for steam. Condensation chambers*

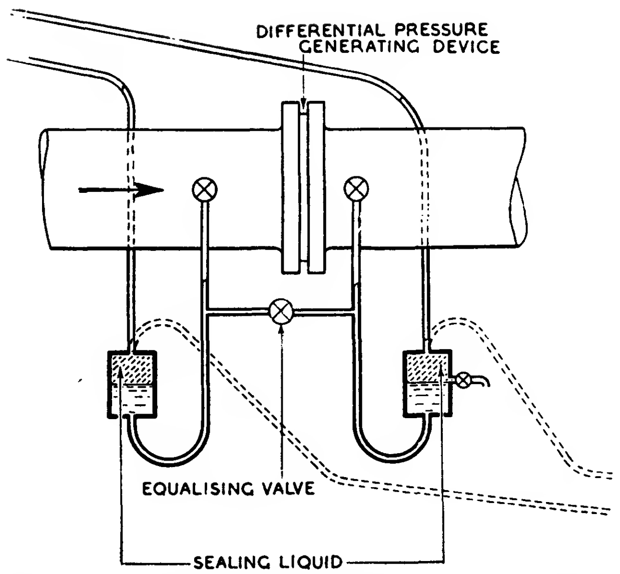


Fig. 85. *Seals. Sealing liquid lighter than metered liquid*

be lagged (preferably to the main) (*Fig.* 84). By this means, false differential pressures, due to the difference in density of the fluid in the main and the fluid in the cooling chambers and pressure transmission pipes, are avoided. Both pressure transmission pipes should be run adjacent to each other so that they are subjected to the same radiation losses.

Sealing Chambers

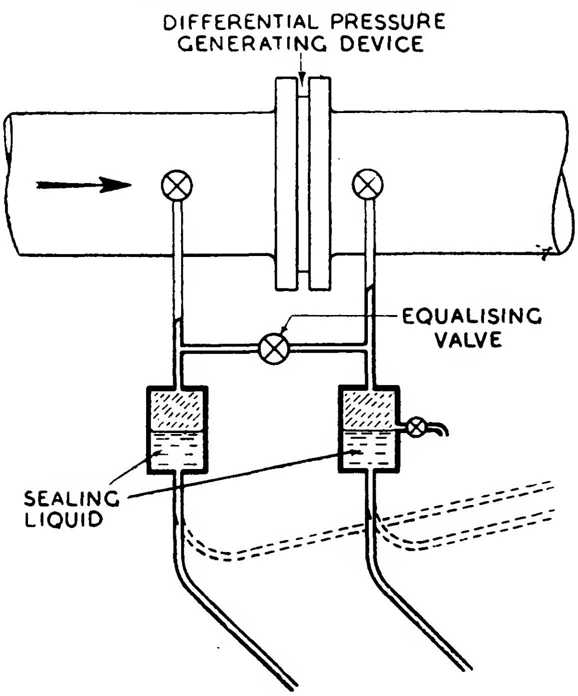
With this system for use with corrosive fluids the measuring instrument is isolated from the differential pressure element by a liquid which is not attacked by the metered fluid and which, in its turn, will not attack any part of the metering system. Oil is frequently used for this purpose. A chamber of the appropriate volume must be mounted immediately adjacent to each pressure tapping on the differential pressure element. An equalizing valve should be fitted between the two chambers. The chambers are sometimes fitted with gauge glasses, which facilitate filling the system and inspection. Alternatively, an overflow plug or cock may be fitted.

Fig. 85 illustrates the arrangement when the metered fluid is lighter than the sealing liquid, and *Fig.* 86 the reverse.

When a sealing liquid is used, it is not possible to vent the metering system extraneous to the detecting element.

The use of these chambers may require the introduction of a correction factor to the measurement.

Fig. 86. *Seals. Sealing liquid heavier than metered liquid*



INTEGRATION FROM DIFFERENTIAL PRESSURE INSTRUMENTS

So far in describing the measurement of such physical factors as pressure, liquid level and fluid flow, indication or recording of the particular factor has been considered. With flow, however, a need for another type of measurement arises. In many plants, fluid is continuously flowing through pipes and it is essential to keep a check on the *total* quantity passing in a given period, e.g., an hour or a day. A particular example is the flow of feed water to a boiler, and the flow of generated steam away from the boiler to the turbines, in a power station. The knowledge of these values over a day is useful in assessing the performance of the steam generating plant. The instruments performing the totalizing operation are known as integrators. They are mainly either mechanical or electrical but electronic versions are available. Typical examples only of these classes will be dealt with.

Integrators

Planimeters

The planimeter enables the integration or totalizing to be carried out away from the flow meter, if the flow has been recorded. The instrument is very familiar, but a brief recapitulation of it is given here. Basically, it consists of two arms pivoted on each other by means of a bearing. In one arm is contained a record wheel and indicating pointer, and, at the extreme end, a tracing point. At the end of the other arm is a similar point termed the pole. In applying the planimeter to flow integration, the pole would be placed at the centre of the chart, and the tracing point made to follow the ink record. In so doing, the recording wheel makes contact with the chart, and rotates, the rotation over the whole ink record being a measure of the total flow over the period for which the flow meter was recording. Special versions exist for dealing with square law charts.

The method has some disadvantages, chief amongst which is that the integration cannot be carried out until the chart is removed from the instrument, i.e., the total flow cannot be seen at a glance at any time during the day; the accuracy of the operation depends to some extent on the skill of the planimeter operator; and the method involves a recording when possibly only integration is desired. An integrator directly linked to the flow measuring instrument is employed in the majority of applications.

Mechanical Integrators

The problem resolves itself into the addition of varying flow rates over varying time intervals. For example, if steam flows at the rate of 5000lb/min for five minutes, at 6000lb/min for 20 minutes, and at 4500lb/min for 35 minutes, at the end of an hour the integrator must show a total of 302500lb of steam.

In other words, the product of the flow rate Q and time t must be continuously integrated or summated.

Note that the problem of the square law relation between differential pressure and flow rate arises again.

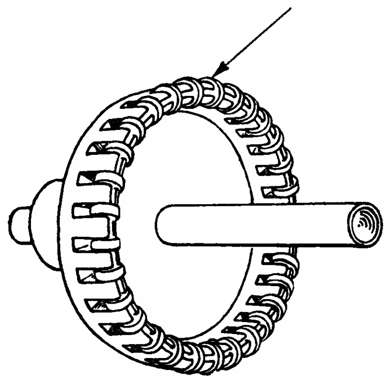
For example, in uncorrected instruments of the U-tube type the float movement will be proportional to the differential pressure and hence proportional to Q^2. If a direct link is made between the float and the counter the latter will integrate Q^2 and not Q. Some correcting device must, therefore, be interposed in such cases between float and integrator. Typical methods have been discussed under Square Law Correction.

The counting part of the integrator is invariably a mechanism of the mechanical digital or counter pattern, although other digital methods are possible. The methods of operating the counter from the basic unit are worthy of attention. With this type of device, two possible ways of operation exist. The counter shaft may be driven at varying rates of revolution proportional to the flow rates, or it can be driven at constant speed but operated for varying periods according to the flow rate. For example, the shaft would be at rest for zero flow, continuously rotating for maximum flow, and operated and at rest for equal periods with 50 per cent flow. Below 50 per cent the operated period would be less than the non-operated in proportion to the flow, and above 50 per cent the operated period would exceed the non-operated, again in proportion to the flow. Both types of mechanism are used in practice. Only one example of each will be described.

Continuously Operated Mechanical Integrators

If a disc is rotated at constant speed, the linear velocity of a point on a radius increases in direct proportion as one proceeds from the centre of the disc to the circumference. A small wheel placed in contact with the disc, its plane at right angles to the latter, will be rotated at rates of revolution proportional to its movement along a radius, assuming no slip. Now suppose the radial movement of the wheel is made

Fig. 87. *Planimeter wheel for disc pattern integrator*



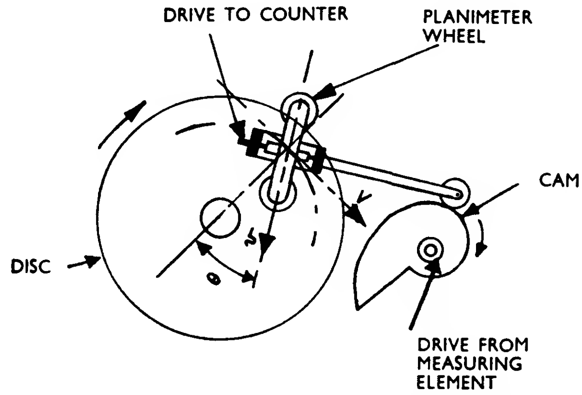


Fig. 88. *Simplified diagram of disc pattern integrator*

directly proportional to the rate of flow, by transmitting its revolutions to a counter a form of integrator is to hand.

A simplified version of an industrial design is shown in Fig. 87. The small wheel is here termed a planimeter wheel. It will be seen that this wheel is fitted with a number of serrated wheels round its circumference which are free to rotate in a plane at right angles to the plane of rotation of the planimeter wheel.

The flowmeter mechanism rotates a cam, such that the angular displacement of this is proportional to the differential head. In contact with the edge of the cam is a roller mounted on a pivoted arm which, as it deflects, causes the planimeter wheel to rotate about its point of contact with the clock-driven disc. Fig. 88 illustrates the operation of a disc pattern integrator. Denoting the tangential velocity of the disc at its point of contact with the planimeter wheel by V and the angular displacement of the plane in which the planimeter wheel rotates from a plane passing through both the centre of rotation of the disc and the point of contact between it and the planimeter wheel by θ, then $v = V \sin \theta$, where v is the tangential velocity of the planimeter wheel. Since the value of V is constant, v is directly proportional to $\sin \theta$. By cutting the cam to a profile which results in $\sin \theta$ beirg directly proportional to the square root of the differential head, the value of v is made directly proportional to the rate of flow.

Through a train of gears, the revolutions of the planimeter wheel are transmitted to the dials of the counter, and these dials register the total quantity passed, in any desired units.

Intermittently Operated Mechanical Integrators

We have seen how the contoured float of Bailey Meters & Controls' flow meter positions a pen arm giving a linear flow rate-movement relation. On the pen assembly is now pivoted an additional member termed the roller arm, because at the extreme right-hand end is a roller which acts as a follower to a heart shaped cam (*Fig.* 89). The cam is driven by a synchronous motor at two revolutions per minute, and the roller is kept in continuous contact by the weighted opposite end of the arm. The roller arm carries a pin which engages with a pawl. The pawl, in turn, mates with a toothed escape wheel driven by means of a friction clutch from the heart shaped cam. The escape wheel is geared to the counter. As long, therefore, as the

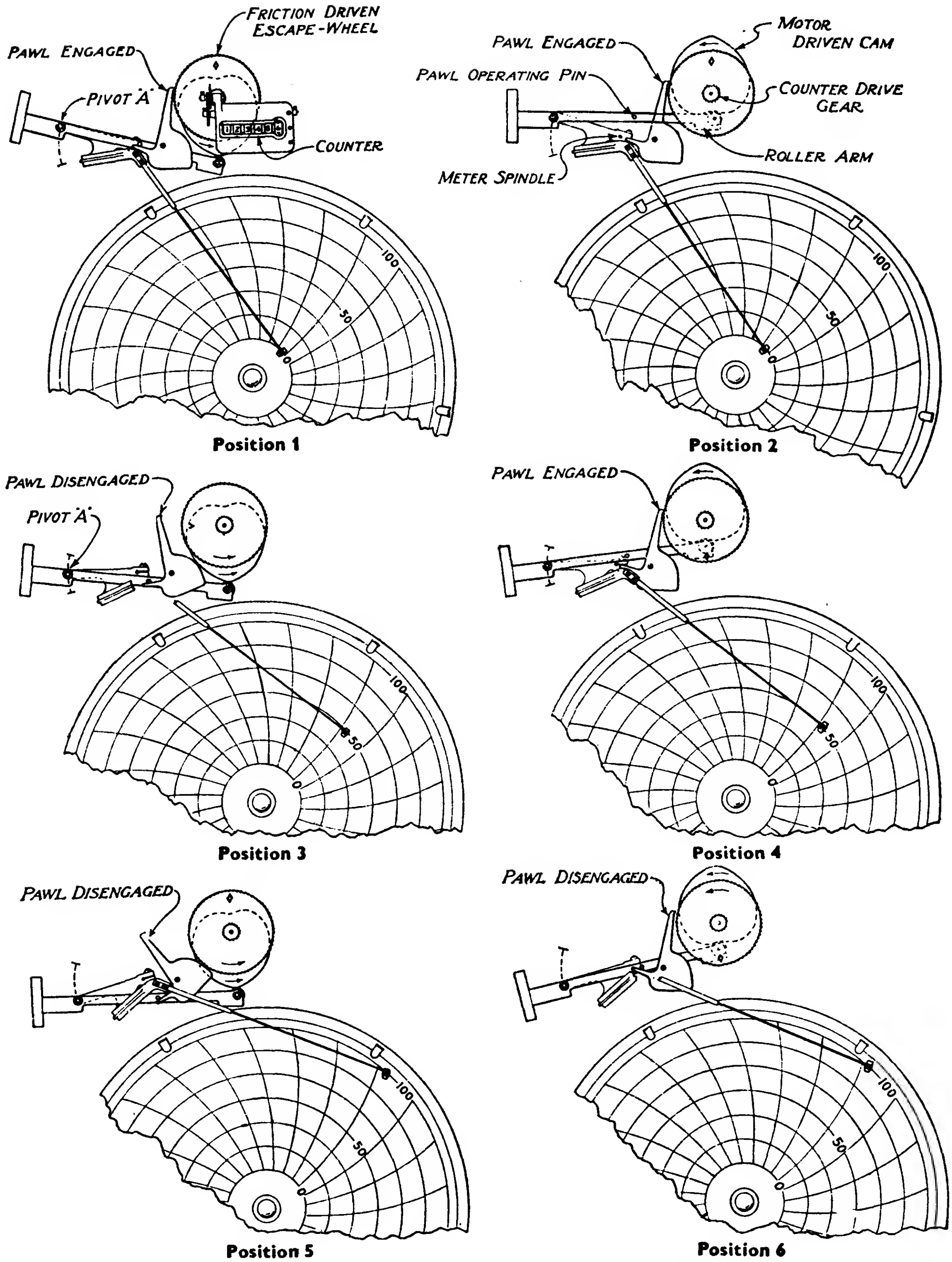


Fig. 89. *Operation of intermittent escapement pattern integrator*

wheel is rotating the counter is operated, and when the wheel is not rotating, the counter is at rest. At zero flow, the counter will not be operated, and at maximum flow it will be continuously operated. With intermediate flows, the periods of operation and non-operation must be adjusted proportionately. The method of doing this is as follows:

The pawl is normally pivoted so that it falls towards and engages the escape wheel. The roller is raised and lowered by the cam as it rotates, but at zero flow the pin on the arm never makes contact with the pawl, due to the position of the arm pivot. The pawl, therefore, is continuously engaged with the wheel and prevents its rotation by means of the slipping clutch (Positions 1 and 2). At maximum flow, on the other hand, the pivot position of the arm is at the opposite end of its arc, and although the pin now engages the pawl and moves it to and fro during the raising and lowering of the arm by the cam, it never allows the pawl to engage the escape wheel. The result is that the latter is

rotated continuously by the friction clutch from the cam and, hence, the counter is in continuous operation (Positions 5 and 6). Now what happens at an intermediate flow? Let us consider a value of 50 per cent maximum. The arm pivot is at the half-way position in the arc, and during one half revolution of the cam the pin engages the pawl, allowing the escape wheel to rotate freely (Position 3). During the other half revolution, however, the pin does not engage the pawl, so that the latter falls towards the wheel and prevents it rotating (Position 4). Thus, the counter will be operated one half the period it was at maximum flow, giving an equivalent integration. At any other flow, it can be seen that the positioning of the pivot of the roller arm will lead to proportionate on and off engagements of the pawl with the wheel, and consequently, proportionate operation of the counter.

Integration is carried out at the rate of four times per minute.

Electrical Integrators

The normal a.c. electrical watt hour or kilowatt hour meter, such as is installed in the average home using electricity, suggests a means for a continuous electrical flow integrator.

The instrument possesses two operating coils or magnets, one connected to the circuit voltage and the other carrying the circuit current. If a metal disc is placed in a given manner in the field of these two coils, eddy currents are induced, and the inter-action of the fluxes due to the two coils and the eddy currents produces a driving torque T_D which rotates the disc.

T_D is given by,

$$T_D \propto V.I.\cos\phi \qquad (80)$$

where V = the circuit voltage
I = the circuit current
ϕ = the phase angle between V and I.

Also equipped on the meter is a permanent magnet between whose poles the disc rotates. This magnet produces a braking or retarding torque T_B which is proportional to the rate of revolution N of the disc.

$$T_B \propto N \qquad (81)$$

When $T_D = T_B$ a steady rate of revolution is produced and

$$N \propto V.I.\cos\phi \qquad (82)$$

Over a period of time t_1 to t_2,

$$\int_{t_1}^{t_2} N.\,dt \propto \int_{t_1}^{t_2} V.I.\cos\phi.\,dt \qquad (83)$$

If the value of the current I is now made directly proportional to Q the flow rate

$$I = k.Q \qquad (84)$$

where k = a constant.

Hence,

$$\int_{t_1}^{t_2} N.\,dt \propto \int_{t_1}^{t_2} V.\,k.\,Q.\cos\phi.\,dt \qquad (85)$$

Gearing the disc, therefore, to a normal type of counter will give us a means of continuous electrical flow integration, since N will be a direct measurement of Q. It will be apparent, however, that the effect of variations in circuit voltage V and the supply frequency must be reduced to a minimum, otherwise errors will be introduced into the measurement.

In this pattern of electrical integrator it is arranged that the potential or voltage magnet is made to govern

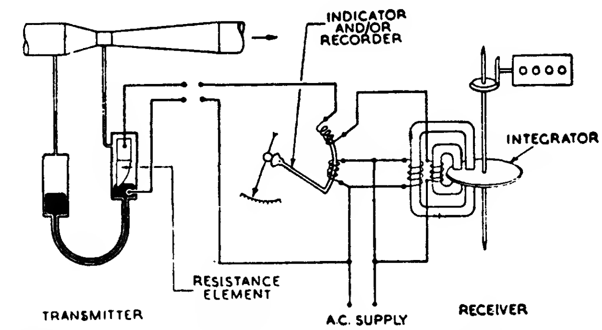


Fig. 90. *Electrical watt-hour pattern integrator*

both the driving torque T_D and the braking or retarding torque T_B, so that a voltage change has the same effect on each. For frequency compensation, a simple network is incorporated in the integrator.

Fig. 90 indicates in a simple fashion the operation of the integrator.

Electronic Integrators

A type of electronic integrator will be described in Chapter 7 in the section on turbine pattern flowmeters.

VARIABLE AREA METERS

Let us revert to one of the fundamental equations for a turbulent fluid flow through an orifice.

Here:

$$Q = C.E.A_2\sqrt{\frac{2gp_d}{\rho_1}} \qquad (86)$$

where Q = the rate of fluid flow
C = the discharge coefficient
E = the velocity of approach factor
A_2 = the orifice area
g = the gravitational acceleration
p_d = the pressure differential across the orifice $(p_1 - p_2)$
ρ_1 = the fluid density.

We have proceeded, hitherto, on the basis that the orifice area A_2 is fixed, and the pressure differential p_d varies with the flow rate Q. Equation (86), however, suggests a possible alternative by keeping the p_d constant and adjusting the orifice area A_2 in proportion to Q. Equation (86), again, lays no restriction as regards the shape of the orifice. For example, we could have an annular pattern formed by the space between a solid disc and the inside of a pipe. Equation (86) would still apply, the values of C and E, of course, being different from the concentric orifice case. Developing the idea still further, consider a vertical tube of conical shape, the area gradually expanding from the bottom to the top. In the tube is a fluid flowing in an upward direction, and in it is placed a disc, free to move, so that it acts as a sort of float in the fluid. An orifice is set up between the disc and the inside surface of the tube, and a pressure drop exists across it. Certain forces act on the disc and these are in equilibrium when it is at rest. A change in flow will affect the pressure drop, altering the relation between inlet and outlet pressure thus upsetting the equilibrium of the forces acting on the disc. The disc will then move up or down the tube adjusting the area of the orifice (due to the conical shape of the tube) until the pressure drop is at the original value, when the

forces are again in equilibrium. The position of the float in the tube is then a measure of the rate of flow. An elementary theory of the variable area meter follows. It should be understood that this is for explanation purposes only, as a very simple equation expressing the operation of the meter is not possible. What are the forces acting on the float in the vertical column of liquid? They are:

(1) The *effective* weight X of the float in a downwards direction.

$$X = V_f(\rho_2 - \rho_1) \qquad (87)$$

where V_f = the float volume
ρ_2 = the float material density
ρ_1 = the fluid density.

(2) The total pressure, Y, acting in a downwards direction on the upper surface of the float

$$Y = p_2.A_f \qquad (88)$$

where p_2 = the pressure per unit area on the upper surface A_f of the float

(3) The total pressure, Z, acting upwards on the lower surface of the float

$$Z = p_1.A_f \qquad (89)$$

Viscous effects are neglected for the time being.

For equilibrium, the upwards acting forces must balance those acting downwards, and

$$Z = X + Y \qquad (90)$$

$$p_1.A_f = V_f(\rho_2 - \rho_1) + p_2.A_f \qquad (91)$$

Imagine that the flow increases, and that the float does not move immediately. An increased differential $(p_1 - p_2)$ results, and the ratio of p_1/p_2 is increased. This means that the force $p_1.A_f$ is now greater than $V_f(\rho_2 - \rho_1) + p_2.A_f$. Since the float is free, however, it will be moved in the direction that $p_1.A_f$ is acting, i.e. upwards. As it moves upwards it increases the orifice area due to the expanding sectional area of the tube, and the pressure differential falls proportionately. The operation continues until $(p_1 - p_2)$ reaches its original value, when the forces, as indicated in equation (90), are in equilibrium again. The new float position must now be a measure of the increased flow value. The operation is reversed on a decrease in rate of flow occurring.

From (91)

$$(p_1 - p_2) = \frac{V_f}{A_f}(\rho_2 - \rho_1) \qquad (92)$$

Substituting in (86)

$$Q = C.E.A_2\sqrt{2g.\frac{V_f}{A_f}\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)} \qquad (93)$$

If the tube is a conical one,

$$D_t = D_i + 2x\tan\frac{\theta}{2} \qquad (94)$$

where D_t = the tube diameter at a distance x from the inlet of the tube
D_i = the inlet diameter
x = distance or height of float from inlet
θ = the cone angle.

Letting D_f the float diameter equal D_i, and neglecting values $x^2\tan^2\frac{\theta}{2}$, as a first approximation,

$$Q = K.C.E.x\sqrt{2g.\frac{V_f}{A_f}\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)} \qquad (95)$$

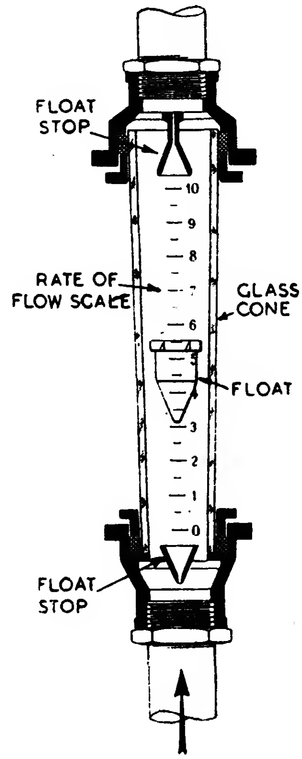


Fig. 91. Diagram of glass body variable area meter

where K is a dimensional constant involving D_i, $\tan\frac{\theta}{2}$, etc. The velocity of approach factor E is without significance, so that (95) can be reduced to

$$Q = K.C.x\sqrt{2g.\frac{V_f}{A_f}\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)} \qquad (96)$$

If it is desired to measure weight flow (e.g. lb/hour) instead of volume flow, since W = $Q\rho_1$,

$$W = K.C.x\sqrt{2g.\frac{V_f}{A_f}(\rho_2 - \rho_1)\rho_1} \qquad (97)$$

These two equations are produced for a particular purpose.

Density Variation and Compensation

The density of a fluid can vary, e.g. with temperature changes, and errors will be introduced into the flow measurement. The force X will be affected since

$$X = V_f(\rho_2 - \rho_1) \qquad (98)$$

This means that the equilibrium of the float will be upset, and it will change to a new position corresponding to the new value of X. Errors will thus be introduced into the flow measurement. We require, therefore, that the change of rate of flow with density be zero. Expressed mathematically,

$$\frac{dQ}{d\rho_1} \text{ or } \frac{dW}{d\rho_1} = 0 \qquad (99)$$

Differentiating (96) with respect to ρ_1, and equating the result to zero, produces the condition that for complete density change immunity, ρ_2 must be infinite. Practically, of course, this cannot be achieved, but the density of the float may be made very large, i.e. many times that of the fluid. This results in reducing the density errors to reasonable proportions.

Differentiating (97) with respect to ρ_1, and equating result to zero, produces the following condition:

$$\rho_2 = 2\rho_1 \quad . \quad . \quad . \quad (100)$$

For liquids, this condition may be achieved practically by making the body of the float either hollow or of solid plastic material. 10 per cent density variations from a mean value do not introduce any significant errors into the flow measurement.

Fig. 96 shows the essential features of a glass tube variable area flow meter with a normal float.

The establishment of two different density compensation conditions may seem a little surprising at first, but it will be apparent that the fluid density, ρ_1, enters into flow measurement in a different manner for volumetric and weight scales, if equations (96) and (97) are examined. ρ_1 appears as a divisor in (96) and as a multiplier in (97). It follows, therefore, that the conditions of compensation will be unlike for the two cases.

Viscous Flow

At low flow rates, laminar or viscous flow conditions may exist. The force of viscous drag must now be taken into account since this is a function of length L_f along the direction of the flow and viscosity η of the fluid. Highly complex theoretical equations have been worked out to explain the action under conditions of viscous flow. (See for example ref (1)).

For reference purposes these are given below:

$$Q = \frac{V_f . g . (\rho_2 - \rho_1)\,(D_t - D_f)^3\,(D_t + D_f)}{24(D_t^2 + D_f^2)L_f\eta} \quad . \quad (101)$$

or

$$W = \frac{V_f . g . \rho_1(\rho_2 - \rho_1)(D_t - D_f)^3(D_t + D_f)}{24(D_t^2 + D_f^2)L_f\eta} \quad . \quad (102)$$

From an examination of either equation the flow is now dependent on the value of η the viscosity of the fluid under measurement in addition to the density ρ_1. We have seen how to compensate for density variation, and it now becomes equally important to render the instrument immune from viscosity changes. In short, we must reduce the effect of viscous drag force D to a minimum. Since this is dependent on the effective length L_f of the float, the reduction of this length should result in a lessening of the viscosity effect. Practically, therefore, the floats take the form shown in *Fig.* 92 or *Fig.* 93. The effective orifice part of the float has been reduced to a sharp edged disc, i.e. with the minimum length along the direction of flow. In yet a third design of float, the "body" is removed to a position above the float out of main flow stream, a thin conical shell acting as the float itself.

Dimensional Analysis

Due to the somewhat complex nature of the variable area meter, it has been found more satisfactory to adopt the method of dimensional analysis in assessing its operation. Certain functions such as $\frac{\sqrt{X . \rho_1}}{\eta}$ and $\frac{Q . \rho_1}{D_f . \eta}$ may be determined from experiments for various values of D_t/D_f. Plotting one against the other, the resulting curves provide more accurate information than the formulae based on theoretical reasoning such as (96), (97), (101), or (102) above. The steps heading to the adoption of such parameters are beyond the scope of this course, but may be obtained from ref (1).

Types of Variable Area Meters

Three typical types are described below.

Glass Type

The basic feature of this type of meter is the conical section glass tube. For accuracy, the diameter of this must be maintained at very close limits. Clear borosilicate glass is used which is highly resistant to thermal shock and chemical action, and the method of its manufacture enables tolerances of 1/10 000 of an inch to be observed. The use of glass introduces the question of a safe working pressure for the fluid being measured. At present this is about 500lb/in^2 and applies to the smaller diameter tubes. For larger sizes the safe working pressure falls from this figure. (The normal diameters range from 2mm to 60mm depending on the flow to be measured.) The tube is normally clamped in a metal frame, the inlet and outlet being sealed into connections as required, e.g. flanged or screwed. Where danger may occur from flying glass resulting from a fracture of the tube, "Armour Plate" glass protection windows encase the instrument.

The standard float shape is indicated in *Fig.* 91, and is perfectly free. Viscosity immune floats, however, may demand a guide, as the float disturbs the equilibrium of the liquid. In one pattern, the guide is a central rod around which the float is made to rotate, so that visual evidence that the float is moving freely is obtained.

The glass tube type measures from 2cc/min up to 3000 litres/min of gas, and 0·5cc to 225 litres/min of liquid.

The pressure drop will depend on the type of float being used and the nature of the fluid, but varies between about 0·2cm (0·078in.) w.g. for small gas flows and 3·5cm (1·38in.) w.g. for liquid flows.

Metal Tube Types

For larger flows than the glass type tubes can accommodate, a conical metal tube pattern is introduced. Here, the metal body is of gunmetal, cast iron or stainless steel with a stainless steel float. The latter is carried on a rod which moves between two guides, one at the lower end and the other at the upper end of the tube. The guide rod passes through the upper part of

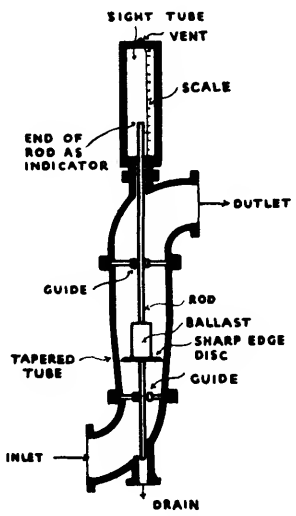


Fig. 92. *Diagram of metal body variable area meter*

the tube into a compartment with a glass scale, the end of the rod acting as an indicator (*Fig.* 92). This type of meter has typically ranges from 250 to 120 000 litres/min of gas flow and 20 to 7000 litres/min of liquid. The maximum fluid working pressure is 500lb/in². When used with opaque liquids, a compressed air supply may be connected to the top of the scale unit and the level of liquid depressed, so that a clear view of the indicator is obtained. Opaque liquids may also be metered by the high pressure version.

Meters for High Working Pressures

Where working pressures higher than 500lb/in² are encountered it is desirable to use a totally enclosed metal tube type of instrument with a magnetic coupling. In the float is embodied a magnet, and as this moves up and down it causes a follower magnet outside the metering tube to move with it. The follower magnet is fixed on to a pivoted pointer assembly, and the pointer is caused to travel over a vertical scale (*Fig.* 93). The metering tube is made of stainless steel, as is the float, and is usually enclosed in a gunmetal casing. This type of meter is suitable for working pressures up to 5000lb/in².

Typical flow ranges are from 30 to 120 000 litres/min of gas and 0·75 to 7000 litres/min of liquid.

There is an electronic version of the magnetic type developed by Rotameter Manufacturing Co. Ltd. in which the follower is linked to a co-axial electronic repeater. This delivers a signal of 0-30mA linearly related to the flow rate.

Plug Type Area Meter

Another form of area meter is the loaded plug type. This area meter is designed for incorporation in a pipeline and comprises a flanged body casting through which the liquid being metered passes. The flow of liquid through the body vertically displaces a metering plug, which can be either spring or weight loaded

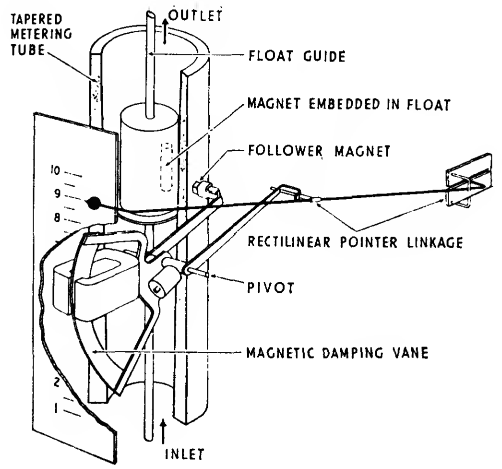


Fig. 93. *Magnetically coupled variable area meter (Rotameter Manufacturing Co. Ltd.)*

according to the nature of the liquid. The metering plug and port assembly is designed to constitute an orifice in the liquid channel whose area varies with the rate of flow in such a manner that a constant pressure drop is maintained. The position of the metering plug is directly proportional to the rate of flow. A differential transformer is employed to produce an electrical signal proportional to the displacement of the plug and to operate a recording or indicating device remotely situated.

Alternatively the plug movement may be coupled to a nozzle and flapper unit to give a pneumatic signal in linear relation to the flow rate.

In one typical pattern the maximum operating pressure is 600lb/in² and various ranges of liquid between 0 and 42 750lb/hr may be obtained using the appropriate weight or spring loaded model.

Books for further reading

LINFORD, A. Flow Measurement and Meters. Chapter 3. E. & F. Spon, 1961.

JONES, E. B. Instrument Technology. Vol. 1, Chapter 3. Butterworths, 1953.

ECKMAN, D. P. Industrial Instrumentation. Chapter 10. Chapman and Hall, 1950.

HOLZBOCK, W. G. Instruments for Measurement and Control. Chapter 4. Chapman and Hall, 1955.

MILLER, J. T. (Ed.) Instrument Manual, Section VI. United Trade Press, 1960.

Questions

1. In an emergency an industrial U-tube possessing a linear relation between float movement and pen movement is used with an orifice to determine the rate of flow in a pipe. If the range is 0-100 000lb/hr, calculate the errors at 70 000lb/hr and 50 000lb/hr.
   (Answer 21 000lb/hr and 25 000lb/hr.)

2. As this chapter has indicated, a flow measuring instrument may be mounted above or below the orifice, nozzle or Venturi tube. Deduce the relation between differential pressure and displacement of liquid in a U-tube mounted above the orifice. Use the nomenclature of *Fig.* 24, Chapter 1.
   (Answer $p_1 - p_2 = h_m (\rho_1 - \rho_2)$)

3. In a variable area meter the ratio of maximum flow to minimum flow on the scale is 10/1. If the ratio of the tube diameter to float diameter is 1·02 at minimum flow, what is it at maximum flow? Take orifice area as directly proportional to the flow rate.
   (Answer 1·185/1.)

4. With the same ratios as in Question (1), what are the maximum and minimum orifice areas using a 1in. diameter float?
   (Answer 0·0316in² and 0·316in².)

5. Using equation (91), if the float diameter is 1in., p_1 = 2·0lb/in², p_2 = 1·95lb/in², ρ_2 = 488lb/ft³, ρ_1 = 62·4lb/ft³, what is the float volume?
   (Answer 0·16in³.)

Chapter 6

FLOW MEASUREMENT IN OPEN CHANNELS

INTRODUCTION

In certain industries and municipal undertakings, the rate of flow of liquids in open channels is important. In so far as this chapter is concerned "open" means open to the atmosphere. In general, the devices used for measurement purposes fall into two classes: weirs and flumes. The first are virtually forms of dams and the second are constrictive devices. Both involve the production of a head of liquid proportional to the rate of flow. Formulae can be deduced theoretically, but for actual measurement these need modifying by a discharge coefficient dependent on several parameters. Conditions of installation can vastly influence the accuracy and from this aspect it is suggested that the reader consults the articles by R. G. West under ref (1) and B.S. 3680.

Let us examine the family of weirs first.

It is very convenient to think in terms of a head of liquid instead of a pressure as in the case of the differential devices of Chapter 4. If p is the static pressure due to a column of liquid of height H and density ρ, then since $p = H\rho$, $H = \frac{p}{\rho}$. In the case of weirs, it is constantly emphasized that the velocity of approach of the liquid to the weir should be kept low. The reason is that an additional factor equivalent to a velocity head $\left(\frac{V^2}{2g}\right)$ may be involved. Here V is the velocity of the stream of liquid and g is the gravitational acceleration. The effect will be shown for one type of weir, the rectangular.

Rectangular Type Weir

This type of weir comprises a metal plate with a rectangular slot cut in it in the manner of Fig. 94. The slot has sharp edges and the plate is installed at right angles to the direction of flow. The slot is narrower than the channel.

In most of the standard text books dealing with flow in open channels it is shown that the velocity of flow at a depth H in a stream is given by the simple formula:

$$V = \sqrt{2gH} \qquad \ldots \ldots \ldots \quad (103)$$

For our first example of weir we will consider the effect of including a velocity of approach factor $\frac{V_m^2}{2g}$. V_m is the mean velocity upstream of the weir.

Equation (103) then becomes

$$V = \sqrt{2g\left(H + \frac{V_m^2}{2g}\right)} \qquad \ldots \quad (104)$$

To obtain a flow rate Q it is necessary to consider an element dH at a depth H in the flow section as in Fig. 95. The flow through the section BdH is given by:

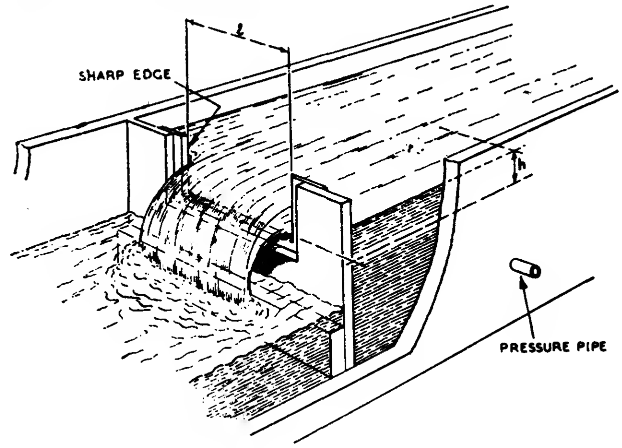
$$dQ = B\sqrt{2g\left(H + \frac{V_m^2}{2g}\right)}dH \qquad . \quad (105)$$

Integrating between O and H_1 to obtain the full flow, Q is found to be:

$$Q = \frac{2}{3} B \sqrt{2g}\left[\left(H_1 + \frac{V_m^2}{2g}\right)^{\frac{3}{2}} - \left(\frac{V_m^2}{2g}\right)^{\frac{3}{2}}\right] \quad . \quad (106)$$

Only if V_m is small can the term $\left(\frac{V_m^2}{2g}\right)$ be neglected.

Fig. 94. Rectangular type weir



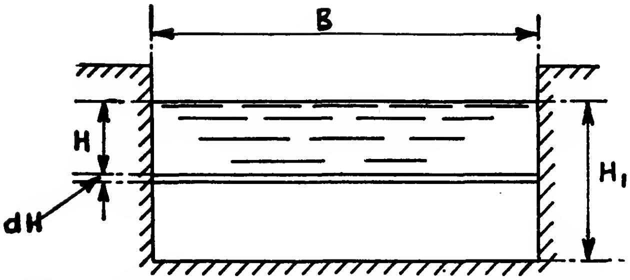


Fig. 95. Method of calculating rate of flow for a rectangular type weir

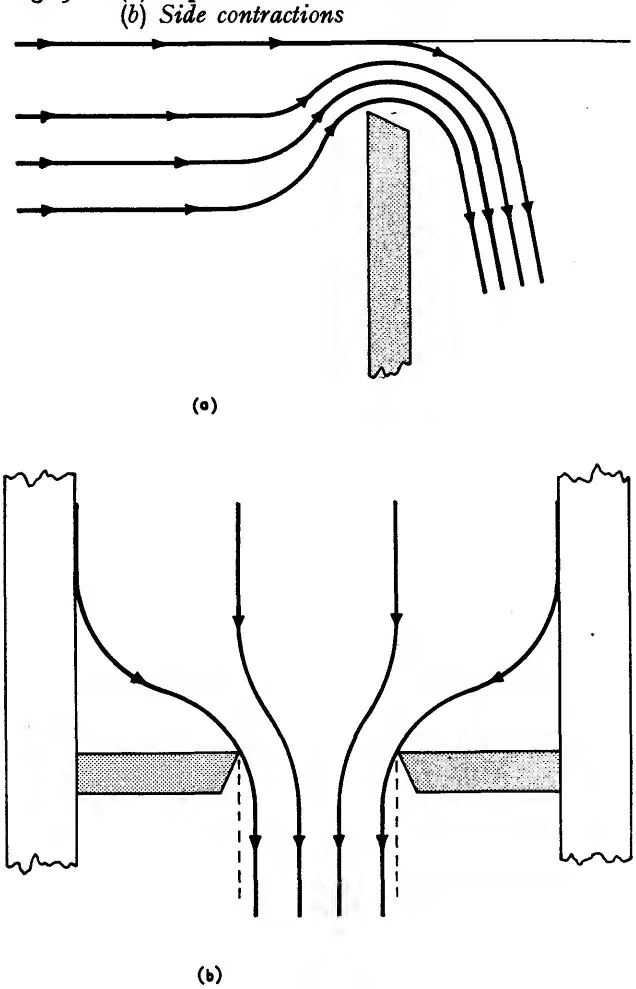
Then, (106) reduces to

$$Q = \frac{2}{3} B\left(\sqrt{2g}\right)H_1^{\frac{3}{2}} \quad . . . . \quad (107)$$

a basic formula which will be found in all relevant text books.

In such a weir, the flow suffers contraction. There is a top and bottom contraction as indicated in Fig. 96a and there is, in addition, a side contraction as shown in Fig. 96b where the jet width after it leaves the weir is less than the weir opening. These factors necessitate the inclusion of a discharge coefficient C. But this coefficient can also be influenced by the dimensions of the approach channel, the velocity of approach, the character of the upstream conditions and the pressure

Fig. 96. (a) Top and bottom contractions
(b) Side contractions



below the nappe and its derivation, therefore, is not a simple matter. With the inclusion of C, equation (107) becomes:

$$Q = \frac{2}{3} BC\left(\sqrt{2g}\right)H_1^{\frac{3}{2}} \quad . . . \quad (108)$$

Practical formulae bear a somewhat different form and a British standard specification, No. 599, "Code for Pump Tests", gives recommended practical formulae for the rectangular and other types of weirs described in this chapter.

One practical formula widely used for rectangular weirs is

$$G = 147{\cdot}9\,(B - 0{\cdot}1H_1)\,H_1^{\frac{3}{2}} \quad . . \quad (109)$$

where G = the rate of flow in gal/hr
B = width of weir slot in inches
H_1 = the head over the weir in inches

Observe that even this formula has a limited practicability, for when $B = 0{\cdot}1\,H_1$, the right-hand side of the equation becomes zero.

V-notch or Thomson Weirs

In this form of weir, a metal plate with a sharp edged V-notch cut in it is installed at right angles to the direction of flow as in Fig. 97.

The formula may be derived, as in the case of a rectangular weir, by considering an element dH at depth H and integrating over the flow section. The simple basic formula is:

$$Q = \frac{8}{15} \tan\frac{\theta}{2}\left(\sqrt{2g}\right)H_1^{\frac{5}{2}} \quad . . . \quad (110)$$

For similar reasons to those given for rectangular weirs, it is necessary to introduce a discharge coefficient C. Then

$$Q = \frac{8}{15} C \tan\frac{\theta}{2}\left(\sqrt{2g}\right)H_1^{\frac{5}{2}} \quad . . . \quad (111)$$

θ is normally between 35° and 120°.

Two notches commonly used are the 90° and the half 90° versions. In the latter, the area is half that of the 90° one. The corresponding formulae are:

90° notch, $G = 117{\cdot}1\,H^{2{\cdot}48}$ . . . . (112)
Half 90° notch, $G = 58{\cdot}6\,H^{2{\cdot}48}$ . . . . (113)

In both cases, G is the flow rate in gal/hr.

Trapezoidal or Cipoletti Weirs

In an effort to reduce the effect of side contraction the trapezoidal or Cipoletti weir is sometimes used. The sides are made sloping in the manner of Fig. 98. The angle θ is obtained from the relation $\theta = 2\tan^{-1}(\frac{1}{4})$ from which θ is approximately equivalent to 14°. A discharge coefficient is still necessary, however, to take care of other parameters and the relation between flow rate and head is given by:

$$Q = \frac{2}{3} CB\left(\sqrt{2g}\right)H^{\frac{3}{2}} \quad . . . . . \quad (114)$$

Suppressed Weirs

The suppressed weir comprises a plate with a sharp edge and extends the whole width of the channel. As its name indicates, it is entirely submerged or suppressed

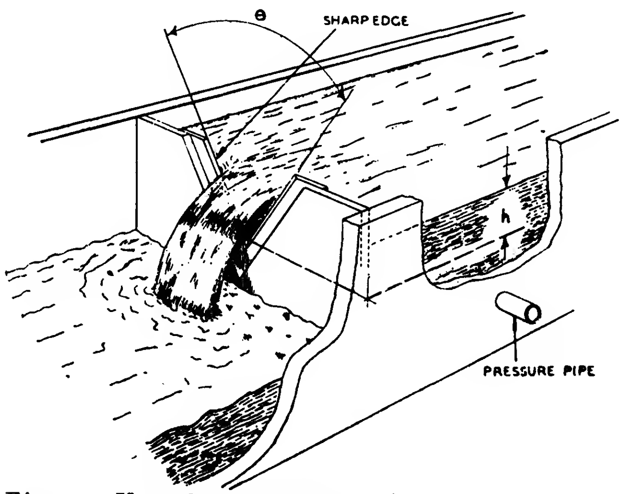


Fig. 97. *V-notch or Thompson weir*

beneath the surface of the liquid. The general arrangement is shown in Fig. 99. Obviously with such a design there is no side contraction effect. The derivation of a suitable formula is not easy and one commonly used is that due to Rhebock:

$$G = 45l\left(3{\cdot}23+0{\cdot}434\frac{H_1}{\beta}\right)\left(H_1+0{\cdot}0408\right)^{\frac{3}{2}} \quad (115)$$

where G = the flow rate in gal/hr
H_1 = head over weir in inches
l = length of crest of weir in inches
β = height of crest of weir above bottom of channel in inches.

The term $\left(0{\cdot}434\frac{H_1}{\beta}\right)$ in the first bracket is equivalent to a correction for velocity of approach. If β is large relative to H_1 the value of the term may be small enough to neglect it.

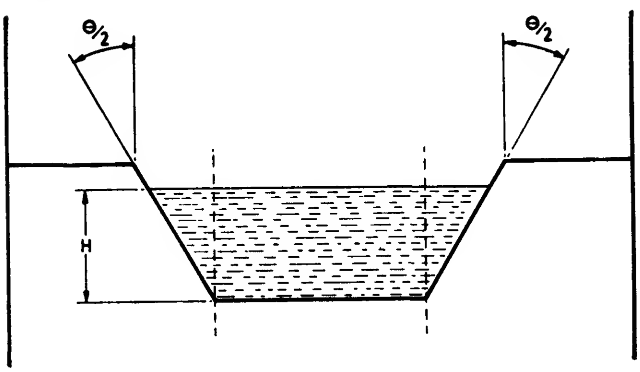
Other Types of Weirs

Another type of weir is the hyperbolic one in which the sides are shaped to produce a linear relation between flow rate and head. Compound weirs are employed in many cases where the installation of one particular type would not meet the operating conditions.

General Precautions to be observed in the Installation of Weirs

1. To assist in uniform velocity distribution, the upstream channel should be relatively wide, deep

Fig. 98. *Trapezoidal or Cipoletti weir*



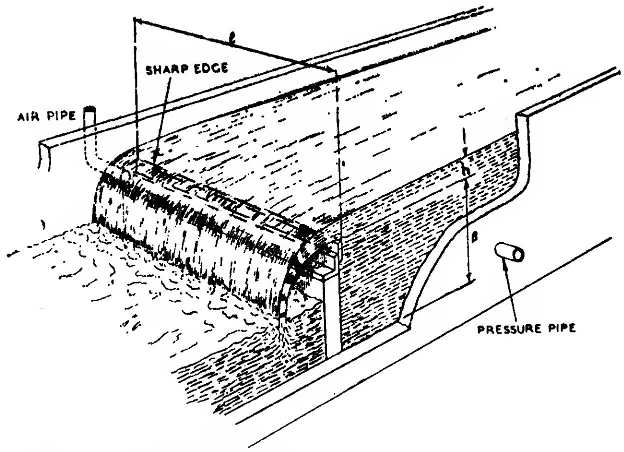


Fig. 99. *Suppressed weir*

and straight and of uniform cross-section. The use of baffles is sometimes recommended to break up currents in the stream. If installed, the nearest baffle should not be less than 10 times the maximum head from the weir plate.

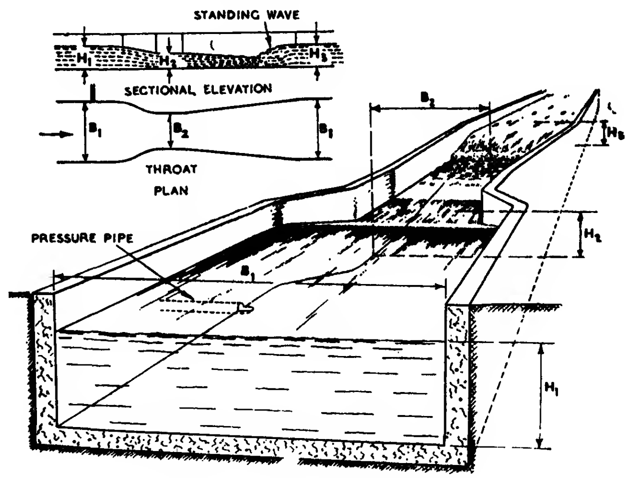


Fig. 100. *Rectangular section Venturi flume*

2. The upstream edge of the notch must be square and sharp.
3. A free overfall must be maintained. This means that the level of liquid on the downstream side of the weir plate must be some distance below the crest of the weir or bottom of a V-notch. The bed and the sides of the channel downstream should not be less than 6in. from the weir and the water level downstream should be at least 3in. below the weir sill.
4. The measurement of the head over the crest of the weir or bottom of a V-notch must be made at some distance upstream from the weir plate. If H is the maximum value of the head, then measurement should be made at a distance of not less than 6H from the upstream face of the weir.
5. The underside of the nappe of the weir must be at atmospheric pressure. This means aerating for suppressed weirs.
6. For suppressed weirs, the sides of the channel must be vertical, smooth and in a plane at right

angles to the plane containing the weir for some distance upstream and downstream of the weir.

7. The conditions of flow may necessitate the installation of a screen upstream from the weir to prevent the piling up of debris at the weir face.

Venturi Flumes

The Venturi flume is virtually a rectangular section Venturi tube with the upper surface removed. Thus we have the inlet contraction, throat and outlet expansion as indicated in Fig. 100.

If V_1 and V_2 are the velocities at the inlet and throat respectively, and H_1 and H_2 are the corresponding hydraulic depths,

$$V_1 = \sqrt{2gH_1} \quad \ldots\ldots\ldots\ldots \quad (116)$$

$$V_2 = \sqrt{2gH_2} \quad \ldots\ldots\ldots\ldots \quad (117)$$

From (116) and (117)

$$H_1 - H_2 = \frac{V_2^2 - V_1^2}{2g} \quad \ldots\ldots \quad (118)$$

If Q is the rate of flow of the liquid through the flume

$$Q = V_1 B_1 H_1 = V_2 B_2 H_2 \quad \ldots\ldots \quad (119)$$

where B_1 and B_2 are the inlet and throat widths respectively.

From (119),

$$V_1 = \frac{V_2 H_2 B_2}{H_1 B_1}$$

Making the necessary substitutions in (118) and introducing a discharge coefficient C,

$$Q = C B_2 \sqrt{2g}\, H_2 \sqrt{H_1 - H_2} \sqrt{\frac{1}{1 - \left(\frac{B_2 H_2}{B_1 H_1}\right)^2}} \quad (120)$$

If $\frac{B_1}{B_2}$ is represented by E and $\frac{H_2}{H_1}$ by x, equation (120) becomes

$$Q = C B_2 \left(\sqrt{2g}\right) H_2 \sqrt{H_1 - H_2} \sqrt{\frac{E^2}{E^2 - x^2}} \quad (121)$$

Equation (121) is not a particularly practical one. It is possible to make the value of E sufficiently large so that the last term becomes substantially constant over a fairly wide range of flow rates. But one is left with an equation which then takes the form

$$Q = K H_2 \sqrt{H_1 - H_2} \quad \ldots\ldots \quad (122)$$

$$\text{where } K = \left[C B_2 \sqrt{2g} \sqrt{\frac{E^2}{E^2 - x^2}}\right]$$

Thus the flow rate is proportional to $\left(H_2 \sqrt{H_1 - H_2}\right)$

The evaluation of the product $(H_2 \sqrt{H_1 - H_2})$ by an instrument mechanism is possible, but the device is neither simple nor cheap. The establishment of the so-called "free" flow or discharge conditions, however, enables the measurement to be much simplified.

Free Flow or Free Discharge Conditions

It may be shown mathematically that when the value of $\left(H_1 + \frac{V_1^2}{2g}\right)$ exceeds the value of $\left(H_3 + \frac{V_3^2}{2g}\right)$ by a specified amount the velocity V_2 in the contracted part of the flume reaches a maximum or critical value. In fact, V_2 then equals $\sqrt{g H_2}$. If this value is substituted in equation (118).

$$H_1 + \frac{V_1^2}{2g} = \frac{3H_2}{2} \quad \ldots\ldots\ldots \quad (123)$$

$$\text{From which, } V_1^2 = 2g\left(\frac{3}{2} H_2 - H_1\right)$$

$$\text{and } V_1^2 = 2g H_1 \left(\frac{3x}{2} - 1\right) \quad \ldots \quad (124)$$

$$\text{Since } V_1 B_1 H_1 = V_2 B_2 H_2,\ V_1^2 = gxH_1 \left(\frac{x}{E}\right)^2$$

From these equations

$$x^3 - 3xE^2 + 2E^2 = 0 \quad \ldots\ldots \quad (125)$$

This is a cubic equation which can be solved in terms of real and imaginary roots, but its chief value here is to show that the ratio $\frac{H_2}{H_1}$, x in the equation, is constant for any one Venturi flume, being dependent solely on the ratio $E = \frac{B_1}{B_2}$.

We can now write xH_1 for H_2 in equation (121) and the final equation is then:

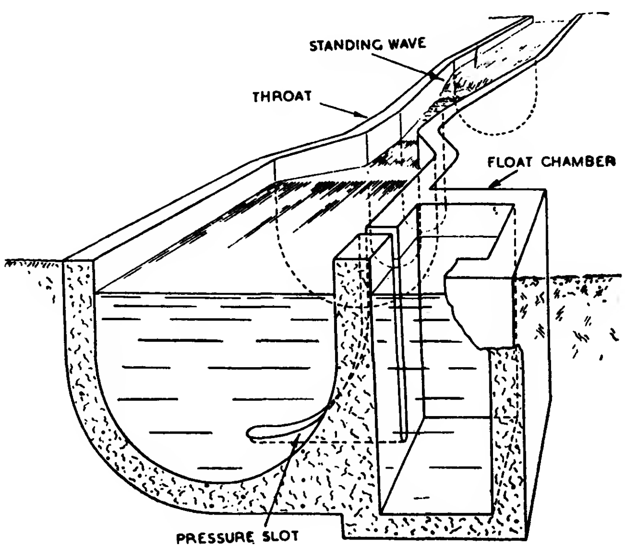
$$Q = C B_2 \left(\sqrt{2g}\right) xH_1 \sqrt{H_1 - xH_1} \sqrt{\frac{E^2}{E^2 - x^2}} \quad (126)$$

$$\text{or } Q = C B_2 \left(\sqrt{2g}\right) H_1^{\frac{3}{2}} K \quad \ldots\ldots\ldots \quad (127)$$

where K is a constant dependent solely on the dimensions of the flume.

Ref. (2) suggests the maximum value $\frac{H_3}{H_1} = 0{\cdot}8$ as a rough guide for the establishment of critical flow conditions in the throat. At values of the ratio below the critical one, a standing wave is formed in the manner of Fig. 100, and the same reference mentioned in the previous sentence suggests a method of penstock adjustment to provide critical flow phenomena with standing waves.

Fig. 101. *U-section Venturi flume*



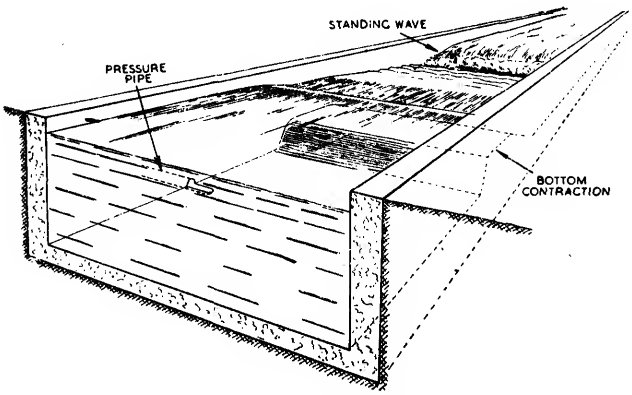


Fig. 102. *Venturi flume with vertical contraction*

When the critical value of the ratio $\frac{H_3}{H_1}$ is exceeded, the flume is operating under "drowned" conditions, and equation (122) must apply. Since this involves the awkward quantity $(H_2 \sqrt{H_1 - H_2})$ which means involved instrumentation, the advantage of ensuring free flow or discharge is considerable.

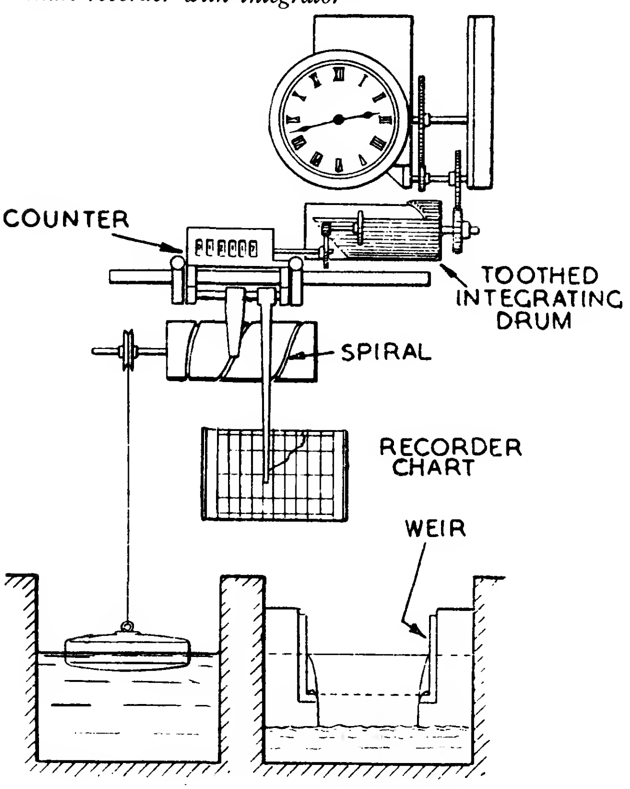
It is not necessary for the cross-section of a Venturi flume to be rectangular. U shaped sections are not uncommon, and trapezoidal forms have been used. Fig. 101 shows a U-section design.

The vertical contraction or hump shown in Fig. 102 is sometimes necessary when the rates of flow in the lower part of the range are such that "drowning" will occur. The introduction of the hump ensures free flow conditions.

Measuring Instruments

We have established that for weirs and flumes, the rate of flow is proportional to a function of the head of liquid either in the weir or in the throat of the flume. A float resting on the surface of the liquid will, therefore, follow any variations in head and so measure changes in the rate of flow. To ensure satisfactory conditions for measurement, the float must be installed in a well or chamber at the side of the main channel, the well being in connection with the channel. Such a well or chamber is shown in Fig. 101.

Fig. 103. *Float operated instruments for weirs. Strip chart recorder with integrator*

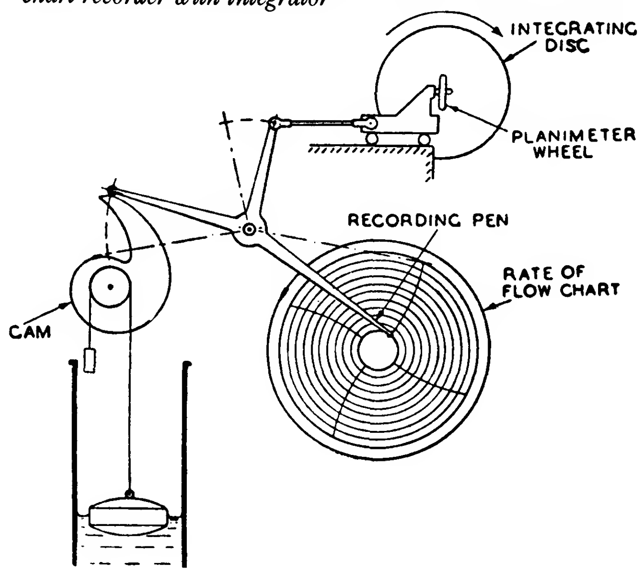


The non-linear relation between rate of flow and head means that correcting devices must be employed between float and measuring or integrating instruments to produce linear conditions. Two means of performing this operation will be discussed.

In the first example shown in Fig. 103, the rise and fall of the float rotates a pulley to the shaft of which is connected a cylinder. On the cylinder is cut a spiral groove, the pitch of which varies in such a manner that linear displacement of the recording pen carriage relative to changes in flow rate is achieved. Fixed to the pen carriage is a mechanical counter whose driving wheel engages with a gear train driven by the toothed integrating drum. The drum is driven at constant speed by a clock mechanism and the teeth on it vary in length in a linear manner as the drum rotates; thus the counter is operated according to its horizontal displacement along the drum. For example, at the zero or left-hand end there are no teeth on the drum for the counter wheel to engage and consequently there are no revolutions of the counter. At the right-hand or maximum end, teeth are continuously cut round the periphery of the drum, and the counter wheel is continuously engaged. Between zero and maximum flow there are teeth and open spaces according to the position along the drum. As a result, the wheel of the counter will be intermittently engaged and at rest for periods bearing a linear relation to the flow rate.

The second example is a variation of the cam-operated design shown in Fig. 88 of chapter five. In Fig. 104 the rise and fall of the float rotates the cam connected to the pulley shaft. The contour of the cam is cut to a law such that the rotation of the recording pen arm and the movement of the planimeter wheel carriage bear a linear relation to the flow rate.

Fig. 104. *Float operated instruments for weirs. Circular chart recorder with integrator*



References for further reading

1. WEST, R. G. Flow and Measurement Problems in Open Channels.
   Part 1, *Instrument Practice*, June, 1962, page 735.
   Part 2, *Instrument Practice*, July, 1963, page 889.
   Part 3, *Instrument Practice*, August, 1963, page 977.
   Boundary Layer and Flow Measurement, March, 1963.
2. LINFORD, A. Flow Measurement and Meters. Chapter 6. E and F Spon, and *Instrument Manual*, Section VI, United Trade Press.
3. B.S. 599 Code for Pump Tests.
4. JONES, E. B. *Instrument Technology*, Vol. 1, Chapter 3, Butterworths.
5. ECKMAN, D. P. Industrial Instrumentation, Chapter 10, Chapman and Hall.

QUESTIONS:

1. In equation (106) if $H_1 = 6$in., calculate the mean velocity V_m which makes $\frac{V_m^2}{2g}$ 2% of H_1. Take g as 32·2ft/sec².

   (Answer: 0·8ft/sec approx.)

2. Using equation (109), calculate the flow rate in gals/hr if $B = 24$in., $H_1 = 10$in.

   (Answer; 107 600gals/hr.)

3. Compare the answer to question 2 with that obtained by using formula (108). Take C as 0·64, g as 32·2ft/sec² and 1ft³ = 6·23 gallons.

   (Answer: 117 000gals/hr.)

4. In equation (125) find the value of E that satisfies the equation for $x = 0·7$.

   (Answer: 1·85 approx.)

British Standard Specifications

B.S.3680 Measurement of Liquid Flow in Open Channels.

Part 4 Weirs and Flumes.

Part 4A Thin Plate Weirs and Venturi Flumes.

Chapter 7

DISPLACEMENT, INFERENTIAL, MAGNETIC AND ULTRASONIC FLOWMETERS

INTRODUCTION

This chapter will be concerned with flowmeters which do not rely on the production of a differential fluid pressure or a static head of liquid for measurement purposes.

DISPLACEMENT METERS

In the displacement type of flowmeter the stream of fluid can operate cyclically a movable member of the meter, providing a means for measuring the total flow over a given period.

Two typical designs only will be described.

Semi-Positive, Semi-Rotary Piston (Oscillating Piston) Type

In this type, a hollow cylindrical chamber A contains a partition piece B which extends along a radius, and runs the whole length or height of the chamber (*Fig.* 105). Concentric with the chamber wall is a

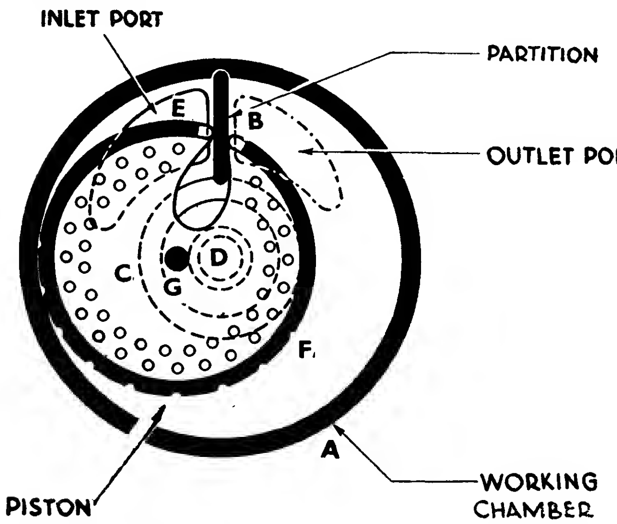


Fig. 105. *Diagram showing the principal parts of a semi-positive, semi-rotary piston flowmeter*

second hollow cylinder C, and within this, a central hub D. In the floor of the chamber is an opening E, E being the inlet port to the meter. Working inside the chamber is a piston F which, in a typical pattern, has a horizontal web, and is fitted with a central peg G projecting upwards and downwards from the web. The underside of the web makes contact with the top surface of cylinder C. The upperside of the web contacts a similar hollow cylinder extending downwards from the lid of the chamber. Small holes are bored in the web to allow liquid to escape from the lower to the upper sides of the piston. The lid of the chamber contains the outlet port H which is on the opposite side of partition B to inlet E.

The parts of the assembly are so proportioned that the peg G makes line contact with hub D; the outer surface of piston F makes line contact with the inner surface of chamber A; and the inner surface of F makes line contact with the outer surface of cylinder C and its counterpart in the lid.

The piston has a pear-shaped cut-out through which the partition B projects. Because of the construction the action of the liquid in entering the chamber via E causes the piston to rotate round the central hub D. In so doing, it slides backwards and forwards along the partition. The operation of the instrument will be better understood if four positions of the piston are considered.

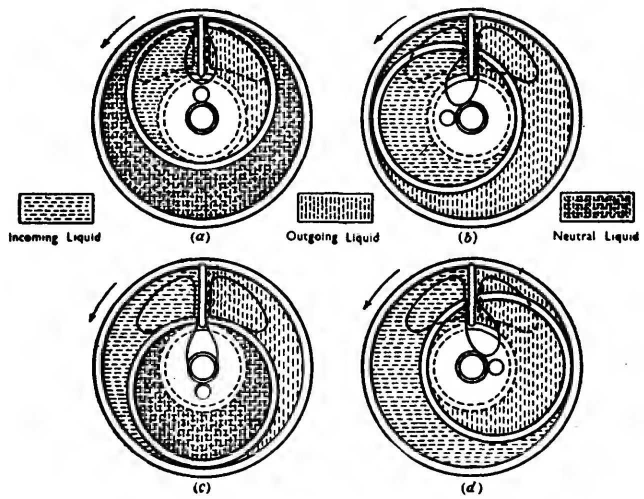


Fig. 106. *Diagram showing the action of the semi-positive, semi-rotary piston flowmeter*

Fig. 106 shows the inlet and outlet ports in communication with the inside of the piston. Liquid is entering by the former and the piston commences its rotary motion. The liquid outside the piston is for the time being cut off from both ports and hence has, for the moment, no part in the operations. It is normally termed, therefore, "neutral liquid". As the piston rotates both ports are gradually uncovered and the neutral liquid becomes the outgoing liquid. Position

Fig. 106*b* is eventually reached where the incoming and outgoing liquids are on both sides of the piston. Further rotation takes place until the piston arrives at the position in *Fig.* 106*c*. Here, the interior is momentarily completely isolated from the two ports, the relevant liquid becoming neutral. The incoming liquid lies between the outer surface of the piston and the left-hand side of the partition, and the outgoing liquid between the right-hand side of the partition and the outer surface of the piston. Observe that the piston has now slid to the opposite end of the partition to that in *Fig.* 106*a*. The rotation now proceeds, and the sliding action reverses its direction, the previous neutral liquid from the interior now becoming the outgoing. *Fig.* 106*d* is arrived at, where the piston is on its return journey along the partition, and the peg has proceeded for three-quarters of a revolution round the hub. The incoming water is beginning to enter the interior of the piston, and the rotational and sliding movement is continued until *Fig.* 106*a* is reached, when the train of operations is repeated.

From this description, it can be seen that the rotation is always in one direction, each semi-revolution permitting a definite volume of liquid to be delivered through the meter.

The peg of the piston is coupled to a cross-bar mounted on a spindle within the chamber, and the spindle is then geared to one or more trains to drive a revolution counter in the upper part of the instrument. This counter has a main dial and pointer and several subsidiary ones, and will normally be calibrated in flow units, e.g., gallons, cubic feet, etc.

Ranges

In view of the extensive use of this type of meter in the water supply field the ranges are extremely high. For example, a typical M type meter of G. Kent Ltd. has a maximum reading of 100 000 000 gallons. A maximum totalizing rate is normally specified. This varies with the meter size, but for a 4in. diameter pattern could be about 7000 gallons/hour. The meters are also suitable for very low liquid flow rates, e.g., 4 gallons/hour.

Semi-Positive, Semi-Rotary Nutating Disc Type

In this pattern a circular flat disc is used in place of the rotary piston. The working chamber in which the disc is situated is of circular shape with hemispherical sides, but has a conical roof and floor. A partition extends along a radius on the incoming and the outgoing sides and virtually divides the chamber into two parts. The disc is split to fit over this partition and is thus prevented from rotating round its own axis, but is free to rock or rotate. To pass from the inlet port to the outlet, the liquid must flow through the chamber, and in so doing displaces the disc. The action of the disc is extremely difficult to describe simply, or to indicate in a clear fashion diagrammatically. If a kind of rolling, tilting action of the disc can be imagined, however, it will be seen that the pin traces out a cone of rotation. The end of the pin, therefore, follows a circular path and is coupled by a cross-bar to a spindle driving the gear assembly and the counter. An alternative design uses a nutating bell.

Ranges

The ranges are similar to the rotary piston type.

Inferential Meters

Fan Type

In this pattern of flowmeter the fluid stream operates a fan device. The fan may be single or multi-bladed. *Fig.* 107 shows a five-blade version in plan view. The blades are fixed to a vertical spindle and the fluid entering the fan chamber impinges on them, imparting a rotary motion to the spindle. The rate of revolution is proportional to the stream velocity, and if the spindle is geared to a counting mechanism, a means of total flow measurement is effected.

Generally speaking, the fan type is restricted to use in pipes up to 2in. diameter. Where an installation for larger diameters is required, say up to 10in., the propeller type is adopted.

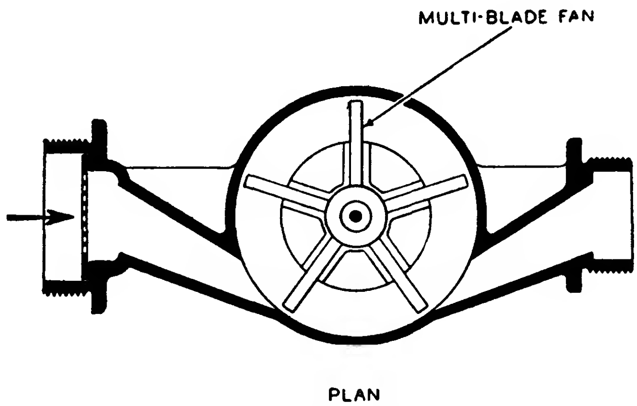


Fig. 107. *Fan type inferential flowmeter*

Propeller Type

The propeller is typically in the form of a helix mounted vertically (*Fig.* 108) or horizontally (*Fig.* 109). In the former type, the propeller chamber must be vertical, and for use in a horizontal pipe line, the inlet and outlet sections assume the curved shapes shown in the figure. The liquid is guided through the inlet (the

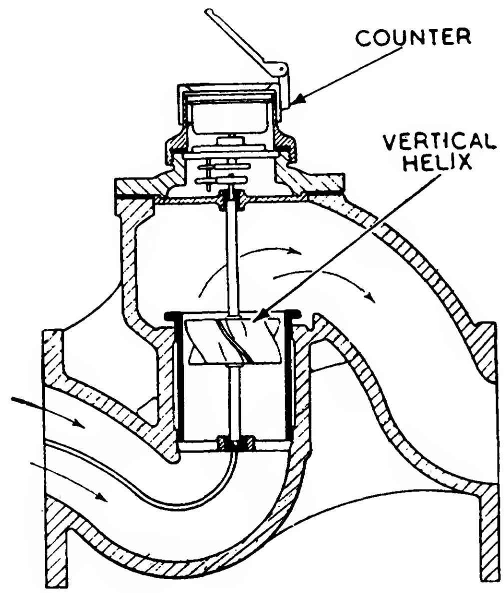


Fig. 108. *Vertical propeller type inferential flowmeter*

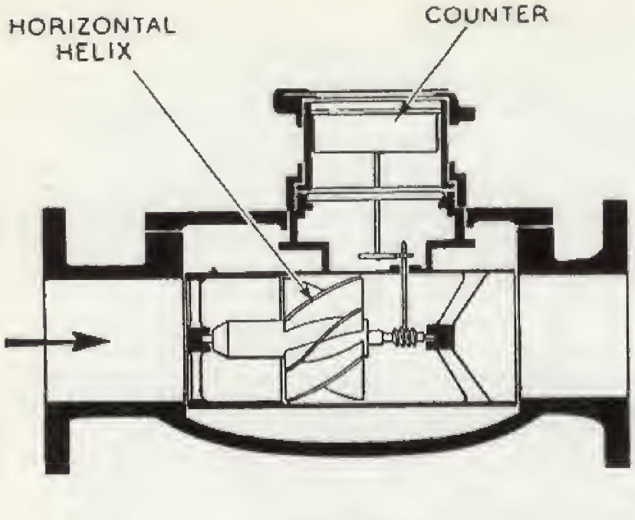


Fig. 109. *Horizontal propeller type inferential flowmeter*

guide is indicated as a double line in the inlet section) and up through the working chamber so as to flow in a smooth fashion. After leaving the working chamber it flows into the outlet section. In a variation of this design, a double helix assembly is used.

With a horizontally mounted helix, on the other hand, the inlet and outlet sections and the working chamber are in the same straight line so that no guide is present.

If Q is the rate of flow, L the pitch of the propeller blades, n the rate of revolution of the propeller, A the net area of the water passage:

$$Q \propto L.A.n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (128)$$

The meter measures total flow, so that between a time t_1 and t_2,

$$\int_{t_1}^{t_2} Q.dt \propto \int_{t_1}^{t_2} L.A.n.dt \quad . \; . \; . \; . \quad (129)$$

Ranges

Similar to piston and disc types.

General Remarks on Mechanical Displacement Meters

Gears

Part or whole of the gear assembly may be in the fluid stream. It is therefore necessary to use materials which are corrosion and wear resistant.

For this reason, the gears are commonly manufactured of nickel, nickel alloys or phosphor bronze with bakelite bearing plates.

Where the gear assemblies are divided into two sections, e.g., undergears and counter gears, the drive into the latter passes through a gland rotating in a stuffing box.

In some designs, the counter gears are immersed in the metered fluid. Any unauthorized opening up of the meter results in flooding.

Materials

Materials for the body of the meter must be chosen with a view to preventing corrosion, electrolytic or otherwise. The body is often composed of one or more brass castings, with a brass working chamber for the rotary piston type. The rotary piston itself is formed from bakelite or ebonite. The multi-piston patterns normally use non-ferrous materials as far as possible—in one or two cases, the pistons can be manufactured from a plastic substance.

Protection Against Freezing

Where the metered liquid is likely to freeze in cold weather some kind of protection is necessary. Rubber pads which compress when the liquid freezes, or thin diaphragms or plates which are deformed when freezing occurs, are the most popular methods.

Pressure Drop

All types of quantity meters will have a pressure drop between the inlet and outlet, the value depending on the type and the rate of flow through the meter.

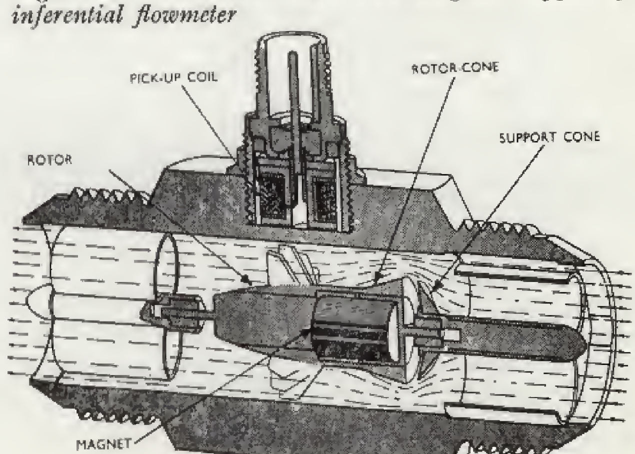
ELECTRONIC TYPE INFERENTIAL METERS

Improved performance by the inferential pattern of flowmeter may be obtained by reducing friction and the amount of work performed by the rotating element. The Pottermeter (manufactured in this country by De Havilland Propellers Ltd.) is designed to take care of these two factors.

The detecting or sensing element consists of a bladed rotor element incorporating a permanent magnet, *Fig.* 110. It rotates within a housing of non-magnetic material. Before discussing the action of the magnet, let us examine the method of operation of the rotor itself.

The bearings are seen supported at the centre of the three circular spring clip units at each end of the housing. Two conical members can be observed: one on the bearing support at the incoming end, and one on the rotor. The fluid is contracted at the support cone. Its velocity is increased, the maximum value being reached at the upstream end of the rotor cone. The maximum diameter of the support cone is slightly less than that of the rotor cone. The fluid decelerates after passing the position of maximum diameter of the latter and eventually the pressure rises to a value nearly that obtaining on the upstream side of the support cone. Now, this pressure is obviously greater than that at the position of maximum conical diameter and since it is effectively acting over the maximum area, the rotor cone tends to move in an upstream

Fig. 110. *Pottermeter—an electromagnetic type of inferential flowmeter*



direction. But due to the difference in diameters of the support and rotor cones, the effect of fluid impact is increased as the rotor cone approaches the support cone. A position is reached where the two forces involved balance one another, and the rotor assumes a position "floating" between the ends of its two bearings. Thrust bearings are thus unnecessary, and thrust friction is eliminated. The fluid is admitted to the bearings through vents so that the rotor shaft operates on a film of the liquid.

The rotor magnet sets up a rotating field which generates a.c. pulses in the pick-up coil seen on top of the housing. The pulse frequency is proportional to the speed of the rotor and the number of poles on the magnet. Further action depends on whether the rate of flow or total flow is to be measured. If the rate of flow is desired, the pulses from the pick-up coil may be fed to a frequency converter unit which delivers a d.c. output proportional to the pulse frequency. The flow rate may thus be indicated directly on a d.c. milliammeter or recorded by an appropriate instrument. If total flow is required, the pulses may be fed to an electronic counter unit and digital indication given.

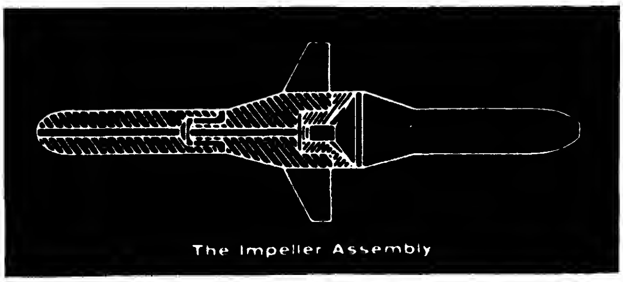


Fig. 111. *The impeller assembly of the Meterflow type of inferential flowmeter*

In a design by Meterflow Ltd., there is no magnet in the impeller assembly as can be seen from *Fig.* 111. Normally, the pressure drop which develops across the rotor blades tends to force the rotor forward on to its outlet bearing, thus producing drag and reducing measuring accuracy. In the Meterflow element, however, this bearing consists of a conical seating which is in direct communication with the inlet to the flowmeter by means of a small drilling which passes through the entire length of the front bearing and the rotor assembly. Fluid at upstream pressure is communicated directly to the rotor bearing, producing a thrust which always compensates that developed across the blades. The system is self-balancing, since the fluid pressure developed on the outlet thrust bearing increases as the rotor moves forward. The value of the pressure developed on the outlet bearing is a function of the relative position of the rotor and the bearing housing, and this position always adjusts itself so that the pressure drop across the rotor equalizes the thrust on the bearing. The rotor, therefore, always floats freely between the two bearings without making physical contact with them.

Screwed on to one face of the flowmeter is a pick-up which contains a magnet and a coil. The passage of the blades through the magnetic flux generates voltage pulses in the coil which can either be used to operate a Dekatron type of display unit or, alternatively, can be utilized to produce a d.c. voltage or current proportional to flow rate by means of frequency d.c. converters.

ANEMOMETERS

Anemometers

Anemometers are basically velocity measuring instruments, suitable mainly for air or gas flow measurement.

Rotating Vane Anemometers

Here, a small vane assembly, similar to a fan, is placed in the air stream. The fan shaft is coupled to a mechanical counter. The rate of revolution is a measure of the velocity, and readings over a given time interval are taken. From these, the velocity of the air flow may be determined. Values up to 10 000 ft/minute can be measured. The lower limit is about 150ft/minute.

Deflecting Vane Anemometer

The stream of air or gas is allowed to impinge on a pivoted vane. The vane is deflected from its normal position by an amount proportional to the velocity of the air. Usually, the deflection of the vane is opposed by a hair spring, and may be magnetically damped. It finds its greatest application in measuring the velocity of air in ventilating ducts.

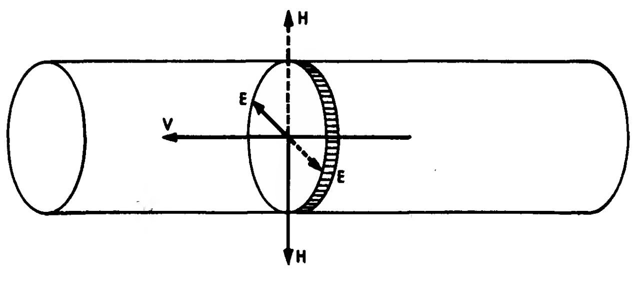
Hot Wire Anemometer

In this type of instrument, a small resistance wire is heated by an electric current and placed in the air or gas whose velocity is to be determined. The stream of gas flowing past the wire exerts a cooling effect on it, dependent on the stream velocity, and so lowers its temperature. The temperature change varies its resistance. The resistance can be measured on a bridge as an indication of the fluid velocity. Alternatively, the current through the wire can be adjusted to keep the resistance constant, when the current itself becomes a measure of the velocity. An N.P.L. design incorporates a thermocouple in the assembly, the millivolts from the latter being a measure of the wire temperature and hence the fluid velocity. Typical ranges are 0-0·4, 0-2·0 and 0 to 5·0ft/second.

ELECTROMAGNETIC FLOWMETERS

The principle of the electromagnetic flowmeter may be understood better if we first consider a very thin disc of an electrically conducting liquid moving with a velocity V along a pipe of internal diameter d. An external magnet system directs a magnetic field of strength H across the section of the pipe so that it acts at right angles to the direction of motion of the disc. Now, by Faraday's law of induction, when an

Fig. 112. *Directions of velocity V, magnetic field H and electromotive force E*



electrical conductor of length L moves through a magnetic field of strength H at a velocity V in a direction at right angles both to the magnetic field and its length, an e.m.f. is generated of value

$$E = k H L V \quad \ldots\ldots\ldots (130)$$

where k = a constant

Our disc of liquid is a conductor obeying the general requirements of Faraday's law, and it can be seen without much difficulty that L in equation (130) is replaced by d, the diameter of the disc. If, now, there is an infinite number of such moving discs contiguous to one another, we have the equivalent of a conducting liquid stream flowing continuously through the pipe (see *Fig.* 112).

The stream will satisfy the following equation

$$E = k H V d \quad \ldots\ldots\ldots (131)$$

In (131) d is constant and if H remains constant

$$E = KV$$

where K = a general constant ..... (132)

Alternatively, since $Q = VA$ where A is the area of the pipe,

$$E = C Q \quad \ldots\ldots\ldots\ldots (133)$$

C being a general constant.

Thus, provided we can physically measure E, a very simple means of determining the flow rate of liquids in pipes is available.

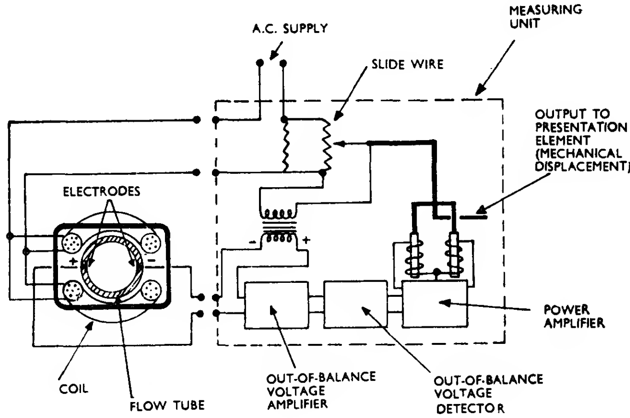


Fig. 113. *Simple block diagram of electromagnetic flowmeter and measuring circuits (Foxboro-Yoxall Ltd.)*

Practical Design

1. In general, the electromagnetic flowmeter takes the form of a metering tube of non-magnetic material. This ensures that the magnetic flux does not go into the tube wall and by-pass the flowing liquid.
2. The tube, if made of conducting material, must have an insulating lining to prevent short circuiting the e.m.f. Although this need only apply in the neighbourhood of the electrodes, the insulation is usually extended the whole length of the tube. Non-metallic tubes will not require this lining unless for reasons of corrosion, etc.
3. The electrodes are usually of point form normally made from stainless steel. Platinum is occasionally used where the liquid handled is severely corrosive. The faces of the electrodes are flush with the lining or tube surface where no lining is used.
4. The magnet coils are energized from an a.c. supply. The advantages and disadvantages of a.c. over a d.c. energized magnet are discussed in a paper devoted to this pattern of flowmeter and need not concern us here (see List of References).
5. The system acts as a source of e.m.f. with a finite resistance dependent on the conductivity of the liquid and the size of the meter. Precautions must be taken in the measuring circuit to avoid anything but an extremely minute current drain from the e.m.f. source.

The measuring circuit is normally a null balance one, a typical one being shown in *Fig.* 113. (Null balance circuits will be described in a later chapter).

Advantages of the Electromagnetic Flowmeter

1. Linear relation between flow rate and measuring signal as compared with the square law relation of differential pressure devices. This results in a rangeability of the order of 100/1.
2. The measuring instrument can be arranged with a centre zero for measuring flow in either direction. Alternatively the electrode leads may be changed over to measure a reverse flow.
3. The only pressure loss is that due to the length of tube, forming the meter. But a pressure loss would be present with the same length of ordinary pipe so that the introduction of the meter cannot be said to involve significant additional pressure losses.
4. There is no obstruction to flow which renders the meter suitable for liquids containing suspended matter. Abrasion may be avoided by choosing a suitable lining material. Wood pulp and paper mill stocks, cement slurries, sewage, food pulp are but a few difficult fluids which may be metered.
5. The design lends itself to the metering of corrosive liquids since parts in contact with the fluids may be made of corrosion-proof materials.
6. It is not affected by velocity profiles, since the e.m.f. is at all points proportional to the velocity of flow across the diameter.

Disadvantages of the Electromagnetic Flowmeter

1. It is not suitable for measuring gas or vapour flows.
2. The normal design is not suitable for hazardous areas.
3. Liquids to be metered must be conductors of electricity.
4. There is a minimum value of conductivity which is related to the lengths of cable leads to amplifiers and the size of the meter. The readings are unaffected by increases in conductivity above the minimum value, but decreases cause the meter to read low.
5. If a concentric build up of deposit of much different conductivity to that of the liquid takes place, significant errors may be introduced. Values will be found in the paper by B. W. Balls and K. J. Brown. Note that it is possible for non-conductive deposits to insulate the electrodes. Where the concentric deposit is of the same conductivity as the metered liquid, the meter continues to read correctly.

A meter has been constructed of $\frac{1}{10}$in. diameter with a flow range 0·002 to 0·2gal/min. In contrast, a typical large diameter meter has been 72in. covering a range 500 to 50 000gal/min.

ULTRASONIC FLOWMETER

1. Consider *Fig.* 114 in which a fluid is flowing at a velocity V. A transducer T_1 transmits a beam of sound to receiving transducer T_2 situated at a distance d downstream. If C is the speed of sound through still fluid, t the time for sound to travel from T_1 to T_2 is

$$t = \frac{d}{C + V} \quad \ldots\ldots\ldots \quad (134)$$

with no flow,

$$t_o = \frac{d}{C} \quad \ldots\ldots\ldots \quad (135)$$

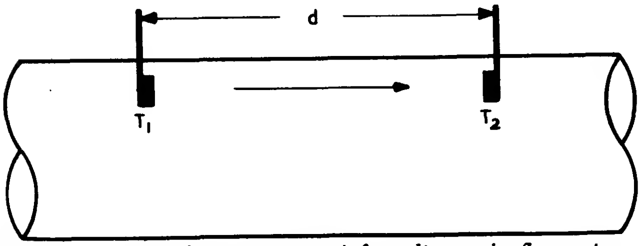


Fig. 114. *Simple arrangement for ultrasonic flowmeter with one transmitting and one receiving transducer. The direction of flow is from left to right*

The difference between t and t_o, Δt, is given by

$$\Delta t = \frac{Vd}{C(C + V)} \quad \ldots\ldots \quad (136)$$

C for most fluids is of the order of 1500 metres/sec whilst V for most industrial applications would be a few metres/sec Equation (136) then reduces to

$$\Delta t = \frac{Vd}{C^2} \quad \ldots\ldots \quad (137)$$

This suggests that Δt could provide a measurement of V. But it involves a knowledge of t_o, not readily measurable, and C, which varies with temperature and pressure. To eliminate t_o, the differential arrangement shown in *Fig.* 115 may be used.

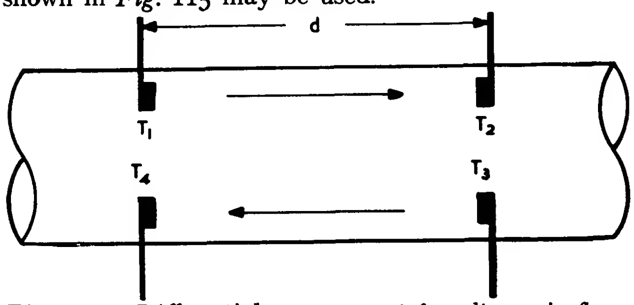


Fig. 115. *Differential arrangement for ultrasonic flowmeter. The direction of flow is from left to right*

2. Two sets of transducers, T_1 and T_2 and T_3 and T_4, are installed in the pipe, the distance between T_1 and T_2 and T_3 and T_4 being d. A beam of sound is transmitted from T_1 to T_2 downstream and from T_3 to T_4 upstream, both beams being of the same frequency. The time for the beam to travel from T_1 to T_2 is

$$t_1 = \frac{d}{C + V} \quad \ldots\ldots \quad (138)$$

where C = the velocity of sound under the temperature and pressure conditions existing in the pipe, and from T_3 to T_4,

$$t_2 = \frac{d}{C - V} \quad \ldots\ldots\ldots\ldots \quad (139)$$

The difference between t_1 and t_2 is

$$t_1 - t_2 = \Delta t = \frac{2\,Vd}{C^2 - V^2} \quad \ldots\ldots\ldots\ldots \quad (140)$$

If V is small compared with C; (140) can be reduced to

$$\Delta t = \frac{2\,Vd}{C^2} \quad \ldots\ldots\ldots \quad (141)$$

3. The measurement of Δt now involves some problems. It may be solved by pulse techniques or a continuous wave beam may be used. In the latter case, the transmitting transducers are driven from a common source and the phase difference between the two received signals measured. The phase difference $\Delta\phi$ is given by

$$\Delta\phi = \frac{2\omega\,Vd}{C^2} \quad \ldots\ldots\ldots \quad (142)$$

where ω = the angular frequency

4. Observe that in all the methods considered, C the velocity of sound is present. This can be eliminated if the method of *Fig* 116 is adopted. A short pulse

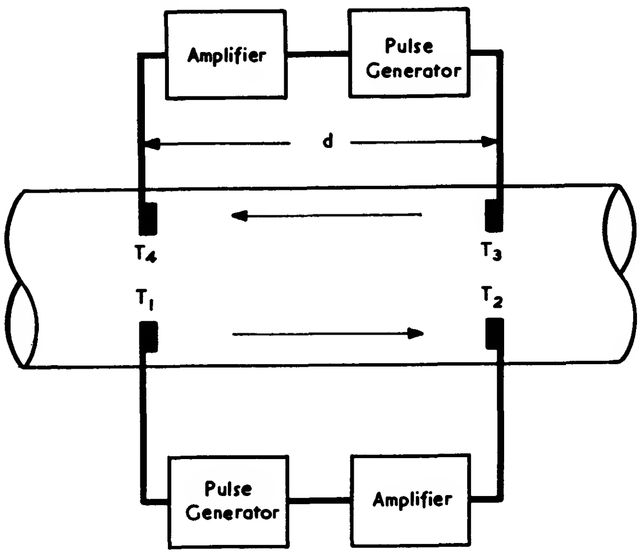


Fig. 116. *Pulse scheme for eliminating the effect of C the velocity of sound. The direction of flow is from left to right*

is emitted from transducer T_1 and is received by T_2. The arrival of the pulse from T_2 triggers another one from T_1. The time between pulses is

$$t_1 = \frac{d}{C + V} \quad \ldots\ldots\ldots \quad (143)$$

The pulse repetition frequency is f_1 and since $f_1 = \frac{1}{t_1}$

$$f_1 = \frac{C + V}{d} \quad \ldots\ldots\ldots \quad (144)$$

A similar pulse is transmitted from T_3 to T_4 and calling the repetition frequency here f_2,

$$f_2 = \frac{C - V}{d} \quad \ldots\ldots\ldots \quad (145)$$

$$f_1 - f_2 = \Delta f = \frac{2V}{d} \quad \ldots\ldots\ldots\ldots \quad (146)$$

Equation (146) is independent of C.

5. A further technique used has been a differential arrangement across the pipe. It can be shown that a beam of sound can be deflected in the downstream

direction in traversing a pipe from one side to the other. The deflection x is approximately given by

$$x = \frac{Vd}{C} \quad \ldots\ldots\ldots\ldots \quad (147)$$

Which of the methods outlined is most suitable for industrial applications?

There are several factors to consider.

1. In the phase difference method the phase difference is given by

$$\Delta\phi = \frac{2\omega Vd}{C} \quad \ldots\ldots\ldots \quad (148)$$

$\Delta\phi$ is proportional to the operating frequency ω and it suggests that ω should be as high as possible.

Measurements of $\Delta\phi$ above 2π are not desirable, from the point of view of interpretation, but $\Delta\phi$ should be as large as possible below this limit. But the higher the frequency the greater the attenuation since it is a function of the square of the frequency. Thus, already we have two conflicting factors. There is yet a third effect: that of the beam width. This is a function of the velocity of sound in the liquid, the radiating area of the transducer and the operating frequency. There may have to be a compromise between all the factors involved.

2. In the frequency difference method, care must be taken to avoid coupling between neighbouring circuits carrying frequencies relatively close to one another. The frequency difference $\triangle f$ is dependent on the flow rate V and may be of extremely low value, e.g. 10c/s or 20c/s unless the flow rate is relatively high.

3. The beam deflection method suffers from the fact that the deflection is proportional to flow rate and, at low flow rates, may not be sufficient for accurate measurement.

For an appraisal of the methods and a detailed examination of principles involved, the paper by R. E. Fischbacher quoted in the references should be read.

QUESTIONS:

No questions are added for this chapter due to the largely descriptive nature of the text.

References for further reading.

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Balls, B. W. and Brown, K. J.	The Magnetic Flowmeter. *Trans. Soc. Inst. Tech.* Vol. 11, No. 2. June, 1959, page 88.
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Fischbacher, R. E.	The Ultrasonic Flowmeter. *Trans. Soc. Inst. Tech.* Vol. 11, No. 12. June, 1959, page 14.

Chapter 8

NON-ELECTRICAL THERMOMETERS

INTRODUCTION

THE most important classes of non-electrical thermometers are:—

1. Liquid expansion.
2. Gas expansion.
3. Vapour pressure.
4. Bi-metallic.

I. LIQUID EXPANSION INSTRUMENTS

In a typical liquid expansion thermometer, a Bourdon tube is connected to a metal bulb by small bore metal tubing known as "capillary tubing" (*Fig.* 117). A completely sealed system is formed. The internal diameter in a typical capillary is 0·01 in. The reason for the smallness of this value will be seen later. The size of the bulb depends on the range of the instrument. The whole of the internal volume formed by the bulb, capillary and Bourdon is filled with a liquid with as great a range as possible between freezing and boiling points. A large coefficient of cubical expansion is also required. A low vapour pressure is desirable since it is only the expansion property we are concerned with using. To avoid any trouble from vapour pressure effects, the thermometer systems are filled and sealed with the liquid at a very high pressure. This may reach a figure of 1000lb/in^2 or more in the mercury filled variety. Thus, the boiling point is raised considerably by being under such a high pressure, and vapour pressure influences on the instrument are negligible.

Apart from the physical properties mentioned, any liquid chosen must not attack the metal parts with which it is in contact.

Mercury is the most popular liquid in use today. Others such as xylene and alcohol are used, but to a lesser extent.

It is convenient at this point to consider the operation of the thermometer. We have seen that the whole system is filled with liquid. Suppose now that the bulb, which has by far the greater part of the total volume, is inserted at the place where the temperature is to be measured. For purposes of explanation, consider that the temperature increases from a certain value. The volume of liquid in the bulb increases, and assuming that the liquid does not become compressed in any manner, the increase is transmitted by the liquid in the capillary tubing to the Bourdon tube. The increase in volume causes the tube to deflect in a similar manner to the pressure instruments described in Chapter 1. It should be clearly understood, however, that it is primarily a volume change which operates the Bourdon tube here, pressure changes being incidental.

The law governing the expansion of liquids may be stated simply as:

$$V_2 = V_1 (1 + a (T_2 - T_1)) \ldots\ldots\ldots (149)$$

where V_2 = the volume of liquid at T_2°C. in cm or in^3

V_1 = the volume of liquid at T_1°C. in cm^3 r in^3

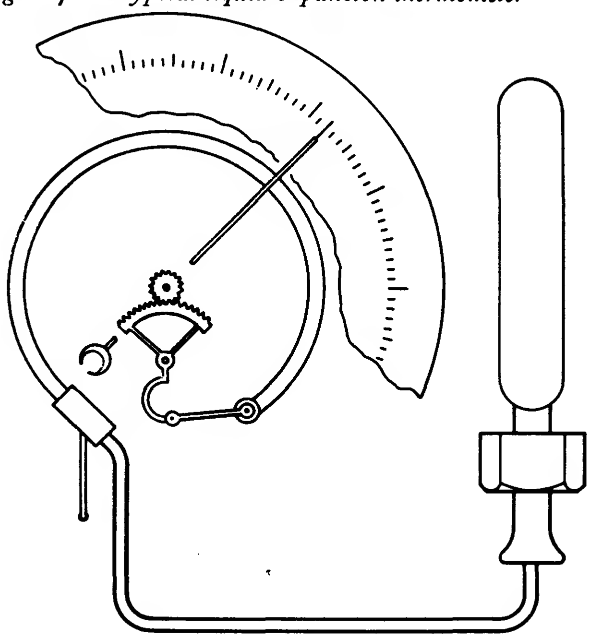
a = coefficient of cubical expansion per °C.

This equation has other terms in the full version, but these need not be considered here. It does show, however, that a substantially linear scale for the instrument may be expected.

The following table indicates the recommended maximum and minimum temperatures and coefficients of expansion of the normal filling liquids.

Liquid	Min. Temp.	Max. Temp.	Coefficient of Cubical Expansion (per °F.)
Mercury	− 38°F. (− 39°C.)	1000°F. (538°C.)	$0{\cdot}101 \times 10^{-3}$
Xylene	− 40°F. (− 40°C.)	750°F. (400°C.)	$0{\cdot}623 \times 10^{-3}$
Alcohol	− 50°F. (− 46°C.)	300°F. (150°C.)	$0{\cdot}635 \times 10^{-3}$

Fig. 117 *A typical liquid expansion thermometer*



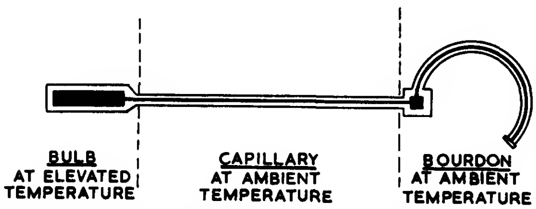


Fig. 118. *The three main sections of a liquid expansion thermometer*

The dependence cn volume change brings one or two disadvantages in its train. If we refer to *Fig.* 118, we see that the system can, in effect, be divided into three sections: the bulb, the capillary, and the Bourdon tube. The operation, being based on a volume change, means that the instrument is sensitive to temperature variations at other places than in the bulb. For example, the temperature of the surroundings of the capillary may alter (i.e. the ambient temperature), which means that the volume of liquid in the capillary varies, affecting the Bourdon in a similar manner to the alteration in bulb liquid volume. The instrument is calibrated on the basis of bulb volume changes, and a change anywhere else in the system can cause errors. Let us see what is likely to be the value of these.

Ambient Temperature Errors

If V is the internal volume of the bulb, and v is the internal volume of the capillary, for a temperature change of t°F. in the capillary, the error x in °F. is given by:

$$x = \frac{v}{V} \times t \quad \ldots\ldots\ldots \quad (150)$$

Now consider an example.

(*a*) $v = 0{\cdot}40$ in^3 (approximately 105 ft of 0·02 in. diameter capillary).

$V = 5{\cdot}00$ in^3.

$t = 15$°C., e.g. a change from 10°C. to 25°C.

Then $$x = \frac{0{\cdot}4}{5} \times 15 = 1{\cdot}2\text{°C.}$$

Suppose the instrument has a range of 100°C.—600°C., then the percentage error in terms of the *range* is

$$\frac{1{\cdot}2}{500} \times 100 = 0{\cdot}24 \text{ per cent}$$

In terms of the *maximum reading* on the scale, i.e. 600°C., it is

$$\frac{1{\cdot}2}{600} \times 100 = 0{\cdot}20 \text{ per cent.}$$

To illustrate how important is the relation between bulb and capillary volumes, assign some new figures to the example.

$v = 0{\cdot}40$ in^3.

$V = 1{\cdot}00$ in^3.

$t = 15$°C.

Then, $$x = \frac{0{\cdot}4}{1} \times 15 = 6\text{°C.}$$

In terms of per cent of *range* the error is:

$$\frac{6}{500} \times 100 = 1{\cdot}2 \text{ per cent.}$$

In terms of per cent of *maximum scale reading* it is:

$$\frac{6}{600} \times 100 = 1 \text{ per cent.}$$

British Standard Specification No. 1041 covering Temperature Measurement allows only ½ per cent of the maximum scale reading for a 30°F. change in capillary and/or Bourdon temperature. We see that the error in example (*a*) is not serious since it is well below ½ per cent. If the conditions of example (*b*) obtain, however, the error is much above the recommended maximum.

Compensating Devices

It is obviously better to avoid correcting devices wherever possible, and to achieve this the capillary bore must be reduced to very small dimensions, e.g. 0·005 in. If, in spite of this, the ambient temperature errors are unacceptable, then some compensating device must be introduced.

Compensating Link

One device is the compensating link of Negretti & Zambra Ltd. This consists of a small chamber containing a core of a metal with a negligible coefficient of expansion such as Invar (A). Between the core and the walls of the chamber is an annular space which is filled with the mercury C. On any change in ambient temperature, the change in volume of the annular space, due to the expansion of the outer casing B, is sufficient to accommodate any variation in volume of mercury in the adjoining length of capillary, and so prevent it exerting an effect on the Bourdon tube. (The general effect is shown at the bottom of *Fig.* 119.)

It would be possible, of course, to adopt precisely the same principle by inserting a length of low expansion wire acting as a core running the whole length of the capillary. The diameter of the core and capillary are then proportioned so that the change in annular volume is sufficient to compensate for the change in mercury volume.

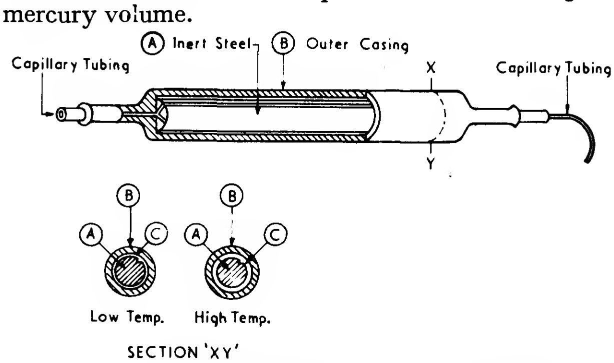


Fig. 119. *A section through the compensating link*

Double Capillary and Bourdon Tube

An alternative compensating device fairly widely adopted is the double capillary and Bourdon tube. In this method, a length of capillary cf the same diameter as that connected to the bulb is run from the neighbourhood of the bulb to the instrument, where it is joined to a second Bourdon tube. This second system is filled with liquid under similar conditions to that of the measuring system, but is made to act on the instrument pointer in an opposite sense to that of the measuring system (see *Fig.* 120).

Since both capillaries and Bourdon tubes are subject to the same conditions it can be seen that the effect of one system due to ambient temperature changes is immediately counteracted by the other and any significant errors are prevented.

Bi-metallic Strip or Spiral

To prevent ambient temperature errors due to the Bourdon tube itself, it is possible to incorporate a bi-metallic spiral with the Bourdon such that the deflection of the spiral due to a temperature change opposes that due to the tube. See Bi-metallic Thermometers for an explanation of the operation of bi-metallic strips.

Other Errors

Barometric Pressure Errors

Since the pressure within a liquid expansion thermometer is very high, e.g. 1000lb/in² and upwards, the effect of any normal barometric change is without significance.

Head Errors

The situation of the bulb relative to the Bourdon tube in the instrument could lead to errors due to the head of mercury above or below the Bourdon tube. If H is the vertical height involved in either case, and ρ is the density of the liquid filling, the pressure due to the head is $H\rho$ lb/in². If the bulb is above the instrument, this pressure causes a positive error, and if below the instrument, a negative error. In the former case, therefore, the reading is too high, and the head error must be subtracted, whilst in the latter case the head error must be added.

Very approximately, one foot of mercury exerts a pressure of 6 lb/in², so that a few feet variation between calibration conditions and installation conditions as regard the relative placing of the bulb and instrument will not cause serious errors. The thermometer is normally calibrated at the works of the supplier,

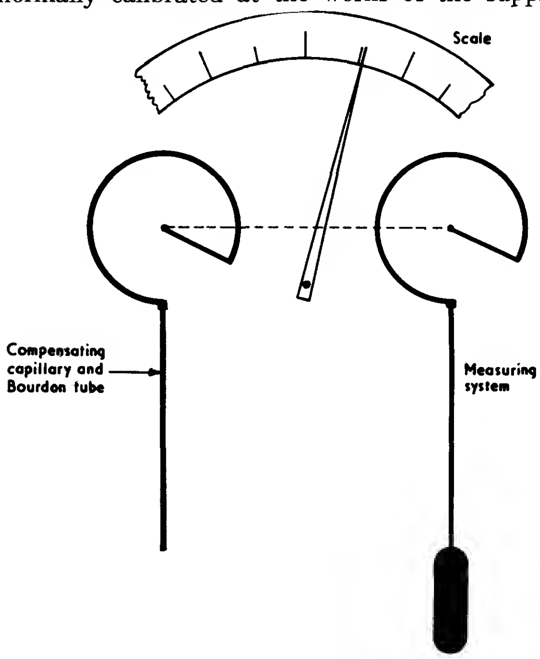


Fig. 120. *Double capillary and Bourdon tube*

with the bulb in the correct position. It is possible, however, to reset the pointer on site for small head errors.

2. GAS EXPANSION INSTRUMENTS

We have the same fundamental elements as for the liquid expansion type of thermometer, i.e. bulb, capillary, and Bourdon, the system so formed being filled with a gas at high pressure. The operation of the instrument, however, depends on a different law. The volume of the metal bulb will vary due to expansion with temperature, but the value of the change is relatively small, and the whole system may be considered as one of constant volume. The gas law then applies (an ideal gas is assumed for explanatory purposes):

$$pv = RT \quad \ldots\ldots\ldots\ldots \quad (151)$$

where p = the absolute pressure in lb/in² of the gas.

v = specific volume $= \dfrac{\text{volume}}{\text{mass}}$ = in³./lb.

R = gas constant for the gas being used, in appropriate units.

T = absolute temperature (°C. + 273 or °F. + 460).

Since the volume and mass of gas remain constant in the gas thermometer, and R is a constant, the pressure p can be regarded as proportional to T. In other words, the action of inserting the bulb at the point where the temperature is to be measured causes a pressure to be set up in the gas proportional to the temperature. The Bourdon tube is acting directly as a pressure responsive device. The deflection is proportional to the pressure and hence to the temperature being measured.

Equation (151) may be extended to show the operation of the thermometer.

If p_2 is the pressure at the maximum temperature T_2 and p_1 is the pressure at the minimum temperature T_1. Substituting in equation (151):

$$p_1v = RT_1 \quad \ldots\ldots\ldots\ldots \quad (152)$$

$$p_2v = RT_2 \quad \ldots\ldots\ldots\ldots \quad (153)$$

and from these,

$$\frac{p_2 - p_1}{p_1} = \frac{T_2 - T_1}{T_1} \quad \ldots\ldots\ldots \quad (154)$$

$T_2 - T_1$ is then the temperature range of the instrument, and $p_2 - p_1$ the corresponding pressure range the Bourdon tube must undergo. If we call Y this range, equation (154) can be written:

$$p_1 = Y \times \frac{T_1}{T_2 - T_1} \quad \ldots\ldots\ldots \quad (155)$$

The maximum and minimum temperatures of the range must always be known.

The above equations indicate that the instrument scale is linear.

Ambient Temperature Errors

As in the liquid expansion type the effect of ambient temperature changes on the capillary and Bourdon tube can produce errors. Compensation is complicated in the gas expansion thermometer since the error for the same ambient temperature change increases as the bulb temperature increases, but decreases as the differential range of the instrument increases. We may, however, still make the bulb volume so large in relation to the capillary and Bourdon tube volume that the errors are reduced within specified limits. Because of the relation between ambient temperature error and bulb temperature, the double system method can only compensate fully at one temperature, and partially at all others.

The pressure of a gas at constant volume increases by 1/273 of its pressure at 0°C. for each degree centigrade change. In the double compensating system

suppose the ambient temperature round the capillaries increases by 5°C. The pressure in the compensating system capillary increases by 5/273 of the pressure at 0°C. In the measuring system, this pressure change will be reduced by gas flowing into the bulb from the capillary until the pressures are equalized. The increase in pressure in this case depends on the relation of capillary volume to that of the bulb, but obviously will be very much less than that of the compensating system.

The error allowed by British Standard Specification No. 1041 is 2 per cent. of the instrument range for a 30°F. ambient temperature change of capillary and/or Bourdon tube.

Using the large bulb method, lengths of capillary up to 300 ft may be supplied, but with the double system with only partial compensation, the length may be reduced to about 100 ft.

Suitable Gases

Nitrogen is almost universally used as the filling gas. It is inert, has a high cubical coefficient of expansion, and is easily obtainable.

Ranges

With gas expansion thermometers the maximum and minimum measuring temperatures recommended by British Standard Specification No. 1041 are:

Minimum Temperature	Maximum Temperature
− 60°F. (− 51°C.)	1000°F. (538°C.)

The maximum limit may be modified by the material of which the bulb is made. It is found at the higher temperatures that certain metals become porous to gases, thereby lowering the pressure in the system. Some manufacturers, therefore, specify a maximum temperature of 800°F. (427°C.) for their products.

Other Errors

Head Errors

The density of nitrogen is extremely small and head errors are negligible in the case of a gas thermometer.

Barometric Pressure Errors

Dependent on the filling pressure, day-to-day changes in barometric pressure may produce small errors. The use of the thermometer at altitudes appreciably different from that at which the thermometer was calibrated can also cause errors.

3. VAPOUR PRESSURE THERMOMETER

Again the closed system of bulb, capillary and Bourdon tube is used. The vapour from a liquid exerts its own pressure, increasing with temperature, and this is particularly evident in a confined space. If, therefore, we introduce a suitable vaporizable liquid into the closed system a means of temperature measurement is at hand, the Bourdon tube again being used as a pressure responsive device.

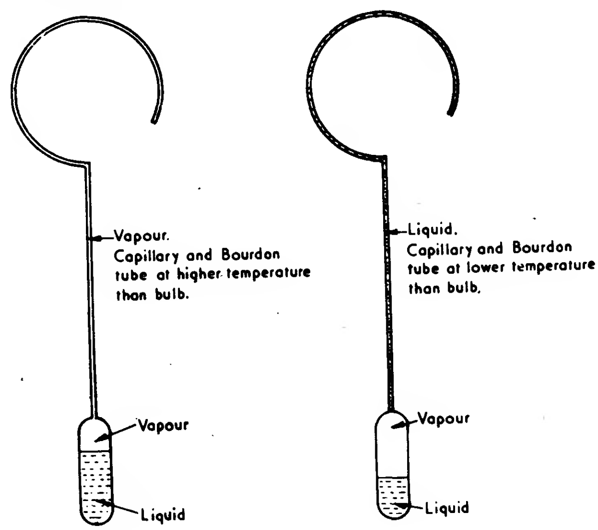
Certain precautions must be observed with this type of thermometer. These are:

1. The vapour must be present in the bulb under saturated conditions. This means that for any one temperature, the maximum amount of vapour has been achieved. The production of any more would cause condensation of vapour into liquid again. Under such conditions, the vapour pressure is independent of the volume of liquid present in the bulb, being dependent only on the temperature. To obtain this state pratically, some liquid must always be present in the bulb over the whole of the temperature range of the thermometer, and the quantity of liquid inserted in the bulb at the filling stage must be sufficient to meet this condition. In other words, there must always be a liquid-vapour surface in the bulb.
2. Two conditions will have to be met, possibly in the same instrument:
   (*a*) Where the bulb is at a higher temperature than the capillary and Bourdon tube (normal temperature measurement).
   (*b*) Where the bulb is at a lower temperature than the capillary and Bourdon tube (refrigeration, etc.).

   In (*a*) the vapour generated from the bulb will condense in the cooler capillary and Bourdon tube to form a liquid which then acts as a transmitting means for conveying the pressure of the vapour in the bulb to the Bourdon tube. The volume of liquid placed in the bulb must be sufficient to fulfil this condition as well as that specified in 1. (*Fig.* 121).

   In (*b*) the reverse phenomena will occur. The capillary and Bourdon tube being at a higher temperature than the bulb will be filled with a superheated vapour, and the liquid will exist only in the bulb. The amount of liquid here again must meet condition 1. The vapour pressure in the bulb will be communicated to the Bourdon tube via the vapour in the capillary (*Fig.* 121).
3. Arising from the points discussed in paragraph 2, it can be seen that it is undersirable to measure ambient temperatures by means of vapour pressure thermometers since the liquid in the capillary and Bourdon tube may be passing into vapour, or the vapour therein may be condensing as liquid. Each time a lag is produced which results in unstable instrument readings.
4. The maximum temperature of the instrument range must be below the critical temperature of the liquid.

Fig. 121. *Conditions 1 and 2 in the vapour pressure thermometer*



5. The minimum temperature of the instrument range must be above the boiling point of the liquid.

Suitable Liquids

Fig. 122 shows temperature-pressure relations for six of the most common vaporizable liquids, and the table gives some properties of these same liquids. It will be observed from the curves that the scale shape will not be linear. The divisions will be smaller at the lower end and larger towards the maximum end.

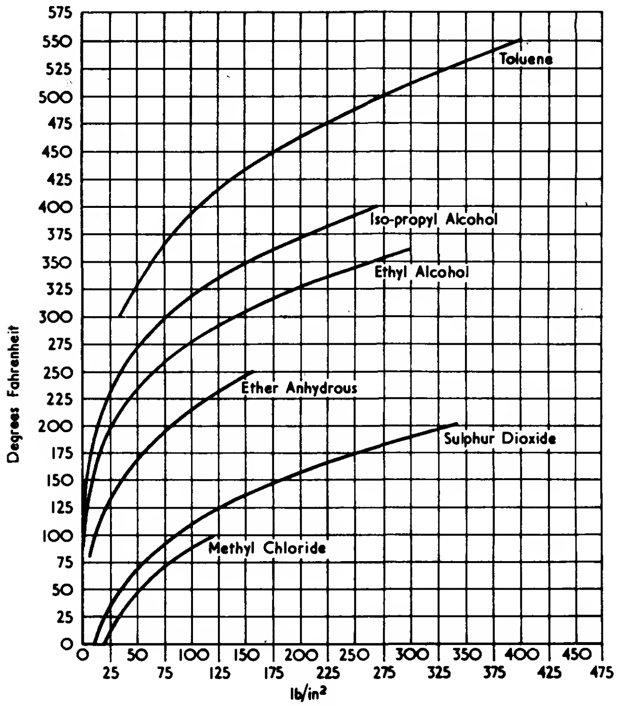


Fig. 122. *Temperature-pressure characteristics for six common vaporizable liquids*

Liquid	Boiling Point		Critical Temp.	
	°C.	°F.	°C.	°F.
Methyl chloride	− 23·7	− 10·7	143·1	289·6
Freon	− 29·2	− 20·6	—	—
Sulphur dioxide	− 10·0	+ 14	157·2	315·0
Alcohol	78·5	173·3	243·1	469·6
Toluene	110·5	230·9	320·6	609·1
Ether	34·5	94·1	193·8	380·9

Ranges

British Standard Specification No. 1041 recommends the following minimum and maximum measuring temperatures:

Minimum Temperature	Maximum Temperature
− 60°F. (−51°C.)	500°F. (260°C.)

Some manufacturers extend the maximum temperature to 650°F. (343°C.).

Ambient Temperature Errors

In condition 1, it was stated that the vapour pressure is independent of the volume of liquid in the bulb. Consider the two most common cases likely to arise.

Capillary and Bourdon Tube Temperature below Bulb Temperature

In such a case, the capillary and Bourdon tube are full of liquid. An increase in ambient temperature will cause the liquid to expand and drive a small amount back into the bulb. A decrease in ambient temperature will cause the liquid to contract, the bulb having to supply a small volume. In either case, however, since condition 1 holds, the temperature is unchanged by any change of liquid volume.

Capillary and Bourdon Tube Temperature above Bulb Temperature

The capillary and Bourdon tube are full of vapour in the superheated condition, and are independent of ambient temperature changes.

For normal temperature changes, therefore, the instrument suffers no errors. If large fluctuations occur in the Bourdon tube neighbourhood, its physical characteristics may be affected.

British Standard Specification No. 1041 allows an error of 1 per cent of the range for a 30°F. ambient temperature change.

Head Errors

When the bulb is at a higher level than the indicating or recording instrument a head error may be introduced. For small values the instrument pointer can be reset on site. If the indicator is to be installed more than 10ft below the bulb this fact should be stated when ordering the thermometer.

Barometric Pressure Errors

Day-to-day changes in barometric pressure and appreciable changes in altitude can cause errors. Pointer resetting is possible if the errors are small.

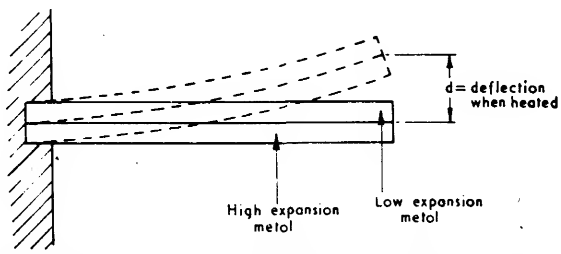


Fig. 123. *Bi-metal strip deflected by unequal expansions of the different metals*

4. BI-METALLIC THERMOMETERS

A type of thermometer widely used is that constructed from a helical element. The helix is formed from a strip of bi-metal. This consists of two metals welded together. One of the pair has an expansion coefficient much greater than the other. Consider one end of the strip to be fixed and the other left free. The application of heat causes the free end to deflect due to the unequal expansions of the metals forming the strip (*Fig.* 123). If the strip is now wound

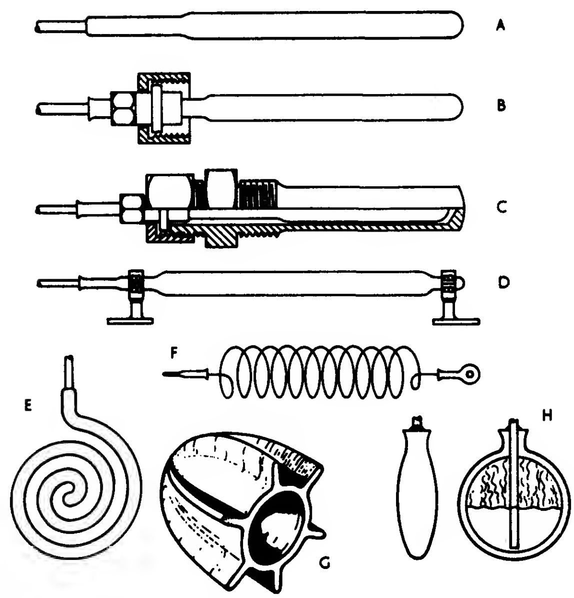


Fig. 124. *Various bulb shapes*

into helical form, again with one end fixed and the other left free, the latter can be made to give a rotary movement to a spindle, when heat is applied. From such an element a thermometer can be constructed since a pointer can be attached to the spindle and a scale added for measuring purposes.

For the low expansion member of a bi-metal strip Invar is commonly used. The high expansion member may be brass or one of the nickel alloys.

The range covered is approximately from $-40°$F. to 600°F.

Accuracy is usually $\pm 1\%$ of the range.

Note one great advantage of this pattern of thermometer—it does not suffer from head cr barometric pressure errors.

RESPONSE

Response times of the thermometers described in this chapter will be discussed in Chapter 15.

BULB SHAPES

Fig. 124 indicates various shapes of bulbs encountered in industry.

Chapter 9

ELECTRICAL THERMOMETERS

THERMOCOUPLES

A thermocouple is not a new device, the thermoelectric effect having been demonstrated as far back as 1821 by Seebeck. In brief, he took two wires of different metals and joined the ends together so that there were two junctions, the wires being separated except at the joints. On heating one junction to a higher temperature than the other he discovered that an e.m.f. was produced causing a current to flow round the loop or circuit formed by the two joined wires (*Fig.* 125). The e.m.f. and current, moreover, were found to vary with the temperature difference between the two junctions.

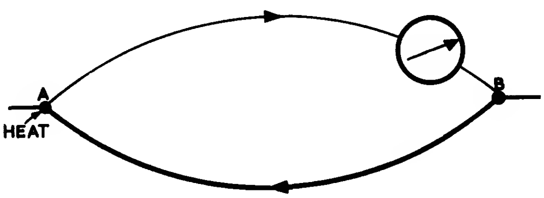


Fig. 125. *Basic thermocouple circuit*

Such an arrangement is called a thermocouple. The junction at the higher temperature is termed the *hot* junction, and that at the lower temperature the *cold* or *reference* junction.

If E is the value of the thermocouple e.m.f., T_1 the temperature of the hot junction, and T_2 that of the cold junction, from theoretical considerations it may be shown that

$$E = a(T_1 - T_2) + b(T_1^2 - T_2^2) \quad \ldots (156)$$

a and b are constants depending upon the metals used. The first term has by far the greater effect.

We have a basis of a temperature measurement since by inserting junction A in the medium whose temperature is to be measured and keeping B at a lower temperature, a measurable e.m.f. is produced, the value depending upon the relative values of T_1 and T_2. But if the e.m.f. is to be a reliable indication of the temperature T_1 of the hot junction, from equation (156) it can be seen that it is essential either to keep the cold junction temperature T_2 constant, or to introduce into the circuit some compensating feature which will nullify a change in T_2. Both schemes are used, the former more for laboratory work and the latter for industrial purposes.

Before discussing practical arrangements there are two laws which should be stated: the Law of Intermediate Temperatures and the Law of Intermediate Metals.

Law of Intermediate Temperatures

The two junction temperatures have been taken as T_1 and T_2 in the foregoing explanation. Let T_3 be a temperature intermediate to T_1 and T_2. By the Law of Intermediate Temperatures the e.m.f. can be considered as the algebraic sum of the two e.m.f.'s produced:

(*a*) With the hot junction at T_1, and cold junction at $T_3 - E_1$

(*b*) With the hot junction at T_3, and cold junction at $T_2 - E_2$.

The law is illustrated in *Fig.* 126, and a numerical example is given below from actual published figures.

(*a*) e.m.f. E, with hot junction at $T_1 = 100$°C and cold junction at $T_2 = 0$°C

$$E = 5{\cdot}09 \text{ millivolts.}$$

(*b*) e.m.f. E_1, with hot junction at $T_1 = 100$°C and cold junction at $T_3 = 20$°C

$$E_1 = 4{\cdot}09 \text{ millivolts.}$$

(*c*) e.m.f. E_2, with hot junction at $T_3 = 20$°C and cold junction at $T_2 = 0$°C

$$E_2 = 1{\cdot}00 \text{ millivolt.}$$

By the Law of Intermediate Temperatures,

$$E = E_1 + E_2$$

$$\text{i.e., } 5{\cdot}09 = 4{\cdot}09 + 1 \text{ millivolts.}$$

$$5{\cdot}09 = 5{\cdot}09 \text{ millivolts.}$$

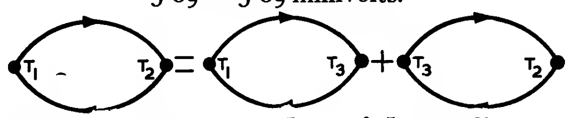


Fig. 126. *Illustrating the Law of Intermediate Temperatures*

Tables of thermoelectric e.m.f.'s and temperatures are published with the cold or reference junction at one fixed temperature, generally 0°C (32°F). When, however, it is desired to refer to another cold junction temperature, say 15°C, the Law of Intermediate Temperatures enables a simple subtraction to be made to achieve this.

Law of Intermediate Metals

Suppose now that we introduce a third wire, BC, of a different metal into our loop at the cold junction end (*Fig.* 127). The Law of Intermediate Metals allows the circuit e.m.f. to remain unaltered provided both of the new junctions B and C, formed by introducing the wire, remain at the same temperature as that of the original single junction. We are not restricted to the addition of a single wire. More than one may be connected in circuit, of varying metals, but provided the temperature of each new junction is the same as that of the original, the e.m.f. is unchanged. The importance of

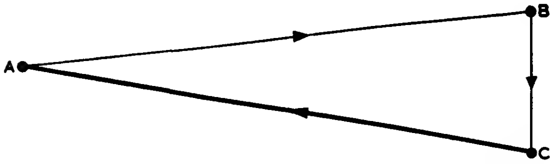


Fig. 127. Illustrating the Law of Intermediate Metals

this can now be seen. Instead of the wire *BC*, we can connect between *B* and *C* a millivoltmeter to measure the circuit e.m.f., and although it may have brass terminals *FF*, copper internal connections with a copper coil *D*, and a manganin resistance bobbin *E* (*Fig.* 128), these do not affect the value of the thermocouple e.m.f. But what is the cold junction? Since *B* and *C* are at the same temperature, the link *BC* can be regarded as providing electrical connection only between *B* and *C*, and the latter path now represents the new cold junction. The importance of all dissimilar metallic junctions in this path remaining at the same temperature can be realized. If one or more should differ in temperature from the others, additional e.m.f.'s will be set up in the circuit, causing errors in measurement.

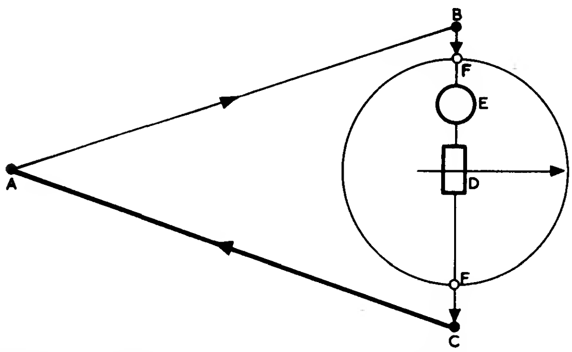


Fig. 128. Practical application of the Law of Intermediate Metals

Positive and Negative Wires

The current produced in the thermocouple circuit will flow from one wire to the other at the cold junction. The wire the current flows from is termed the positive wire and the wire into which the current flows the negative one.

Thermocouple Materials

There are two broad groups of thermocouples: rare or precious metal and base metal.

RARE METAL GROUP

Positive Wire	Negative Wire	Recommended Maximum Working Temperatures (B.S. 1041, Part 4)	
		Continuous	Spot Readings
Alloy of 90% Platinum 10% Rhodium	Platinum	1400°C	1650°C
Alloy of 87% Platinum 13% Rhodium	Platinum	1400°C	1650°C

BASE METAL GROUP

Positive Wire	Negative Wire	Recommended Maximum Working Temperatures (B.S. 1041, Part 1)	
		Continuous	Spot Readings
Copper	Constantan (or Eureka). Alloy of 40% Nickel 60% Copper approx.	400°C	500°C
Iron	Constantan	850°C	1100°C
Chromel* Alloy of 90% Nickel 10% Chromium	Alumel* Alloy of 94% Nickel, 2% Aluminium Silicon and Manganese in varying amounts	1100°C	1300°C
Nickel-Chromium alloys	Nickel-Aluminium alloys	1100°C	1300°C

*—Chromel and Alumel are registered trade names of the Hoskins Manufacturing Co. (British agents: R. G. McLeod).

Other Classes of Thermocouples

Considerable research has been made into the subject of combinations of pure metals and alloys other than those quoted above. There is, for example, the tungsten/tungsten 26% rhenium thermocouple for which the extremely high upper limit of 4200°F (2330°C) is claimed. A development in the platinum-rhodium group is the platinum 30% rhodium/platinum 6% rhodium thermocouple with a limit for continuous operation of 2732°F (1500°C) and intermittent operation 3272°F (1800°C). In addition, combinations using molybdenum and iridium have also been tried in the field. Reference (5) summarizes the characteristics of the newer versions, and B.S.1041 Part 4 gives some data.

Effect of Atmosphere on Thermocouples

Thermocouples cannot be used indiscriminately. Some of those described suffer oxidation at elevated temperatures, and must be used in neutral or reducing atmospheres. The suitability of any thermocouple for a particular application must always be checked from this aspect.

Industrial Types of Thermocouples

For industrial work the thermocouple is constructed on the lines of *Fig.* 129. The two metal wires are twisted together at one end and welded to form the hot junction *A*. From the hot junction the wires pass through ceramic insulators *D* to terminals *B* and *C* on an insulating head *E*. A thermocouple may be used

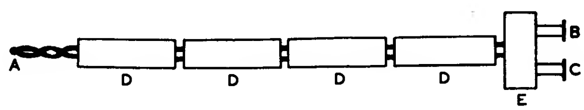


Fig. 129. General form of construction of industrial type thermocouple

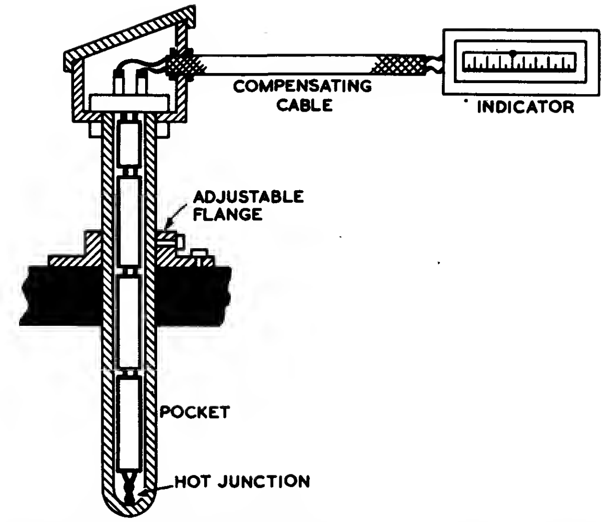


Fig. 130. *Basic arrangement of thermocouple installation in industrial plant*

bare, but in the majority of applications it is necessary to insert it in a metallic pocket with a protecting head. The pocket can take the form of *Fig.* 130 which is equipped with an adjustable flange to allow the pocket position to be varied, or it can take the form of a screwed pattern pocket similar to that shown in *Fig.* 124 of Chapter 8.

Measuring Instruments

Measuring instruments fall into two main classes. In one, moving coil movements are used as millivoltmeters to measure the thermocouple e.m.f. They are calibrated, of course, directly in temperature units. In the other, the potentiometer circuit is used as a basis of measurement particularly in the null-balance type of instrument. The potentiometer circuit is discussed under Cold Junction Compensation, and a brief description of null-balance instruments appears at the end of the chapter. A fuller treatment will be found in Chapter 13 on Feedback.

Unless precautions are taken errors can be introduced in the thermocouple circuit, and the moving coil pattern of instrument has been chosen to illustrate these.

Cold Junction Location

The measuring instrument may be a large number of feet from the thermocouple. Supposing that the terminals of the thermocouple shown in *Fig.* 129 are connected to the instrument by copper wires. The conditions are then of the basic circuits, *Fig.* 127 or *Fig.* 128. The link BC represents the cold junction. But B and C are relatively near the hot junction A and could, therefore, be at a moderately high temperature, thus reducing the thermocouple e.m.f. In addition, the presence of the cold junction near the hot one could lead to severe fluctuations in temperature. But if the cold junction is transferred to the instrument itself,

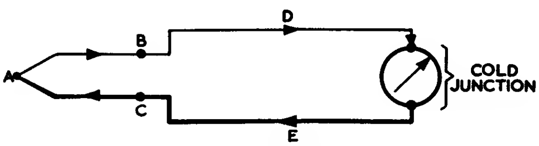


Fig. 131. *Cold junction location in thermocouple installation*

usually mounted well away from the hot junction in a location at normal room temperature, a bigger temperature difference is realised and the cold junction temperature is less liable to severe fluctuations.

To achieve this the connecting leads D and E of *Fig.* 131 must be of the same materials as the thermocouple or be of materials possessing the same or similar thermoelectric properties as those of the thermocouple. For copper/constantan thermocouples leads of the same materials are used. For the nickel chromium/nickel aluminium type of thermocouples, the same materials may be used depending on the length involved. If this should be fairly long, leads of iron and a copper-nickel alloy can be substituted to reduce the cost. In the case of precious metal thermocouples, the use of the same materials would prove prohibitive from the cost point of view and leads of copper and a copper-nickel alloy (or copper and nickel) are installed.

In general, such connecting leads are called *compensating leads* or *cables.*

The following is a table of thermocouple wires and corresponding compensating leads:

Thermocouple		Compensating Leads	
Positive Wire	Negative Wire	Positive Wire	Negative Wire
Copper	Constantan	Copper	Constantan
Iron	Constantan	Iron	Constantan
Chromel	Alumel	Chromel	Alumel
Chromel	Alumel	Iron	Copper-Nickel Alloy
Platinum-Rhodium (10 or 13%)	Platinum	Copper	Copper-Nickel Alloy

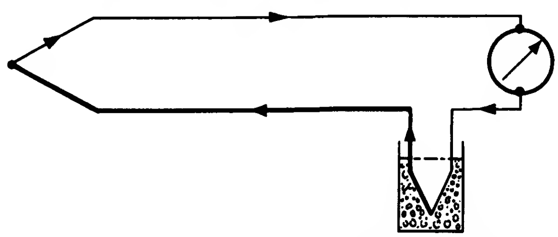


Fig. 132. *Cold junction located in melting ice*

Cold Junction Temperature Variation

The millivoltmeter used for thermocouple work is of the moving coil type in which the moving coil deflection is proportional to the current flowing through it, and, hence, to the thermocouple e.m.f. The latter, as we have seen, is proportional to the temperature difference ($T_1 - T_2$). If we desire the e.m.f. to be a true measure of T_1 it is imperative either to keep T_2 constant or to introduce into the system some compensating device which will nullify any variations in T_2.

Fixed Cold Junction Temperature

Proceeding on the first basis, the cold junction T_2 could be maintained in a chamber of melting ice, i.e., at 0°C, which would give us an accurately known fixed temperature. The instrument is connected in circuit as indicated in *Fig.* 132. Alternatively the cold junction could be kept in a temperature-controlled chamber.

Such devices are available for industrial purposes. But normally recourse is made to the second scheme of introducing a temperature compensating device. In the vast majority of cases the device is a bi-metallic strip.

Bi-metallic Compensation

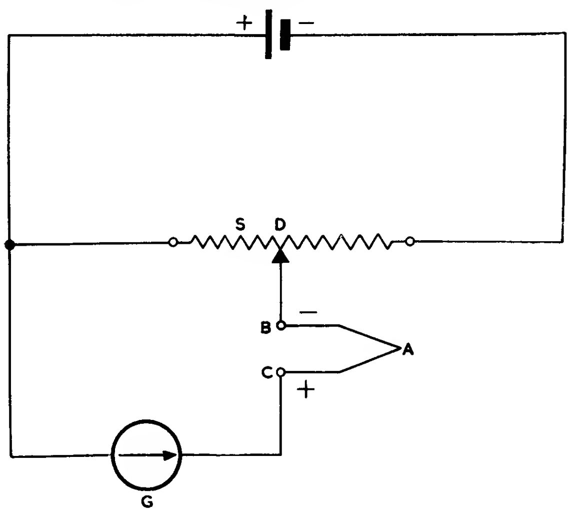
We have seen in Chapter 8 that the strip consists of two metals, bonded together, of unlike expansion coefficients. A loop of bi-metallic material is introduced into the hairspring system of the moving coil in the millivoltmeter. When the ambient temperature changes, the thermocouple e.m.f. varies, and normally would affect the reading of the instrument. The bi-metallic strip deflects at the same time, however, in such a direction as to oppose that of the moving coil. By knowing the values of the torque exerted by the coil and the strip, it is possible to arrange for satisfactory compensation for ambient temperature variation. An instrument equipped with such a device is said to have automatic cold junction compensation.

Potentiometer Circuit

It is not proposed to discuss the fundamental principles of the potentiometer, but to explain how it is used in thermocouple work. The basic circuit is shown in *Fig.* 133. The e.m.f. due to the couple is balanced against the voltage drop along the slide wire *S* until the instrument *G* reads zero. The position of the contact arm *D* is then a measure of the thermocouple e.m.f. and, hence, the temperature. Automatic cold junction temperature compensation is introduced into the circuit by means of a nickel resistance spool *N*, indicated in *Fig.* 134. *M* helps to reduce the current taken from the battery to 1 or 2 milliamps in this branch. *A* is a scale suppression resistance which enables the instrument scale to commence from some other temperature than zero, whilst *B* reduces the current through the slide wire *S* to 1 or 2 milliamps. *R* is a voltage adjusting resistance. *A*, *B*, *S*, and *M* are of manganin or similar material

As the temperature of the instrument changes, the resistance of the nickel spool varies, and injects into the circuit a small e.m.f. increase or decrease to counter-balance that produced by the cold junction temperature change.

Fig. 133. *Basic potentiometer circuit*



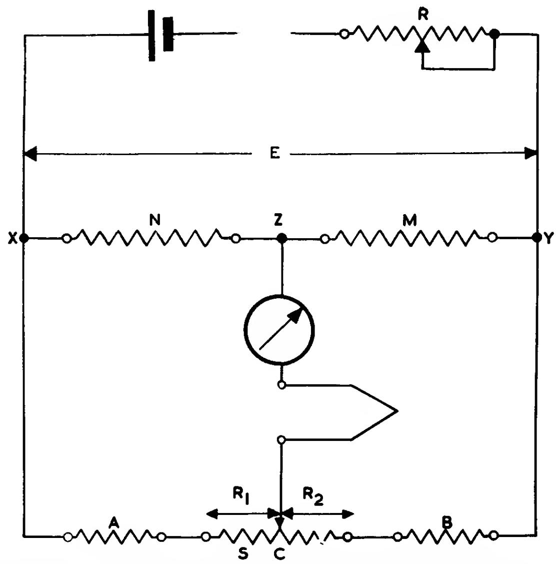


Fig. 134. *Potentiometer circuit with cold junction compensation*

Variation of Resistance of Instrument Coil with Temperature

The coil of the millivoltmeter type of instrument used in thermocouple work is made from copper or aluminium wire. The resistance of this varies with temperature, and it is necessary to examine the value of this in the light of introducing an error into the measurement. A typical coil may have a resistance of 50 ohms, and the e.m.f. for a full-scale deflection about 5 millivolts. This means the current is limited to 1/10 of a milliamp. Let us suppose that the range of

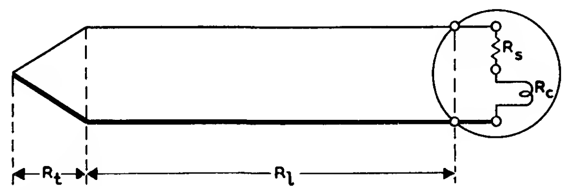


Fig. 135. *Thermocouple circuit indicating resistances involved*

the instrument is 0 to 700°C and that iron-constantan thermocouples are used. With the cold junction at 0°C the millivolt span is 38·21 millivolts, taking some typical figures. 38·21-5 millivolts must be absorbed outside the coil itself and a resistance in series with the coil is necessary (sometimes referred to as the swamp resistance). From the electrical standpoint the circuit now becomes *Fig.* 135.

In this figure, R_t = thermocouple resistance
R_l = resistance of lead wires
R_c = coil resistance
R_s = series or swamp resistance

R_s is wound from a wire possessing negligible resistance change with temperature.

Giving practical values to these resistances,

R_t = 1 ohm
R_l = 5 ohms (10yd of iron-constantan cable)
R_c = 50 ohms
R_s = 326·1 ohms
Total resistance = 382·1 ohms.

Considering a copper coil, the variation in resistance with temperature is given by:

$$R_T = R_{15}(1 + a(t - 15)) \quad . \; . \; . \; (157)$$

where R_T = the resistance at temperature T°C

R_{15} = the resistance at 15°C

a = 0·00426 (the temperature resistance coefficient).

15°C has been taken as an average room temperature. Suppose the temperature increases to 20°C, a reasonably higher limit of variation. The resistance at 20°C of a 50 ohm coil is, using formula (157), 51·065 ohms.

The total circuit resistance has now increased to 383·16 ohms, a percentage change of ¼ per cent approximately. This would be the percentage error on the indicating or recording instrument, since the deflection is proportional to the current, and the current has been reduced by ¼ per cent. Such an error is not particularly serious. It can be seen, however, that if the swamp resistance should become too low in value the percentage error would increase in proportion. With R_s equal to 100 ohms, the error is nearly 1 per cent. Paragraph 106 of British Standard Code No. 1041, "Temperature Measurement", suggests that for millivoltmeters with a range up to 0-20 millivolts the maximum error should not exceed ¼ per cent for a change of 1°C in the ambient temperature. It is desirable on this basis to keep the internal resistance as high as possible.

External Resistance Variation

In the preceding section, we have seen how the change in internal resistance of the instrument can affect the readings. External resistance changes can also introduce errors. Referring to *Fig.* 135, we see that the external resistance comprises R_t and R_l. Of these, R_t is usually very small compared with the other circuit resistances and any change in its resistance is not so important as that of the compensating leads R_l. If E is the thermocouple e.m.f. in millivolts, the millivoltage e appearing at the terminals of the instrument is

$$e = E\left(\frac{R_s + R_c}{R_s + R_c + R_t + R_l}\right) \quad . \; (158)$$

The current through the instrument is

$$i = \frac{E}{(R_s + R_c + R_t + R_l)} \quad . \; . \; (159)$$

Any alteration in R_l will, therefore, affect the current i and the instrument reading.

If we wish to fix a maximum percentage error a in the current or instrument reading,

$$a = \frac{\delta R_l \times 100}{(R_s + R_c + R_t + R_l)} \quad . \; . \; (160)$$

where δR_l is the maximum variation in R_l allowable so that this percentage error is not exceeded.

$$\delta R_l = a\left(\frac{R_s + R_c + R_t + R_l}{100}\right) \quad . \; (161)$$

As an example let us take the preceding figures for the resistance values, and stipulate that a shall not exceed ¼ per cent.

$$\delta R_l = \frac{1}{4} \times \frac{382 \cdot 1}{100}$$

$$\delta R_l = 0 \cdot 956 \text{ ohm.}$$

It is customary to mark the external resistance on indicating or recording instruments.

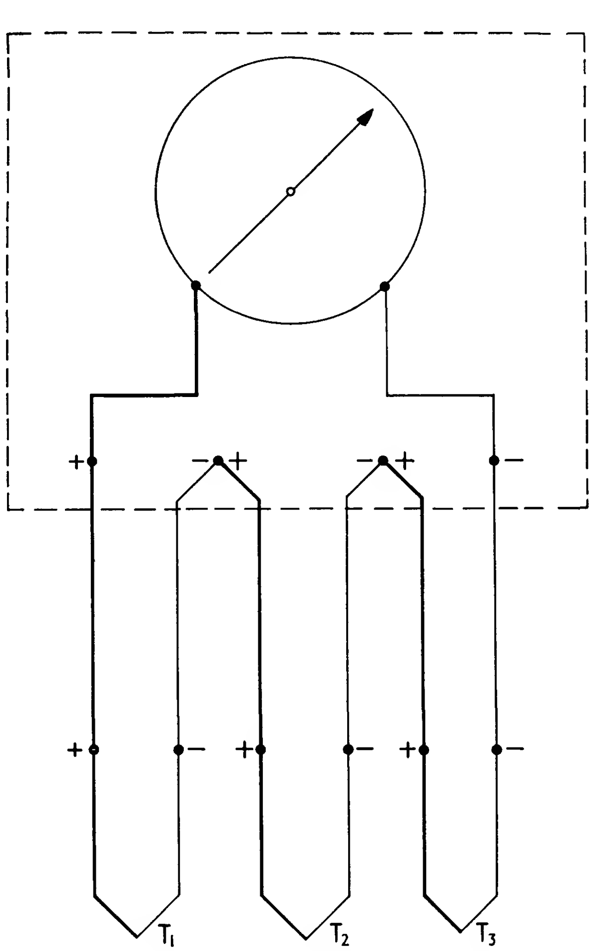


Fig. 136. *Measuring average temperatures by means of thermocouples*

Measurement of Average Temperature by Thermocouples

If we have a series of temperature measurements whose millivolt outputs are e_1, e_2, e_3, etc., the true average is

$$E_{av} = \frac{e_1 + e_2 + e_3 + \ldots e_n}{n} \quad . \; . \; . \; . \; . \; . \; . \; . \; (162)$$

Since n is constant, however, the sum of $(e_1 + e_2 + e_3 + \ldots)$ is a measure of the average temperature. The thermocouples are therefore connected in series (*Fig.* 136) and the instrument calibration based on the sum of the e.m.f.'s. The individual compensating cables must be carried up to the instrument.

Measurement of Temperature Difference by Thermocouples

For measuring temperature difference it is only necessary to connect the thermocouples in opposition (see *Fig.* 137). If e_1 and e_2 are the two e.m.f.'s in question, t_1 and t_2 are the corresponding hot junction temperatures, and t_0 is the cold junction temperature,

$$e_2 - e_1 \, \alpha(t_2 - t_0) - (t_1 - t_0) \quad . \; . \; . \; . \; (163)$$

or,

$$e_2 - e_1 \, \alpha(t_2 - t_1) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (164)$$

It will be observed that the cold junction term has disappeared. In consequence, the connections of *Fig.* 137 may be used with a saving of compensating cable since EH and GJ may be of copper.

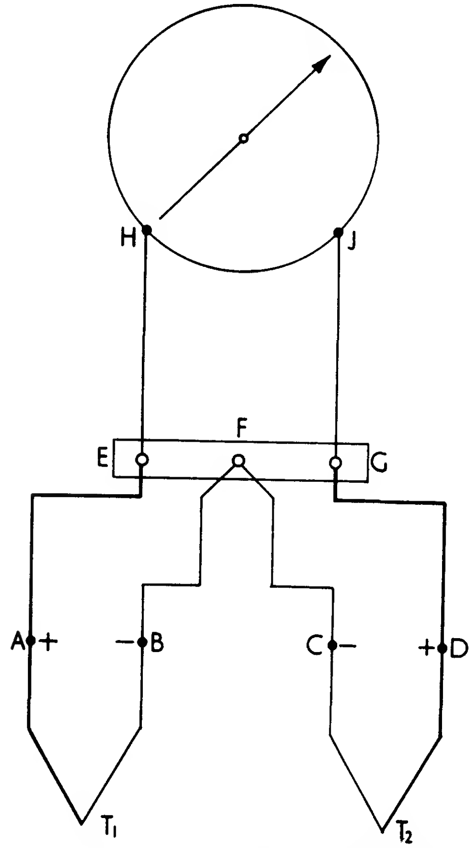


Fig. 137. *Measuring temperature difference by means of thermocouples*

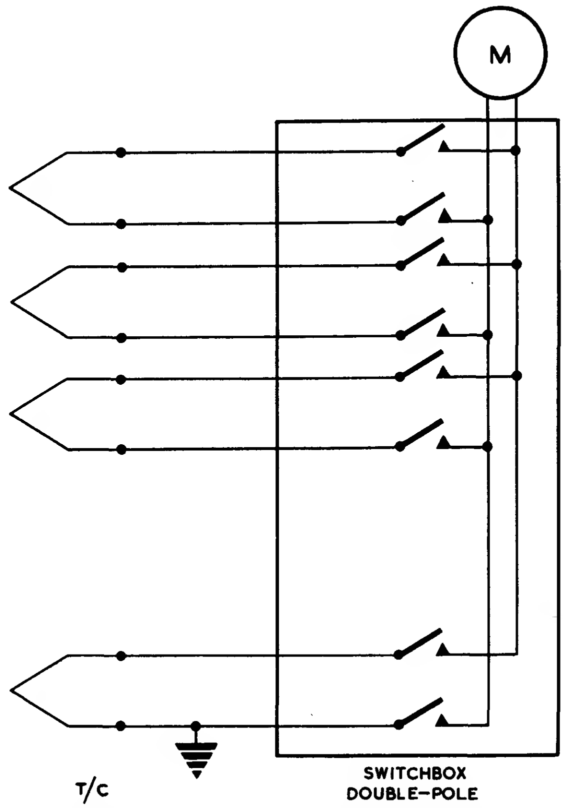


Fig. 139. *Multipoint switching for thermocouples utilizing double pole switching*

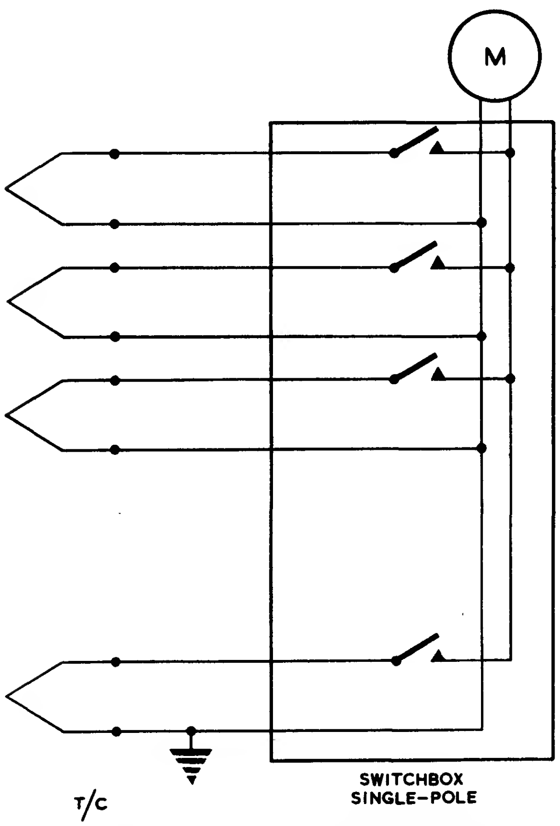


Fig. 138. *Multipoint switching for thermocouples utilizing single pole switching*

Multipoint Installations

Where is it desired to measure the temperature at a number of points for indicating purposes a multipoint switch box is installed, and for recording purposes the switching is performed automatically by mechanism operated by a small motor. The contact resistances of the actual switches are of some importance as any non-conducting films on the contacts are liable to cause trouble, since only a very small millivoltage is available to break down the films. The devices are numerous and range from rotary stud patterns to leaf spring types and mercury switches. Since two leads from the couple are brought to the switch box, double pole switching is logical, but is not always adopted. Single pole switching with a common lead to the millivoltmeter is often used. This has the disadvantage that an earth on this lead appears on all of the couples. *Fig.* 138 indicates the basic circuit for single and *Fig.* 139 for double pole switching.

In many installations, the switching is performed manually, but on modern scanning systems, the switching is carried out automatically by mechanical switches of the uniselector type, by relays or by solid state switching circuits.

Response of Thermocouples

The thermocouple is invariably inserted into a metal pocket of sufficient thickness to withstand pressure and temperature conditions existing in the medium whose temperature is to be measured. Between the inside of the pocket wall and the couple itself an air space may exist. The dimensions of the pocket wall and the air space tend to introduce a lag into the

response of the couple to a temperature change. This is more serious in a control system than when indicating or recording, and will be discussed when dealing with instrument and element response in later chapters.

Materials for Thermocouple Pockets

The materials used for thermocouple pockets must be specified with care, due to the high temperature and corrosive nature of some applications.

The following is a short list of commonly used materials with the approximate limiting temperatures:

Brass	250°C
Copper	500°C
Steel	800°C
Stainless Steel	800°C
Heat Resisting Steel	1000°C
*Nichrome	1050°C
Fireclay	1200°C
Porcelain	1300°C
Sillimanite	1500°C
Mullite	1600°C

The above figures may be modified by the nature of the gases or liquids whose temperature is to be measured.

**Registered name British Driver Harris Co. Ltd.*

RESISTANCE THERMOMETERS

The variation of electrical resistance of materials with temperature affords a very convenient basis for temperature measurement. If we except thermistors and semiconductors, the materials used are metal wires.

The general requirements are:

1. As large a change in resistance as possible for a given temperature range, i.e., a large temperature-resistance coefficient.
2. A melting point well above the highest temperature likely to be measured.
3. Ability to produce a pure metal so that reproducibility is easily effected.
4. Physical and mechanical properties to be such that excessive manufacturing difficulties are not encountered.

Three metals are used industrially. They are platinum, nickel and copper. All three may be manufactured in a very pure condition. Platinum and nickel predominate in this country. The following table shows the melting points and the normal maximum measuring temperature:

	Melting Point	Normal Maximum Temperature (B.S. 1041 Part 3)
Platinum	1773·5°C	600°C
Nickel	1455°C	350°C
Copper	1083°C	250°C

Resistance-Temperature Relation

The resistance-temperature relation takes the following form:

$$R_T = R(1 + aT + bT^2 + cT^3 + \ldots) \quad \ldots (165)$$

where R_T is the resistance at T°C or F.

R is the resistance at some lower temperature, usually specified as 0°C or 32°F.

$a, b, c,$ are constants.

The number of constants involved depends on the temperature to be measured but, generally speaking, two would be sufficient up to 540°C.

Fundamental Interval

The difference in resistance of a thermometer between 100°C and 0°C is termed the fundamental interval, and is sometimes used in specifying the requirements for a thermometer. If R_{100} is the resistance at 100°C and R_0 the resistance at 0°C, $(R_{100} - R_0)$ is the fundamental interval. It appears in the equation stated by Professor Callendar:

$$T = \left[\frac{R_T - R_0}{R_{100} - R_0}\right]100 + \delta\left[\frac{T^2}{100^2} - \frac{T}{100}\right] \quad (166)$$

where T = the temperature in °C.

R_T = the resistance at T°C.

Comparing equations (165) and (166)

$$a = \frac{R_{100} - R_0}{100R_0}\left(1 + \frac{\delta}{100}\right) \quad \ldots\ldots (167)$$

$$b = -\frac{(R_{100} - R_0)\delta}{100^3 R_0} \quad \ldots\ldots\ldots\ldots (168)$$

The International Practical Scale of Temperature (1948) is suggested by B.S. 1904. Here:—

$$T = \frac{1}{\alpha}\left(\frac{R_T}{R_0} - 1\right) + \delta\left(\frac{T}{100} - 1\right)\left(\frac{T}{100}\right) + \beta\left(\frac{T}{100} - 1\right)\left(\frac{T}{100}\right)^3 \quad \ldots (169)$$

between—183°C and °C

$$T = \frac{1}{\alpha}\left(\frac{R_T}{R_0} - 1\right) + \delta\left(\frac{T}{100} - 1\right)\frac{T}{100} \quad \ldots (169a)$$

between 0°C and 630°C. α, β and δ are constants

Forms of Industrial Resistance Bulbs Resistance Value

As indicated in *Fig.* 140, a resistance spool is often located in the head of the thermometer. Known as the ballast resistor, it serves to adjust the temperature-resistance relation.

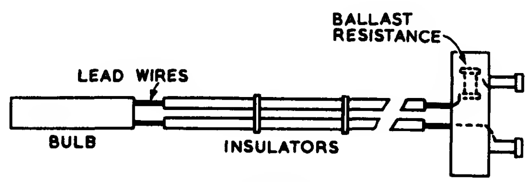


Fig. 140. *General construction of resistance pattern thermometer*

B.S.1904 specifies a ratio of R_{100}/R_0 of 1·3910. When the temperature does not exceed 600°C the fundamental interval is 38·50 ohms. Above 600°C, the fundamental interval becomes 38·50, 10·000, or 1·0000 ohms as required.

Power Dissipated in Thermometer

Any effects due to self-heating, i.e., heating due to the current flowing through the thermometer, must be minimized. For this reason, the power dissipated by the current in the thermometer should not exceed a small fraction of a watt e.g. 10 milliwatts.

Construction

Invariably the thermometer bulb consists of a coil of wire wound on a former. Since the thermometer is an electrical one, it is essential that the turns be insulated from each other, and from earth, at all temperatures likely to be encountered. With a few exceptions bare wire is used so that the former material should be non-hygroscopic.

The gauge of wire will depend to some extent on the resistance value, but values between 0·002in. and 0·008in. diameter are common for platinum.

A design widely used comprises a hollow cylindrical ceramic former about 2½in. long and ⅜in. diameter. On the outer surface of this a threaded groove is moulded. Bare wire is wound into the grooves, and to make as rigid an element as possible and also to insulate the exposed surface of the wire, a coating of heat resisting cement, porcelain glaze, or other high temperature insulating material is placed over the wound former. Just above the former, the platinum is sealed, preferably by welding, to leads extending down from terminals on an insulating head. The lead wire materials are of nickel or similar metal of much stouter gauge than the platinum wire. These must be insulated from one another and from any surrounding metal pocket. It is customary, therefore, to pass them through insulators of cylindrical or disc pattern. The latter have the advantage of occupying nearly the full diameter of a pocket, and of helping rigidity. In addition, they tend to prevent convection currents of air inside a pocket. The complete element takes the form of *Fig.* 140.

Attention must be drawn to another class of bulb, the Dynatherm of Foxboro-Yoxall Ltd. Here a metal core is used. On to the core is wound the resistance wire, in this case insulated nickel. The lower end of the metal core makes contact with metal foil, itself in thermal contact with the bottom of the protecting pocket. Such an arrangement produces a reasonably fast rate of heat transfer from medium to element. This particular design is suitable up to a maximum temperature of 600°F (315°C).

A Bristol design also uses a metal former. It is an aluminium corrugated tube. The resistance element which is in the form of a helix is wound spiral fashion into the corrugations. The crests of the corrugations make contact with the inside surface of the pocket ensuring a rapid heat transfer. The maximum temperature is 600°F (315°C).

The process of manufacture may introduce certain stresses into the platinum wire, and the element should be stress relieved by annealing at a higher temperature than the maximum value likely to be encountered in use.

Measuring Circuits

The common measuring circuit is the Wheatstone bridge. For resistance thermometer work it can be applied in two ways.

In the circuits which follow the bridge supply is shown as a battery, V. This should be regarded as symbolic since there are many ways of obtaining bridge supplies in addition to batteries (e.g. Zener diodes).

Out of Balance Method

Referring to *Fig.* 141, the bridge is balanced by R_3 for one value of the thermometer resistance T and the out of balance current taken as a measure of the resistance at all other values. If a suitable moving coil instrument G is included in arm BC, knowing the temperature-resistance relation of the thermometer, it may be calibrated directly in temperature units.

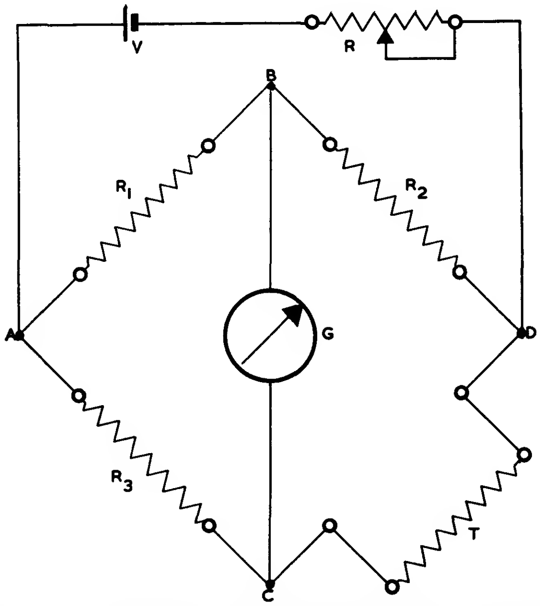
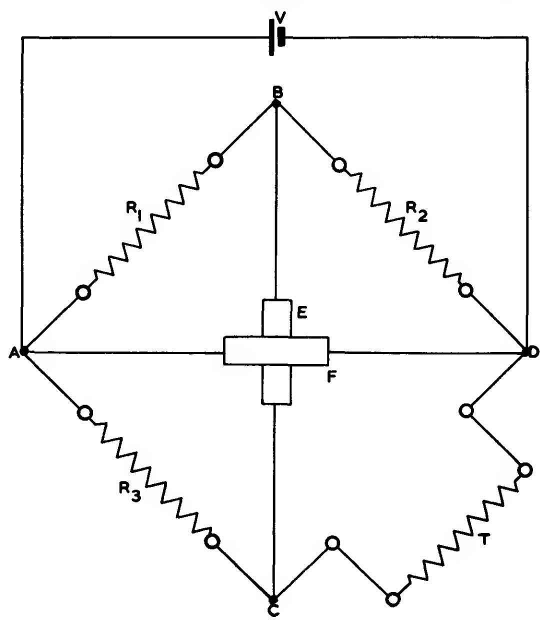


Fig. 141. *Basic bridge circuit for resistance thermometer*

Using the bridge in such a fashion entails certain precautions. Any variation in the bridge voltage supply V affects the out of balance current, and so introduces an error into the instrument reading. One method to eliminate the error over a reasonable voltage change is to use a cross coil movement. In this arrangement the coil assembly in the indicating instrument carries two coils with their planes at an angle to one another. We may term one the measuring coil, E, and the other the control coil, F, in *Fig.* 142. The torques due to the currents flowing in the coils oppose one another, and the deflection, therefore, depends on the values of these two currents, which from *Fig.* 142 are affected equally by a change in bridge voltage V. Since

Fig. 142. *Bridge circuit utilizing cross coil movement*



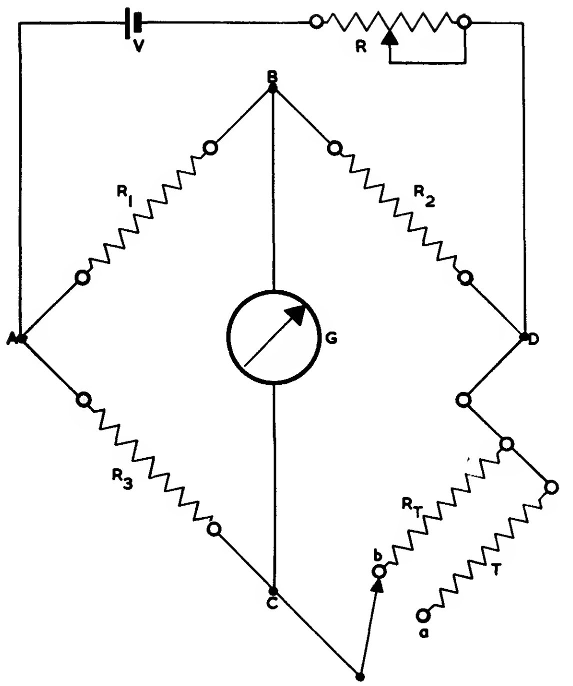


Fig. 143. *Bridge circuit incorporating test resistance*

the torques are opposing, by adjusting the current in the control coil, the effect of a change of V on the one coil can be made to cancel that on the other.

If V consists of a d.c. supply from batteries, and the single coil movement instrument is used, it is desirable to check the battery voltage at regular intervals. This may be carried out conveniently by arranging for the bridge to operate at a specific voltage value, and including in the bridge circuit a test resistance (*Fig.* 143). The switch is thrown to b for checking purposes, and resistance R in the supply lead adjusted until the instrument pointer reaches a particular test point on the scale above the maximum reading. If this is done before observations are taken, the bridge voltage will always be at the same value. An a.c. supply with a step-down transformer and rectifier in the secondary circuit gives an equivalent d.c. source.

Alternatively, a constant voltage supply may be installed. One method is to use an a.c. supply with a constant voltage step-down transformer. In such an arrangement a variation over a wide range of voltage, e.g., 190–260 volts, on the primary side causes only a ½ per cent–1 per cent variation in the secondary voltage. The circuit on the latter side must include a rectifier for producing d.c. bridge supply voltage.

In some supply units barretters are used for stabilization purposes.

But there are many types of stabilizing systems and newer techniques have involved the use of semiconductor circuits.

Null-balance Method

In the second method the Wheatstone bridge is used as a null-balance instrument. This would correspond, in *Fig.* 141, to adjusting R_3 in the bridge, every time a temperature observation was required, so that G showed no deflection. Then,

$$\frac{R_1}{R_2} = \frac{R_3}{T} \quad \ldots\ldots\ldots\ldots\ (170)$$

or

$$R_3 = \frac{R}{R_1} \cdot T \quad \ldots\ldots\ldots\ (171)$$

R_3 thus becomes a measure of T and hence the temperature.

In one design of hand-operated null-balance instrument resistance R_3 is made to alter the position of a pointer over a scale or, *vice versa*, the movement of a scale against a fixed index mark. In either case, the scale is calibrated directly in temperature units. R_3 is adjusted from the front of the instrument by means of a knob. Coupled to the resistance knob shaft is the mechanism for moving the pointer or scale. The knob is turned until the value of R_3 balances the bridge, observed by zero deflection on the galvanometer pointer. The scale or pointer is then at the value of the measured temperature.

But rebalancing of the bridge may be carried out by automatic means involving servo motors. These are described later in this chapter.

Lead Resistance

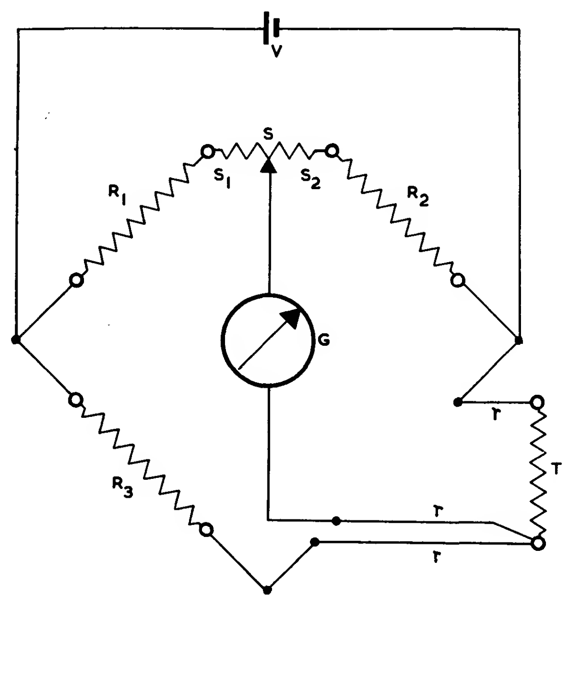
Two Lead Circuit

The thermometer may be a considerable distance from the instrument which contains the bridge circuit. In *Fig.* 141 it can be observed that there are two leads from the thermometer to the instrument. The resistance of these leads is of some significance since the bridge measures the total resistance between C and D. The gauge of the conductors in the connecting cable should be as large as possible, and 3/0·036 or 3/0·029 is usually specified. 3/0·036 copper has a resistance of 0·008ohm/yd and 3/0·029, 0·012ohm/yd. B.S. 1041 recommends the resistance of the thermometer itself should be at least 30 times the total lead resistance to avoid ambient temperature effects on the leads.

Three Lead Circuit

By using three leads from the thermometer it is possible by an arrangement, due initially to Sir W. Siemens, to include one lead in the thermometer arm and one in the opposite arm of the bridge. The third

Fig. 144. *Bridge circuit incorporating three wire lead from thermometer*



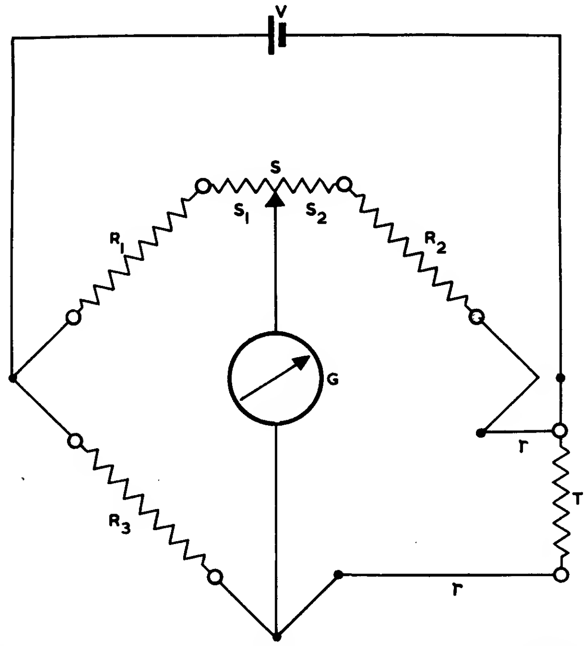


Fig. 145. *Alternative bridge circuit incorporating three wire lead from thermometer*

completes the necessary connection to the galvanometer arm (*Fig.* 144). An alternative version includes the third lead in the supply branch of the circuit (*Fig.* 145).

A further refinement is observable in the diagram. A variable resistance S has been connected between R_1 and R_2 for balancing purposes. This removes contact resistance variation from an actual resistance arm where it could affect the measurement.

At balance,

$$\frac{R_1 + S_1}{R_2 + S_2} = \frac{R_3 + r}{T + r} \qquad \ldots\ldots\ (172)$$

The only condition where the circuit is independent of a change in r is for

$$R_3 = T \qquad \ldots\ldots\ (173)$$

In other words, the circuit is independent of lead variation at one temperature of the thermometer. At other temperatures, however, the effect of change in r for any ambient temperature change is negligible.

If the alternative circuit is chosen (*Fig.* 145) the condition for complete independence is

$$T = R_2 + S_2 \qquad \ldots\ldots\ (174)$$

leading to a similar result, as before, of independence at one temperature only.

Four Lead Circuit

In this circuit one pair of wires from opposite arms of the bridge is brought to the thermometer. One set is actually connected to the latter, and the other two are looped at the thermometer. The arrangement means that practically the same lead resistance is in each arm at all temperatures, and accurate compensation results (*Fig.* 146).

Multi-point Installations

A single indicator or recorder can be installed for multi-point measurement. A switch box enables the individual thermometers to be connected to the instrument in similar fashion to the thermocouple arrangement of *Fig.* 138. Single or double pole switching may

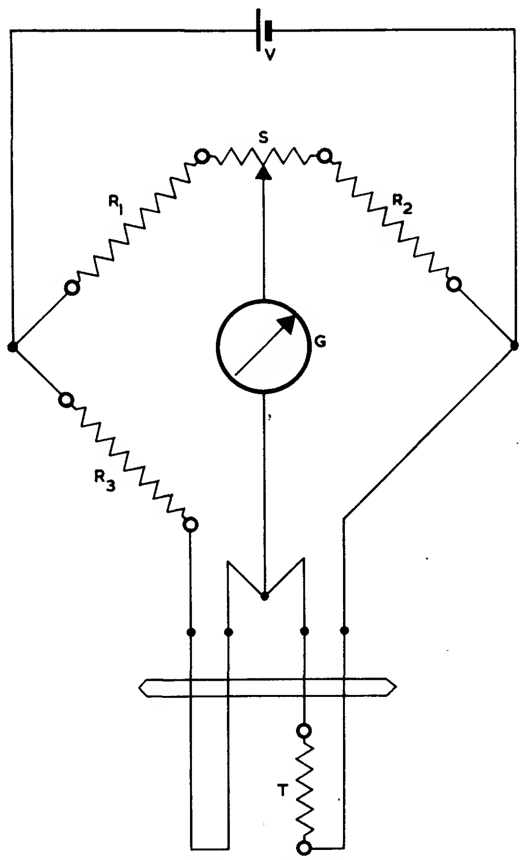


Fig. 146. *Bridge circuit with four wire lead arrangement*

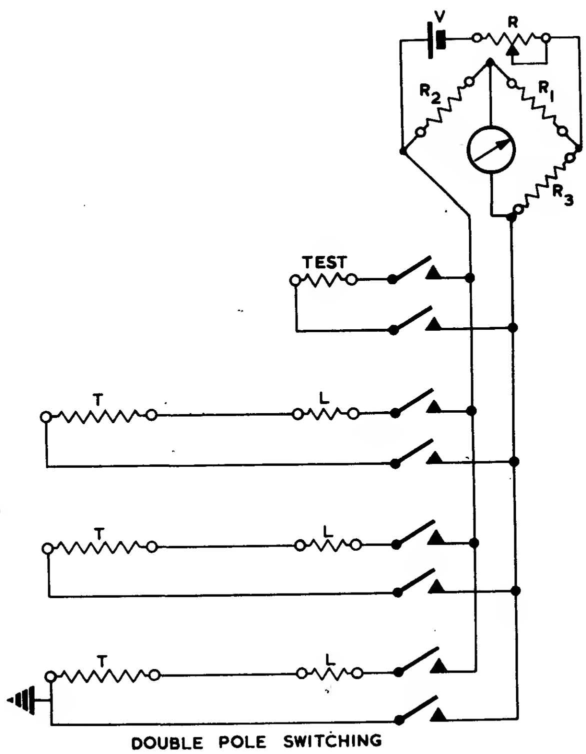
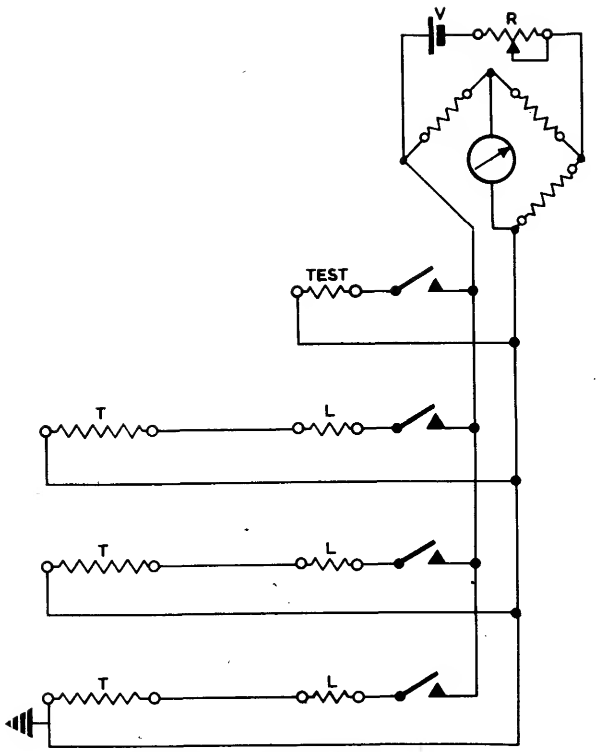


Fig. 147. *Multi-point switching for resistance thermometers utilizing double pole switching*



SINGLE POLE SWITCHING

Fig. 148. *Multi-point switching for resistance thermometers utilizing single pole switching*

be used, but the possibility of earthing troubles with the former exist as with thermocouples. Observe that each line carries a line balancing resistance L. This consists of a few turns of wire, giving a very small resistance, with which the circuits are adjusted to a set value when installation takes place. The basic circuits for single and double pole switching are shown in *Figs.* 147, 148.

Measurement of Temperature Difference by Resistance Thermometers

For the measurement of temperature difference by resistance thermometers it is customary to connect the two thermometers in opposing arms. Whilst this is satisfactory, a modification, shown in *Fig.* 149, results in a simple equation for balance. The modification is to insert a resistance S, equal to that of the slide wire, in the circuit.

The equation for balance at any moment is

$$\frac{T_1 + (S - S_1)}{T_2 + (S + S_1)} = \frac{R}{R} = 1 \qquad \ldots\ldots\ (175)$$

which gives

$$S_1 = \frac{T_1 - T_2}{2} \qquad \ldots\ldots\ (176)$$

This means that the portion S_1 of the slide wire resistance is at any moment directly proportional to the temperature difference.

Measurement of Average Temperature by Resistance Thermometers

For measuring average temperature of several points, the most satisfactory method is to connect all the resistance thermometers T_1, T_2 and T_3 in series as one arm of the bridge. It is usually more convenient if the resistance values are reduced in proportion to the number of thermometers involved. For example, if the

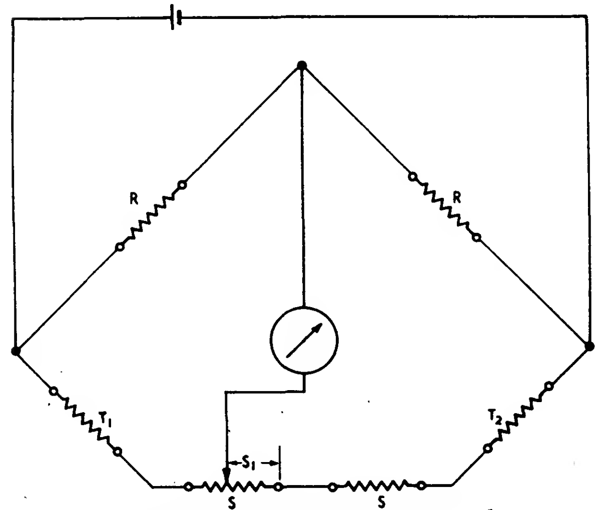


Fig. 149. *Measuring temperature difference by means of resistance thermometers*

normal value of a single point resistance thermometer at 0°C is 100 ohms, and three points are now to be averaged, each thermometer will now be made $\frac{100}{3}$ ohms, i.e., 33·33 ohms. *Fig.* 150 shows the method of connection for two lead circuits.

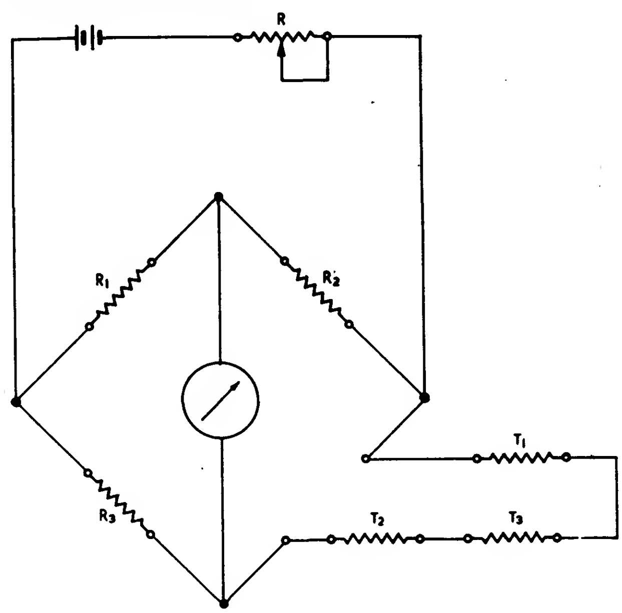
THERMISTOR THERMOMETERS

Thermistors are composed of metallic oxides which are compressed and fired at elevated temperatures. They may be produced in the form of small beads, discs, rods and washers. They possess negative temperature-resistance relations, a typical value of which is about −4% per °C at 20°C. The general resistance-temperature relation is given by:

$$R = a.e^{\frac{b}{T}} \qquad \ldots\ldots\ (177)$$

where R = the resistance
T = the absolute temperature
a and b = constants.

Fig. 150. *Measuring average temperature by means of resistance thermometers*



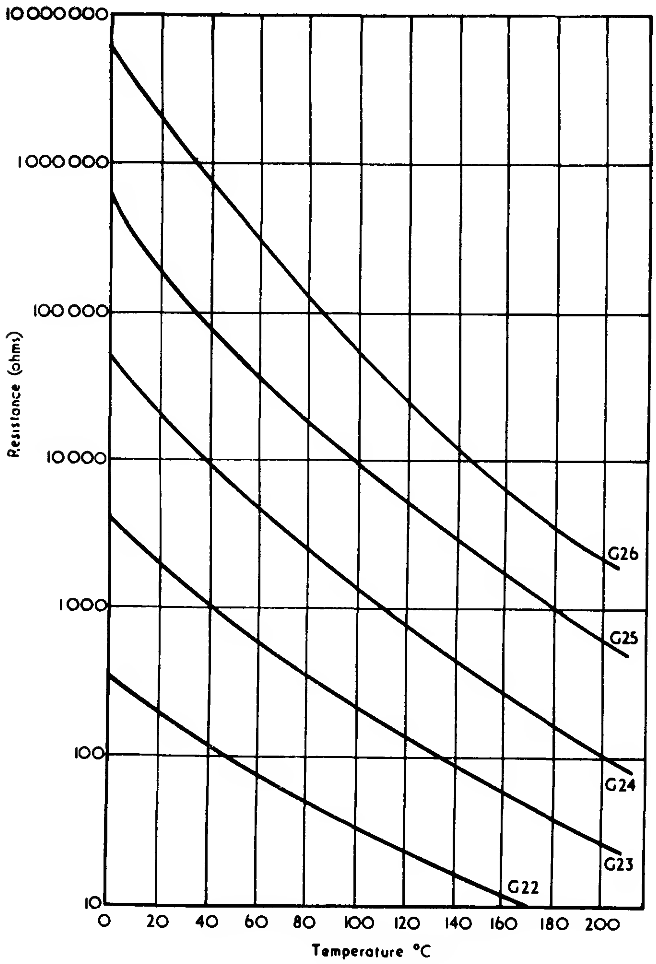


Fig. 151. *Temperature resistance relation of typical thermistors*

Typical curves for the resistance-temperature relations are shown in *Fig.* 151 which are for thermistors of Standard Telephones and Cables manufacture.

The temperature range over which thermistors are satisfactory as temperature measuring devices depends on the type, but experience suggests 0° to 300°C as a reasonable value.

Two types are available: the directly heated and the indirectly heated. In the directly heated version, current is allowed to pass directly through the thermistor element. In the indirectly heated design, the current is fed into a small heating winding round the thermistor element, and does not, therefore, pass through the element itself. Both types have their own applications. The directly heated ones have tended to be used more as temperature measuring devices, whilst the indirectly heated ones have found numerous applications in servo circuits.

Fig. 152 shows a basic bridge circuit for temperature measuring purposes with a directly heated thermistor, *F*23 in the figure. It forms one arm of the bridge. Balancing is carried out by the 1000 ohm resistor for coarse adjustment and by the 120 ohm resistance for fine adjustment. The range is 90°F to 120°F.

Fig. 153 indicates a directly heated thermistor Th_1 being used as a temperature measuring device, but with somewhat different circuit arrangement to *Fig.* 152. It is actually a control circuit, the object being to maintain the temperature of the oven, shown at the right-hand side of the figure, at a constant value. It can be seen that Th_1 and R_1 form a potential dividing line. R_1 acts as a resistor for setting the desired value of the oven temperature. If the oven temperature is at the desired value, the resistances of Th_1 and R_1 are equal. For the purposes of explanation, however, consider that the temperature of the oven has fallen below the desired value. The resistance of Th_1 is then higher than that of R_1. A positive signal is now fed to the control stage of the transistor amplifier. This increases the current through the oven winding, and, hence, causes the oven temperature to rise. But Th_1, measuring the oven temperature, has a negative temperature-resistance relation so that the resistance of Th_1 decreases as the temperature increases. The action continues until the resistance of Th_1 is equal to that of R_1 when the error signal is zero and the temperature is back again to the desired value.

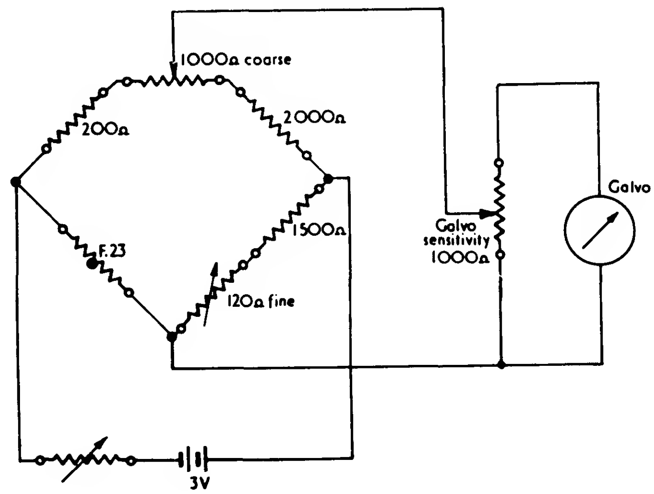
The temperature - resistance characteristic of a thermistor may be modified by connecting resistances in series and parallel with it. Such a property gives rise to two possible applications. One is that the non-linear temperature-resistance relation may be made substantially linear over a moderate temperature range, e.g., 50°C. The other is that in the use of a thermistor as a compensating device, the resistor networks may be so designed as to give the thermistor exactly the form of compensating curve required.

References 3 and 4 at the end of the chapter should be consulted for full details of thermistor applications.

SEMICONDUCTOR THERMOMETERS

This is the name loosely given to a class of thermometers constructed from pure germanium or pure silicon crystals. Taking a typical design, the temperature measuring element is a practically pure crystal of silicon. The range can be from −320°F (−195°C) to 600°F (315°C) overall, but a smaller range from −50°F (−45°C) to 350°F (177°C) is suggested in practice. The general action is that of a resistance thermometer and indeed the resistance change is stated to be relatively high. In published data the change from −50°F (−45°C) to 500°F (260°C) is given as 40% to 50% per 100%. The characteristic temperature-resistance is positive and is only slightly non-linear. Response with the bare element inserted in the medium whose temperature is to be measured is

Fig. 152. *Bridge circuit incorporating a thermistor*



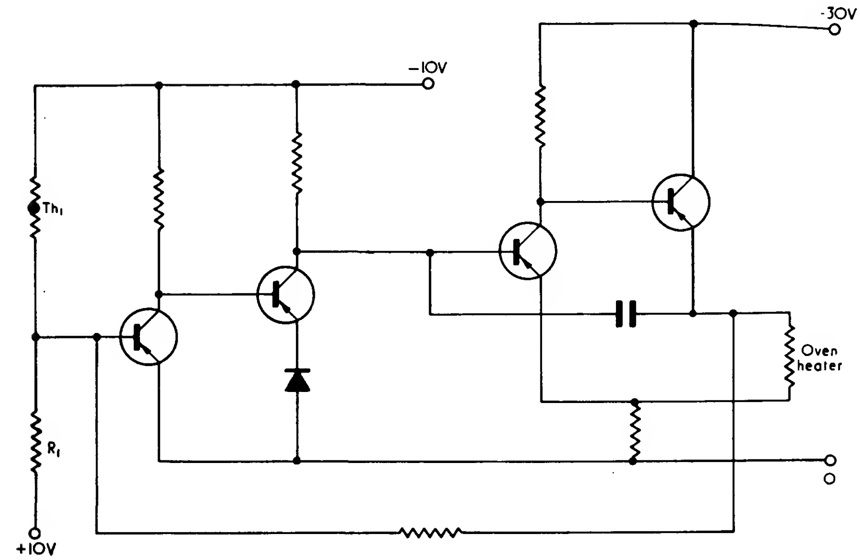


Fig. 153. *Temperature control circuit utilizing a thermistor*

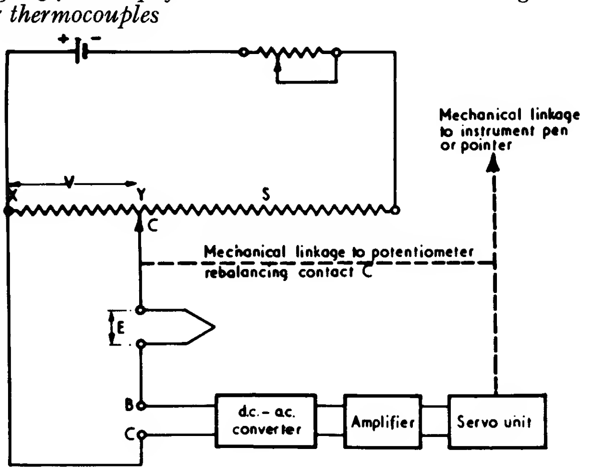
claimed to be extremely fast. To reach 63% of the final temperature value when a change has taken place requires only 15 millisecs and to reach 90% of the final value 40 millisecs.

NULL-BALANCE INSTRUMENTS

Both the examples of null-balance instruments described here are intended to illustrate the principles involved only and, hence, are very much simplified.

Considering the thermocouple version *Fig.* 154 first, in the normal potentiometer circuit a null-balance detecting instrument is connected between *B* and *C*. In the automatic null-balancing instrument, however, between *B* and *C* is connected a train of circuits. These comprise a d.c.-a.c. converter, an amplifier and a servo unit. When the potentiometer circuit is balanced the potential drop *V* along the section *XY* of the slide wire *S* is equal to the thermocouple e.m.f. *E*, and no signal is fed into the converter. Consider that a change in measured temperature occurs, *V* is no longer equal to *E* and a state of unbalance exists. There is now an input signal to the a.c.-d.c. converter which converts the d.c. unbalance signal to a corresponding a.c. one. This is amplified and the output signal from the amplifier is fed into the servo unit. This latter may take one or two forms. It may typically be a small two-

Fig. 154. *Simplified automatic null-balance arrangement for thermocouples*



phase motor with windings spaced electrically 90° apart or it may take the form of the double solenoid device shown in *Fig.* 113 of Chapter 7. The actual type

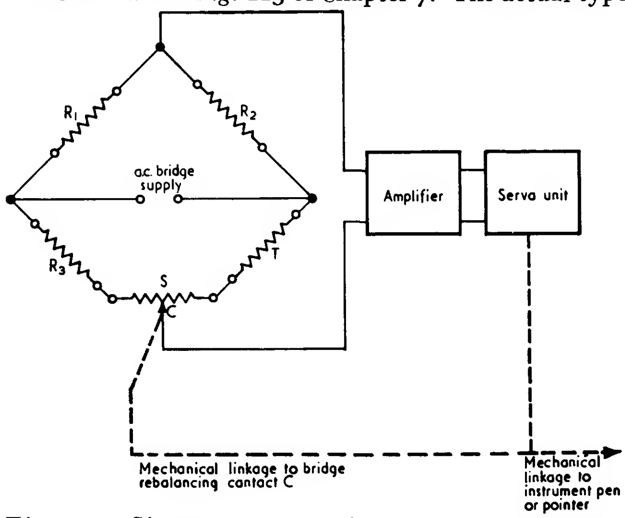


Fig. 155. *Simplified automatic null-balance arrangement for resistance thermometers*

need not concern us in this chapter, the function of the servo unit being the important factor. The output shaft of the servo device is coupled by means of linkage to the movable contact of slide wire *S* and to the pointer or pen of the instrument. The reception of the amplified unbalance signal by the servo unit sets the shaft in motion. Movement of the shaft adjusts the position of the contact on the potentiometer slide wire *S*, and the movement is always in such a direction as to rebalance the circuit. The operation continues until the circuit is actually rebalanced, i.e., when the new value of *V* equals the new value of the e.m.f. *E*. At the same time by virtue of the linkage to the pen or pointer, the position of which are adjusted at the same time, the new value of temperature is indicated on the scale or chart of the instrument.

In the resistance thermometer design, the bridge supply is an a.c. one as indicated in *Fig.* 155. When the bridge is balanced no signal appears at the input of the amplifier. If the temperature changes, the bridge becomes unbalanced and an input signal is supplied

to the amplifier. As in the thermocouple example, the amplified signal causes the servo unit to adjust a contact, in this case contact C of the variable resistance S between the two arms of the bridge R_3 and T. The operation persists until the bridge is rebalanced. The pen or pointer of the instrument is adjusted by means of linkage in a similar manner to the thermocouple system to indicate a new value.

Books, etc., for further reading

1. ECKMAN, D. P. *Industrial Instrumentation*. Chapters 2, 3, 4 and 5. Chapman and Hall, 1950.
2. JONES, E. B. *Instrument Technology*. Chapter 4. Butterworths, 1953.
3. ATKINS, P. A. and SETTERINGTON, R. A. Thermistors in Temperature Measurement and Control. *Instrument Practice*, Vol. 13, No. 10, Oct., 1959, pp. 1042-1047.
4. HARRISON, D. N. Recent Developments Using Thermistors. *Instrument Practice*, Vol. 17, No. 9, Sept., 1963, pp. 937-942.
5. MILLER, J. T. High Temperature Measurement with Thermocouples. *Instrument Practice*, May, 1960.

British Standard Specifications

The thermocouple and resistance pattern of thermometers have the following specifications devoted to them:

B.S.1904 Commercial Platinum Resistance Thermometer Elements.
B.S.1826 Reference Tables for Platinum Rhodium v. Platinum Thermocouples.
B.S.1827 Reference Tables for Nickel Aluminium v. Nickel Chromium Thermocouples.
B.S.1828 Reference Tables for Copper V Constantan Thermocouples.
B.S.1829 Reference Tables for Iron V Constantan Thermocouples.
B.S.1041 Code for Temperature Measurement.
Part 3 Industrial Electrical Resistance Thermometers.
Part 4 Thermocouples.
Part 7 Temperature/Time Indicators.

Questions

1. For a Copper-Constantan thermocouple a in equation (156) is $37{\cdot}54 \times 10^{-3}$ millivolts/°C, b is $0{\cdot}045 \times 10^{-3}$ millivolts per °C. If T_2 is 0°C, and T_1 is 100°C, calculate the values of the two parts of the equation and hence the total E.M.F. in millivolts. (Answers 3·75, 0·45, and 4·20 millivolts.)

2. The e.m.f. of an Iron - Constantan couple is 44·39 millivolts at 800°C with the cold junction at 0°C. If the latter alters to 15°C, what is now the corresponding temperature, assuming that the temperature-millivolts relation is a straight line and the variation of e.m.f. with temperature is 0·06 millivolts per °C.
(Answer 785°C.)

3. A mistake is made in allowing for the length of compensating cable in an installation. Originally 30yd were specified, and the new value is 60yd. If the resistance is 0·5 ohm/yd, what is the percentage error introduced into the instrument reading if the thermocouple resistance is 1 ohm, coil resistance 50 ohm and series or ballast resistance 400 ohm? (Answer 3·22 per cent.)

4. Using equation (167) what is the value of δ: if $R_{100}=150{\cdot}3$ ohm, $R_0=110$ ohm, $R_T=273$ ohm and $T=444{\cdot}6$ °C?
(Answer 1·513.)

5. In Fig. 141, R_1, R_2, R_3 and T have the same value at one particular temperature. At this value not more than 0·01 watts must be dissipated in T. If the bridge is balanced, and the voltage across AD is 2volts, what is the value of T?
(Answer 100 ohm.)

6. Assume that an installation has 100yd of 3/0·036 from thermometer to instrument. What is the change in the value of $\frac{R_3 + r}{T + r}$ in equation (172) if $R_3=100$ ohm, $T=120$ ohm, for an ambient temperature change from 15°C to 20°C? Take copper as changing its resistance 0·43 per cent, per °C.
(Answer 0·8344 to 0·8347.)

Chapter 10

RADIATION PATTERN THERMOMETERS OR PYROMETERS

INTRODUCTION

In Chapters 8 and 9 the liquid expansion, gas expansion, vapour pressure, bimetallic, thermocouple and resistance patterns of thermometers have been described. It might be thought that all temperature measuring requirements in industry could well be covered by one or other of these. But the thermometers have one factor in common—they all require the element itself to be inserted in the medium whose temperature is to be measured, either directly or in a protecting sheath or pocket. This is not possible in all processes and it is necessary to look for some other means of measurement. One or two additional considerations demand a fresh type of thermometer. Temperatures in excess of 1500°C are not uncommon in industry, and this represents approximately the limit for the platinum/platinum rhodium group of thermocouples. The newer developments such as the tungsten/tungsten 26% rhenium or the iridium/iridium 60% rhodium thermocouples extend the range to 2000°C and above, but there is a need for instruments which can measure temperatures well above the upper limit of any thermocouple. In any case, even if a thermocouple can be considered suitable for measurement, the problem of a pocket material which can stand up to elevated temperatures must be solved. Finally, the conditions of operation may mean that the thermocouple is liable to oxidation or embrittlement. The provision of a reducing or inert atmosphere can safeguard the thermocouple against oxidation but not every process lends itself to the installation of equipment for producing such atmospheres.

Whilst it may sound paradoxical, some system is needed in which the temperature measuring means can be installed external to the process. This brings us to the consideration of radiation pattern thermometers or pyrometers.

SOME BASIC LAWS OF RADIATION

The instruments to be described will be understood more clearly if some basic laws of radiation are considered in a simple manner.

All hot bodies emit radiant energy, the intensity of which bears a relation to the absolute temperature of the emitting surface. Heat radiation is a wave phenomena, analogous to light, and occupies a definite place in the spectrum, extending from about 1/100 of a micron ($0{\cdot}01\mu$) to 100 microns (100μ) wavelength (1 micron (1μ) is equal to 10^{-4}cm or 10^{-6}metres). A hot body, in general, emits radiation embracing several wavelengths.

Prévost's Theory of Exchanges

If we have two bodies A and B, of which A is the hotter, both emitting radiation, there is a net transfer of energy between them. In other words, each body radiates to and absorbs energy from the other. A emits more than it absorbs, whilst B absorbs more than it emits.

The law has a bearing on the measurement of the temperature of a hot gas in an enclosure, for example, a furnace. The walls of the enclosure may be at a lower temperature than that of the gas itself. Suppose a thermometer is placed in the gas to measure its temperature. It receives heat by radiation from the walls, but if it tries to assume the higher temperature of the gas by Prévost's law it commences to radiate more heat than it receives and, hence, may register a lower value than the true temperature of the gas.

Emissivity and Absorbtivity

Bodies or surfaces which possess good absorbing powers to thermal radiation are found to be good emitters of radiation. In contrast, bodies or surfaces which possess poor absorbing powers are poor emitters. Bright metal bodies are poor absorbers and emitters, whereas black bodies are good absorbers and emitters. A simple proof of this is given in most text books on physics and heat in the descriptions of the experiments with the Leslie cube and the U-tube with black and bright bulbs. It may assist in the assimilation of the fundamental characteristics of thermal radiation if a brief account of the experiments are given here. In the Leslie cube one face is blackened, one is highly polished, one is left unpolished and a fourth is covered with paper. The cube is filled with hot water and a sensitive thermal radiation detector is placed near it. The faces of the cube are rotated so that each, in turn, faces the detector. It is found that more heat is radiated from the black surface than from any of the others, and the polished surface radiates the least heat.

In the absorption experiment which is carried out with the U-tube, each limb of the tube ends in a bulb. The liquid in the tube must be non-volatile. Referring to Fig. 156—one bulb A is blackened and the other B is covered with some highly polished material such as silver foil. The apparatus is placed so that the bulbs are at the same distance from a source of radiant heat. It is found that the liquid in limb A with the black bulb moves downwards. This is due to the black bulb A absorbing more heat than B with the silver foil. It causes a greater expansion of air in A than in B, so pushing the liquid in A down.

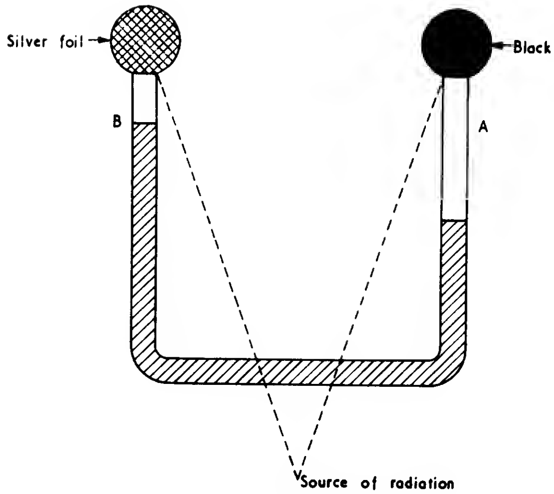


Fig. 156. *Experiment to indicate absorptivity*

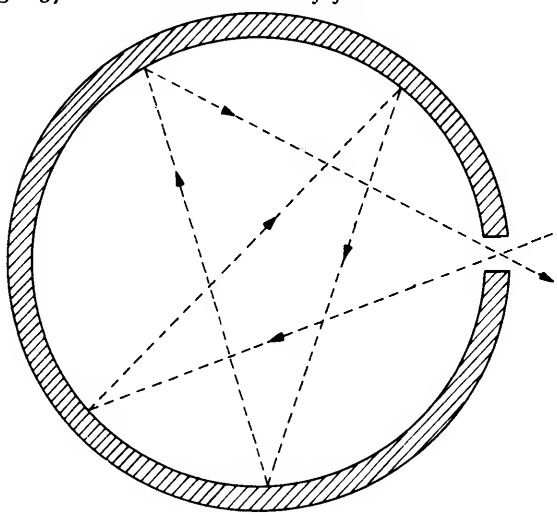
Since there is no better absorber or emitter of thermal radiation than a black body it is taken as a sort of standard, and its emissivity is given the value 1. Emissivity may be defined here as the ratio of the total radiation from a non-black body to that from a geometrically similar black body at the same temperature.

It is suggested by British Standard Specification No. 2082, Code For Disappearing Filament Optical Pyrometers, that the term "full radiator" be used instead of "black body radiator". The former term will, therefore, be used in this chapter.

How do we achieve full radiator conditions in practice?

A large hollow enclosure of any material whose internal surface is not totally reflecting is brought to a *uniform* temperature. A small hole is provided through the wall and the radiation emitted via this hole is a very close approximation to full radiation. One proviso is that the area of the hole must be very small compared with the total internal surface area of the enclosure. The action can be realized if the behaviour of a ray falling on the aperture is followed (*Fig.* 157). In its travel it strikes the surface of the enclosure and, since we have stipulated that the enclosure material is not perfectly reflecting, the ray suffers part reflection

Fig. 157. *The establishment of full radiator conditions*



and part absorption. The reflected portion strikes another part of the surface and again suffers part reflection and part absorption. The action continues until the ray is virtually completely absorbed. But we have seen that a perfect absorber is also a perfect radiator and the enclosure, when acting as a radiator does so under full radiation conditions. Note that the emissivity of the enclosure material can be less than 1 in fulfilling these conditions.

It will be apparent that we have described a state of affairs approximating to an industrial furnace. In some cases it may not be desirable to have an aperture in communication with the outside atmosphere. It is permissible then to insert a metal or ceramic tube with a closed end into the furnace. The closed end is heated to an average furnace temperature and the radiation emitted corresponds, to a close degree of approximation, to that from a full radiator. We can carry the uniform temperature enclosure idea one step further. Consider a body placed inside. The body partly absorbs radiation from its surroundings and partly reflects it. But the body must radiate the same amount of energy that it absorbs, otherwise temperature equilibrium is not established. When viewed from outside the enclosure

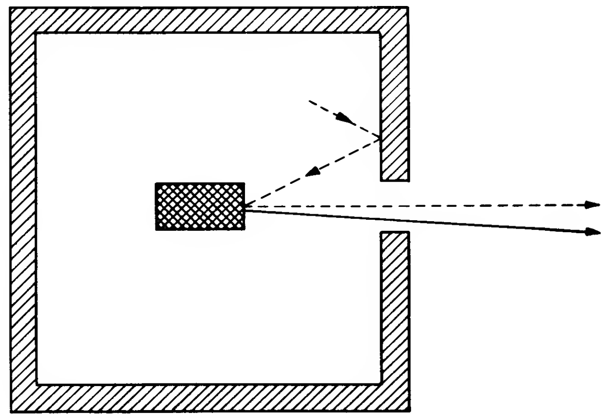


Fig. 158. *Conditions for full radiation in a furnace*

the absorbed as well as the reflected parts of the energy are received (*Fig.* 158). Since these two parts together are full radiation, the heated body inside the furnace at the same temperature can be considered to approximate very closely to a full radiator.

Stefan-Boltzmann Law

An important law first enunciated by Stefan, and later deduced theoretically by Boltzmann, states for full radiator conditions that

$$J = \delta T_1^4 \qquad \ldots\ldots\ldots\ldots \quad (178)$$

where J = the energy radiated from unit area of the body's surface per second. Normally in ergs/sq.cm/second, it is an energy density.

δ = Stefan's constant, $5{\cdot}77 \times 10^{-5}$ ergs/second/sq.cm/(degree)^4.

T_1 = the temperature of the emitting surface in °C absolute.

If a surrounding or neighbouring body is at a temperature T_2°C absolute, the net emission of energy from the original source is modified to:

$$E_1 = \delta(T_1^4 - T_2^4) \qquad \ldots\ldots\ldots \quad (179)$$

This follows from Prévost's Theory of Exchanges.

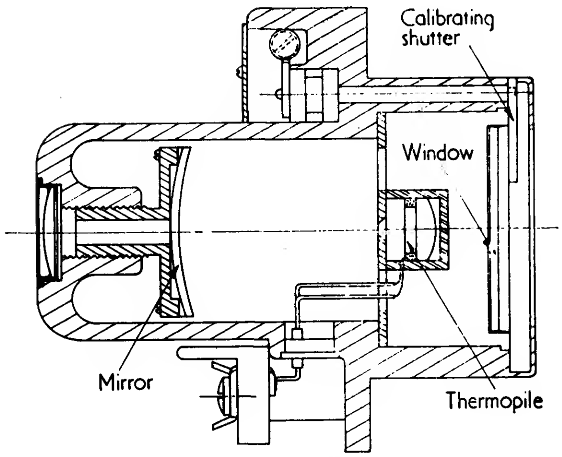


Fig. 159. *A mirror type total radiation pyrometer*

The total energy radiated, W, equals $E.A$, A being the surface area of the body, so that

$$W = A\delta(T_1^4 - T_2^4) \quad \ldots\ldots\ldots (180)$$

If the body is not a full radiator, its emission will be a fraction E of the full radiation and equation (180) becomes,

$$W = A\delta E(T_1^4 - T_2^4) \quad \ldots\ldots (181)$$

In general it is written

$$W = K(T_1^4 - T_2^4) \quad \ldots\ldots\ldots (182)$$

where K is an aggregate constant.

We now have to consider methods of measuring W.

Expression (182) is one for total radiation and any pyrometers based on it are termed Total Radiation Pyrometers.

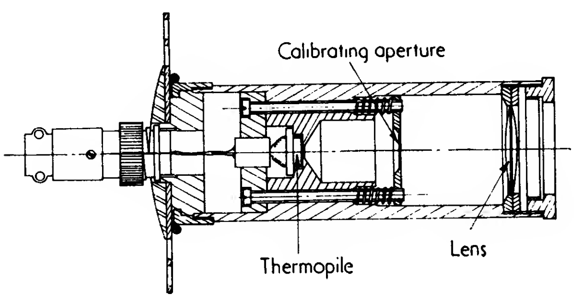
TOTAL RADIATION PYROMETERS

The basic features of one form of total radiation pyrometer are indicated in *Fig.* 159. This pattern is sometimes known as the Fery pyrometer named after one of the pioneers of radiation pyrometry. The radiation from the source whose temperature is to be measured enters by a window of glass or (to a much less extent) silica or fluorite. The radiation falls on to a metal mirror which focuses it on a thermopile. The latter is typically constructed from a number of fine wire thermocouples connected in series. An alternative design in *Fig.* 160 dispenses with the mirror and uses a lens system for focusing. The heat generated by received radiation causes the thermocouples to produce an e.m.f. bearing a relation to the temperature of the emitting source.

Certain precautions must be observed with this pattern of instrument[4]:

(1) *Cold junction compensation.* The cold junction is usually designed to be quite close to the hot junction.

Fig. 160. *Lens type total radiation pyrometer*



For example, in the enclosed type of couple, the cold junction could be at the base pins which plug into a holder. The terminals of the latter may be connected by copper cable to the indicator. The adjacent position of hot and cold junctions ensures that both are equally, or very nearly equally, affected by any ambient temperature change. The cold junction must be shielded, however, from radiation from the source, and conduction of heat from the housing to the couple can be reduced by filling the glass envelope with an inert gas. (Some designs evacuate the envelope.) The use of very fine thermocouple wire reduces conduction losses from the couple itself.

Where excessive temperature rise of the pyrometer housing is likely to be encountered, it should be enclosed in an asbestos lined metal casing, or in a water-cooled jacket.

If cold junction compensation is necessary, a nickel resistance spool is connected as a shunt across the thermocouple leads at the cold junction end. The variation of the resistance with ambient temperature is such that it is sufficient to compensate the thermocouple e.m.f. for temperature change.

In another design, a bi-metallic compensator cuts off part of the incoming radiation if the temperature of the housing increases.

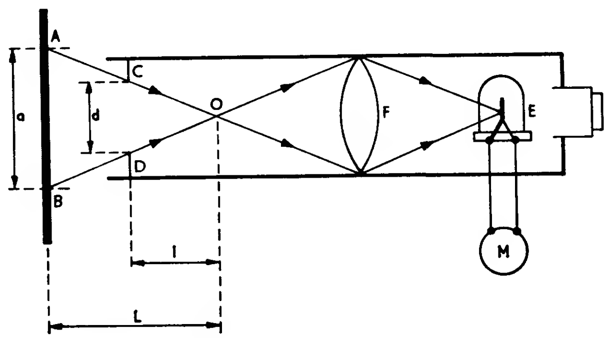


Fig. 161. *"Fixed focus" conditions*

(2) *Distance between the source of radiation and the pyrometer.* This is important as the radiation must always be focused on to the receiving thermopile.

There are two methods of ensuring this, by mirror or lens.

In the first, the position of the mirror may be adjusted, so that with varying distance of source from mirror the radiation is always focused to the same point. In a typical design, a concave mirror of polished stainless steel is adjustable by a rack and pinion. Two small mirrors, inclined to one another, are placed near the receiving disc. Viewed through the eyepiece, an image of the source is seen. When the instrument is correctly focused by adjusting the position of the concave mirror, the image will appear as a whole. When incorrectly focused, a split image can be seen.

In a second method, the arrangement shown in *Fig.* 161 is used. Here, the position of the lens or mirror is fixed, but the cone of radiation must always be such as to fill aperture CD. CD then becomes a secondary source, and provided it is completely covered the radiation is always brought to the same focus. The instrument becomes independent, within limits, of the distance from the source, and is termed a fixed focus pyrometer. From the geometry of *Fig.* 161, it can be seen that the relation between the distance L, the

aperture diameter d, l the distance between aperture and O, and source diameter a, is

$$L = \frac{al}{d} \quad \ldots\ldots\ldots\ldots\ldots (183)$$

If α, the angular aperture, is known,

$$l = \frac{d}{2 \sin\frac{\alpha}{2}} \quad \ldots\ldots\ldots (184)$$

and

$$L = \frac{a}{2 \sin\frac{\alpha}{2}} \quad \ldots\ldots\ldots (185)$$

Thus, provided certain minimum requirements are observed, the radiation will always fill aperture CD.

The pyrometer geometry is usually arranged so that the diameter of the source of radiation, a, must be at least $\frac{1}{10}$ of the distance L.

(3) *The atmosphere between the source and pyrometer.* This should be kept clear of any absorbing media such as fumes or smoke likely to cause errors. (See also Two Colour or Ratio Pyrometry.)

(4) *Lens or mirror* must be kept clean, and the mirror in a polished condition.

(5) *Corrections* must be made where the source does not fulfil full radiator conditions.

(6) *The Stefan-Boltzmann law* as regards the fourth power does not necessarily hold for every pyrometer. A truer statement would be

$$W = K(T_1^{b} - T_2^{b}) \quad \ldots\ldots\ldots (187)$$

where b is an index which Burgess and Foote (*Characteristics of Radiation Pyrometers*, National Bureau of Standards) found to vary from 3·28 to 4·26 for 22 tested instruments. The average was 3·89. The fourth power law should not be used indiscriminately to extend an existing calibration. A typical range for total radiation pyrometers is from 500°C-2000°C.

PHOTOCELL PYROMETERS

Photocells have been introduced as radiation detecting devices in pyrometry. The lead-sulphide cell, because of its characteristics, has been widely used. It is thought that a description of one development will illustrate the potentialities of photocells in pyrometry. This is an infra-red pyrometer.

The pyrometer comprises two units: the viewing head and the control unit. The infra-red sensitive element, housed in the viewing head, is a lead-sulphide cell, located behind a rotating "chopper", or serrated disc, in such a manner that it is irradiated alternately by the heated body under measurement and by a tungsten-filament comparison lamp. An alternating voltage is thus produced across the cell and the value of this voltage is zero if the level of radiation from the two sources is the same.

The circuit in the control unit adjusts the comparison lamp current so that the alternating voltage across the lead-sulphide cell is kept at a minimum. This current adjustment is entirely automatic, requiring no attention from the operator. It is the lamp current, indicated on a meter at the front of the unit, which is a function of the temperature of the body being measured. The cell is thus used as a "null" detector, and the small inherent drift in its sensitivity is unimportant.

The incoming infra-red radiation is focused on to the lead-sulphide cell by an objective lens, mounted at the inner end of the lens tube. The sensitivity of the pyrometer is such that it is necessary when measuring high temperatures to attenuate the incoming radiation to bring the meter current within the range of the instrument. For this purpose, two removable "stops", or masks, are provided for insertion into the lens tube; one reduces the aperture area by ten and the other by one hundred. Further attenuation can be obtained by interposing heat-absorbing glass (i.e., an infra-red filter) between the surface under test and the viewing head.

Use of the stops provides three temperature ranges: 150°C-400°C on full aperture; 300°C-600°C on the medium aperture stop; and 400°C-1200°C on the small aperture. The last range can be extended to 2000°C by the use of an infra-red filter. The lowest range is used for bodies at "black" heat.

The smallest area which can be viewed is a circle 0·4in. diameter; for this, the lens tube, fully extended, must be as close as possible without actually touching the surface under test. The diameter of the circular area increases by 1in. for each foot increase of distance.

As the reading depends on so many independent variables, the instrument must be calibrated for each application under similar conditions to those in which it is to be used. Once the instrument has been calibrated for a given material and surface finish, at a given distance, no further calibration is necessary unless any of these conditions are changed.

RADIANT ENERGY AND WAVELENGTH

If a heated body becomes luminous, the luminosity can become a measure of the temperature of the body, since a relation exists between the two. Unfortunately different materials have different luminosities at the same temperature. A factor ε_λ called the spectral emissivity is involved. This is defined as the ratio of energy radiated per unit area at a certain wavelength to the energy radiated per unit area by a full radiator at the same temperature. To take some practical examples from B.S.2082, at wavelength $\lambda = 0{\cdot}65\mu$, for clean molten steel ε_λ is 0·35, but for solid oxidized iron ε_λ is 0·98 (at 800°C).

A useful equation connecting wavelength, emissivity, temperature and radiant energy is due to Wien:

$$J_\lambda = C_1\lambda^{-5} \, [\exp \, (C_2/\lambda T)]^{-1} \quad \ldots (188)$$

where J_λ = the energy radiated at wavelength λ.

T = the absolute temperature of the body in °K (degrees Kelvin).

C_1, C_2 = constants $C_2 = 1{\cdot}438$cm degree.

λ = wavelength of radiation in cm.

For reasons given in the first paragraph the temperature as measured by the luminosity is not necessarily the true temperature. If S is the luminosity temperature and ε_λ the spectral emissivity,

$$\begin{aligned} J_\lambda &= \varepsilon_\lambda C_1\lambda^{-5} [\exp \, (C_2/\lambda T)]^{-1} \\ &= C_1\lambda^{-5} [\exp \, (C_2/\lambda S)]^{-1} \end{aligned} \quad \ldots. (189)$$

From which,

$$\varepsilon_\lambda = \exp \, (C_2/\lambda T - C_2/\lambda S) \quad \ldots. (190)$$

or

$$\frac{\lambda \log \varepsilon_\lambda}{C_2} = \frac{1}{T} - \frac{1}{S} \quad \ldots\ldots\ldots\ldots (191)$$

B.S.2082 includes a series of curves based on equation (191). Values of the correction (T-S) in °C which must be added to the observed temperature (S°C) are plotted for values of ε_λ ranging from 0·05 to 0·95. The wavelength selected is 0·65μ.

If such corrections are to be applied to a measured temperature the value of ε_λ must be known reasonably accurately. If ε_λ is not known, a value may be obtained from B.S.2082. Note, however, that the figures given in this specification are for certain conditions only and the latter may not apply to those of the actual measurement.

We can now deal with instruments for measuring luminance temperature, i.e., optical pyrometers.

OPTICAL PYROMETERS

Disappearing Filament Type

A common form of pyrometer is the disappearing filament pattern, as shown in *Fig.* 162.

It measures the intensity of a monochromatic beam of the visible light radiated by a hot body.

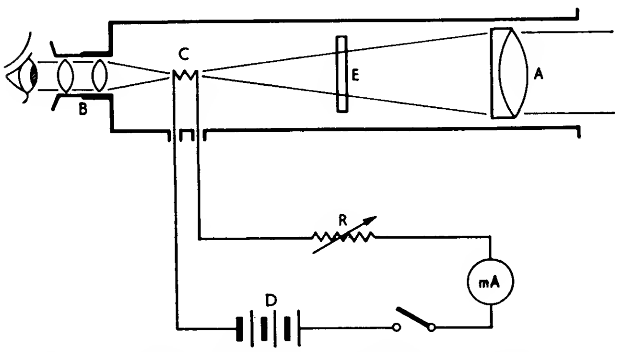
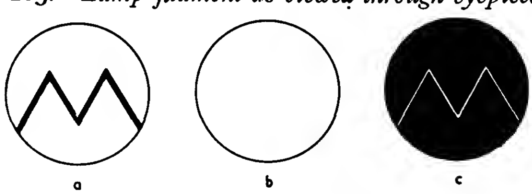


Fig. 162. *Basic features of an optical pyrometer*

In one type, the radiation source is viewed through a telescopic system consisting of objective lens A and eyepiece B. Inside the telescope is a small lamp C heated by battery D. The current through C is adjustable by resistance R and a milliammeter is connected in the heating circuit. A red optical filter is interposed between eye and lamp. On looking through the eyepiece, the source is seen as a bright circle, square, rectangle, or other shape, and in the centre of it is the image of the filament of the lamp. The resistance R is adjusted until the brightness of the filament is equal to that of the radiation. When this occurs, the filament image appears to merge into the radiation image and present a uniform picture to the eye. This is indicated in *Fig.* 163*b*. If the filament is not as bright as the source image, it appears dark against a lighter background (*Fig.* 163*a*). If, on the other hand, the filament is brighter than the source it appears as a light band against a dark background (*Fig.* 163*c*). The transition points between *a*, *b*, and *c* are definite, and different observers can record the same values within small limits of personal error.

Fig. 163. *Lamp filament as viewed through eyepiece*



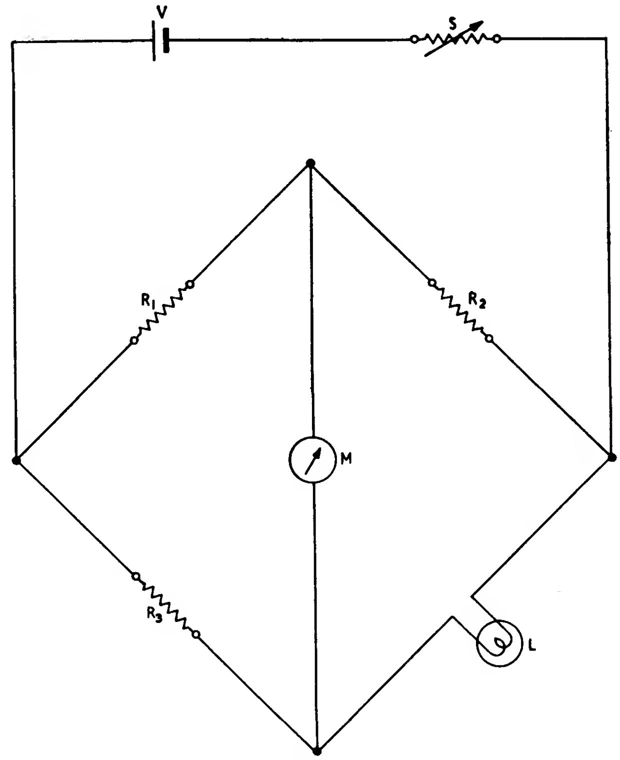


Fig. 164. *Alternative circuit for optical pyrometer*

In another design, popular in portable versions, the battery V is located in the handle. The milliammeter M, calibrated directly in °C or °F, is situated just below the telescope section, where it is easily observable by the operator as he looks through the eyepiece. In *Fig.* 164 the lamp L is connected as one arm of a Wheatstone bridge with fixed resistances R_1, R_2, R_3. The variable resistance S, adjustable by means of a milled ring on the telescope body, is in the battery circuit. As its value is altered, the current through the lamp changes until its brightness is at the right value.

TWO COLOUR OR RATIO PYROMETER

Consider Wien's equation for two different wavelengths, λ_1 and λ_2

$$J_1 = E_1C_1\lambda_1^{-5}\,[\exp\,(C_2/\lambda_1T)]^{-1} \quad \ldots \quad (192)$$

$$J_2 = E_2C_1\lambda_2^{-5}\,[\exp\,(C_2/\lambda_2T)]^{-1} \quad \ldots \quad (193)$$

The ratio of the two radiations is

$$R = \frac{J_1}{J_2} = \frac{E_1C_1\lambda_1^{-5}\,[\exp\,(C_2/\lambda_1T)]^{-1}}{E_2C_1\lambda_2^{-5}\,[\exp\,(C_2/\lambda_2T)]^{-1}} \quad \ldots \quad (194)$$

$$R = \frac{E_1\lambda_2^{5}}{E_2\lambda_1^{5}}\exp\left[\frac{C_2}{T}\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)\right] \quad \ldots \quad (195)$$

It has been suggested that for certain applications the emissivities E_1 and E_2 could be assumed equal. Equation (195) could then be reduced to:

$$R = C_3\exp\,(C_4/T) \quad \ldots\ldots \quad (196)$$

where $C_3 = \dfrac{\lambda_2^{5}}{\lambda_1^{5}}$

$$C_4 = C_2\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$$

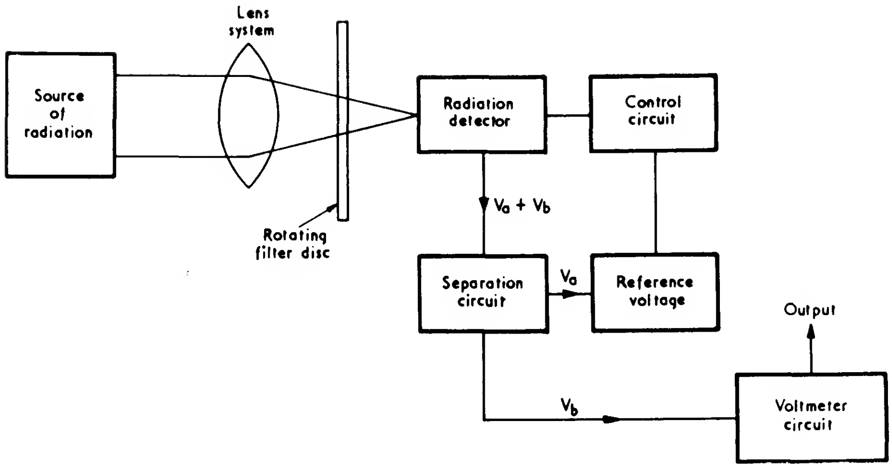


Fig. 165. Block diagram of Latronics ratio pyrometer (Originally known as the Shawmeter)

Equation (196) implies that by a measurement of the ratio of two radiation intensities at given wavelengths the temperature of a body may be determined. This has given rise to the name two colour or ratio pyrometry. Since the particular wavelengths chosen are normally both in the infra-red region, ratio would seem a better description than two colour.

Observe that the ratio concept renders the measurements less susceptible to dust or smoke in the atmosphere between the radiation source and pyrometer, assuming, of course, that the effect of the dust and smoke is the same at both wavelengths.

Provided the radiation fills the same viewing area at both wavelengths it is not necessary for it to occupy the whole of an inlet aperture as in the other optical pyrometer described.

But the most significant feature of the ratio pyrometer must be the minimizing if not the elimination of the effects of emissivity values and variations. This can be seen from equations (195) and (196).

A practical design of a ratio pyrometer is not easy and the number of models available is not large. Space does not permit of a detailed description of each, but brief particulars of the principal types follow.

A paper by T. P. Murray and V. G. Shaw[5] describes the Latronics ratio pyrometer. The instrument utilizes a photomultiplier as a radiation detector. The basic arrangement is shown in *Fig.* 165. The radiation is focused into the radiation detector by a lens system. Interposed between the radiation source and the detector is a rotating disc containing two filters which transmit wavebands in different parts of the spectrum. Let us call these for descriptive purposes "*a*" band and "*b*" band. The output from the photomultiplier cell is in the form of square-edged pulses as indicated in *Fig.* 166. V_a, the output from the cell corresponding to the "*a*" band, is compared with a reference voltage and is maintained at a constant value by the closed loop control circuit.

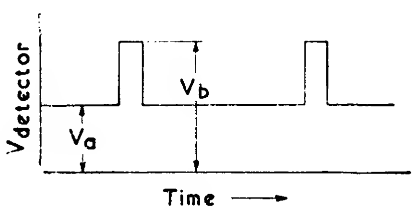


Fig. 166. *Signal waveforms of Latronics ratio pyrometer*

V_b is the cell output voltage corresponding to the "*b*" band of radiation. This is not kept constant, but is allowed to vary. V_b must then be proportional to the ratio of the radiation energies in the two bands.

If the original paper is consulted, it will be found that the instrument achieved excellent results in a steel works.

On the British side, T. Land[6] makes reference to designs of his own in a paper to the Society of Instrument Technology. These use two detectors instead of one as in the Shawmeter. The radiation beam is split into two parts. One is transmitted to a detector sensitive to one waveband in the spectrum and the other part is transmitted to a second detector sensitive to another waveband. One design utilizes a silicon cell and a thermopile as detectors. Another design incorporates a silicon cell and a selenium cell as detectors.

A design by Joseph Lucas Ltd. has been developed to measure rapidly varying temperatures, from 150°C upwards, on moving surfaces of varying emissivity. It uses indium antimonide photocells and infra-red filters to obtain electrical signals proportional to intensity in two different wavebands. The instantaneous ratio of these two intensities is computed by means of logarithmic function generators followed by a differential amplifier. Calibration charts are plotted on paper of log intensity against reciprocal of absolute temperature.

PYROMETERS FOR MODERATELY LOW TEMPERATURE MEASUREMENT

An increasing need for measurement of the temperatures of surfaces at moderately low values has led to the development of a number of designs. In addition to the Lucas model just described, two more are selected as typical. The temperature range involved is of the order of 50°C to 200°C.

In the first example the lead-sulphide photocell has been used as the radiation detecting device. This particular instrument is due to the British Scientific Instrument Research Association[7]. *Fig.* 167 indicates the principles. A heater emitting full (black body) radiation is placed in the vicinity of the surface whose temperature is to be measured. This radiation falls on the surface and some is reflected back to the photocell. If r is the reflectivity of the surface, the radiation reaching the cell can be represented by rQ_1, where Q_1 is the radiation emitted by the heater. The surface emits radiation due to its own temperature. If e is the emissivity of the surface then radiation eQ_2 reaches the

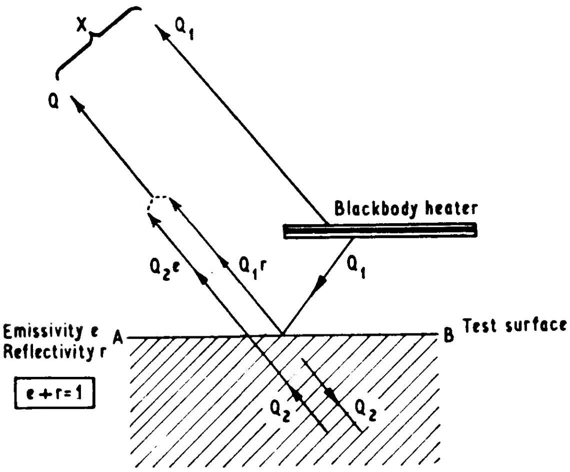


Fig. 167. *Principle of BSIRA pyrometer*

cell from this source. But in addition there is direct transmission of radiation from the heater to the cell of Q_1. We have virtually two beams incident on the cell:

$(rQ_1 + eQ_2)$ from the surface,

Q_1 direct from the heater.

Now the heat supply to the heater is adjustable by a servo system, and each of the two beams is modulated alternately by a suitable chopper. If the radiation in the two paths is not equal, a signal of modulation frequency is produced. This can be used to operate the servo system to adjust the heat supply until the temperature of the heater is the same as that of the surface. Then the following condition holds:

$$Q_1 = Q_2 = Q \text{ (heater radiation)} \quad \ldots \quad (197)$$

For specular reflection $(r + e) = 1$, hence

$$(rQ_1 + eQ_2) = Q\,(r + e) = Q \quad \ldots\ldots \quad (198)$$

Under these conditions, the alternate chopping of the two paths produces no signal of modulation frequency at the cell. A pen mechanism coupled to the servo system enables a recorder to be used. The accuracy is stated to be ±2°C under good conditions and ±5°C under less favourable conditions.

The second example uses a sensitive heat flowmeter and is due to the T.N.O. and T.H., Delft[8]. The heatflow unit is shown in *Fig.* 168. It comprises a Teflon tape wound with half coppered constantan wire. In this manner a series of thermocouples is obtained with the hot junctions on one side and the cold junctions on the other. The pyrometer unit is placed very close to the surface whose temperature is to be measured. The front of the heatflow unit is presented to the surface whilst the back surface is heated by a heater. The electrical heating supply is adjustable by a Variac

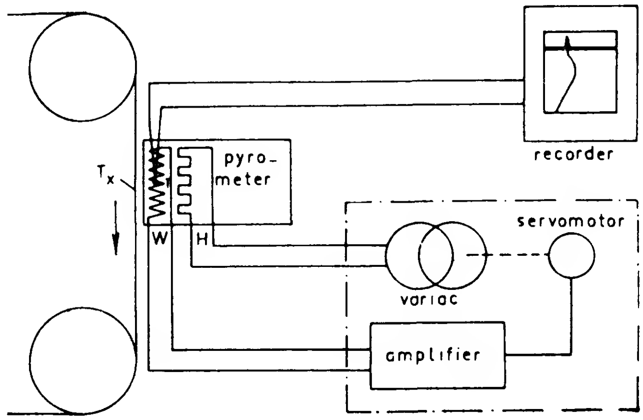


Fig. 169. *Circuit of TNO pyrometer*

pattern transformer. The adjustment is carried out by means of a servo motor (*Fig.* 169). Now if a difference in temperature exists between the front and back of the heatflow unit, the thermocouple arrangement produces a signal which is supplied to the amplifier. The amplifier output operates the servo motor which adjusts the heat supply until a heat balance is achieved and the temperatures at the front and back of the unit are equal. It is then considered that because of the close proximity of the unit to the surface the front temperature is the same as that of the surface. A thermocouple inserted between the Teflon tapes enables the temperature to be recorded or indicated.

Books, etc., for further reading

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2. JONES, E. B. *Instrument Technology.* Chapter 4. Butterworths, 1953.
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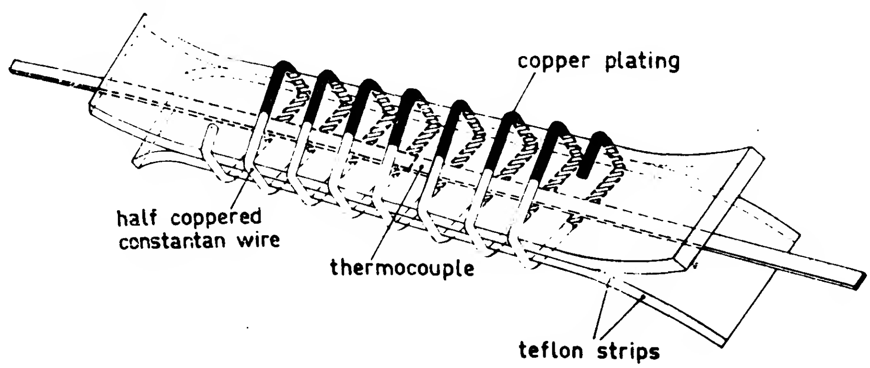


Fig. 168. *Construction of TNO pyrometer element*

Chapter 11

MEASUREMENT OF MOISTURE CONCENTRATION IN AIR AND OTHER GASES

INTRODUCTION

A KNOWLEDGE of the concentration of water vapour in air and other gases is of considerable importance in a variety of industries. Two well-known examples are tobacco and textile manufacture. The concentration is measured in terms of relative humidity, absolute humidity, dew point and weight concentration.

Let us commence with a consideration of relative and absolute humidity.

RELATIVE HUMIDITY

For the purposes of explanation let us restrict ourselves to the atmosphere which normally contains a percentage of water vapour. The vapour itself exerts a pressure which we will call p_v. The air exerts a pressure which we will call p_a. The *total* pressure P of the moist atmosphere is:

$$P = p_v + p_a \qquad (199)$$

Under such circumstances each constituent is said to exert a *partial pressure*. The definition of the latter is the pressure which the constituent would exert if it alone occupied the volume.

The relative humidity H_R is the ratio of the partial pressure p_v of the water vapour actually present in a given volume of air to the partial pressure p_s of the water vapour required to saturate the same volume of air at the same temperature. It is normally expressed as a percentage:

$$H_R = \frac{p_v}{p_s} \times 100 \qquad (200)$$

If water vapour is considered to obey the gas laws

$$p_v V = W_v RT \qquad (201)$$

and

$$p_s V = W_s RT \qquad (202)$$

where V = the volume of moist air.

p = the partial pressure due to the water vapour present in V.

p_s = the partial pressure which would be exerted by the water vapour required to saturate V.

W_v = the weight of vapour present in V.

W_s = the weight of vapour required to saturate V.

T = the absolute temperature.

R = a constant.

From (201) and (202),

$$\frac{p}{p_s} = \frac{W_v}{W_s} \qquad (203)$$

so that,

$$H_R = \frac{W_v}{W_s} \times 100 \qquad (204)$$

Although air only has been mentioned, the argument applies to gases in general.

METHODS OF MEASURING RELATIVE HUMIDITY

The names hygrometer or psychrometer are frequently applied to relative humidity measuring instruments. One type universally used is the wet and dry bulb hygrometer, and this is described first.

Wet and Dry Bulb Hygrometer

When water changes from the liquid state to the vapour, i.e. as it evaporates, it requires heat in the form of *latent heat* to perform the operation. The supply of heat by the water tends to lower its temperature. The rate of evaporation is dependent on the amount of moisture in the atmosphere. Upon these basic facts it is possible to construct a humidity measuring instrument known as the wet and dry bulb hygrometer.

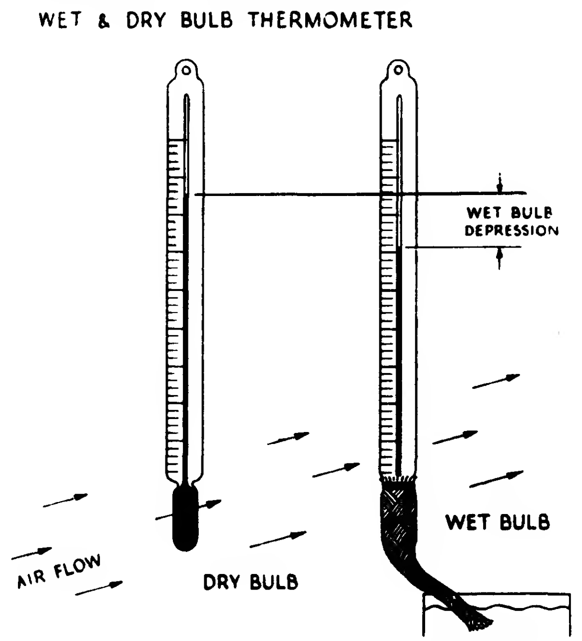


Fig. 170. *Mercury-in-glass type of wet and dry bulb hygrometer*

It comprises two thermometers and a simple design is shown in *Fig.* 170. Round the bulb of one thermometer is a tight fitting cotton wick which dips into a vessel of water. This is termed the wet bulb thermometer. The second thermometer, in an adjacent position, is normal and known as the dry bulb thermometer. The action of the water evaporating from the wick of the wet bulb thermometer cools the bulb, and its temperature, T_W, falls below that of the dry bulb,

T_D, by an amount depending upon the relative humidity of the air. A true theoretical explanation of the process has been sought but to some extent the equation relating the various factors remains empirical:

$$p = p_s - AP\,(T_D - T_W) \quad . \quad . \quad . \quad (205)$$

where p = the vapour pressure due to the vapour actually present in the air.

p_s = the saturation vapour pressure at the wet bulb temperature T_W.

A = a constant. It is found to be dependent on the dimensions of the wet bulb if the velocity of air stream past the bulb is less than about 10ft/second. Precautions should be taken to ensure that the air is in movement at this or greater values, if accurate readings are to be obtained.

P = the atmospheric pressure.

T_D = the dry bulb temperature.

T_W = the wet bulb temperature.

Rearranging equation (205),

$$\frac{p}{p_s} = 1 - \frac{AP}{p_s}\,(T_D - T_W) \quad . \quad . \quad (206)$$

$$H_R = \frac{p}{p_s} \times 100 = \left[1 - \frac{AP}{p_s}(T_D - T_W)\right]100 \quad . \quad (207)$$

Tables based on the above formulæ are issued by various manufacturers and the Meteorological Office. These are normally in terms of the wet bulb depression ($T_D - T_W$) and the dry bulb temperature T_D, i.e. the temperature of the air at which the humidity requires to be known. Usually the table values are calculated for P = 760mm of mercury, but corrections can be made for other values of atmospheric pressure. Unless the change is large, errors are not serious.

Two developments of the wet and dry bulb hygrometer in which an adequate flow of air past the wet bulb is ensured are described below.

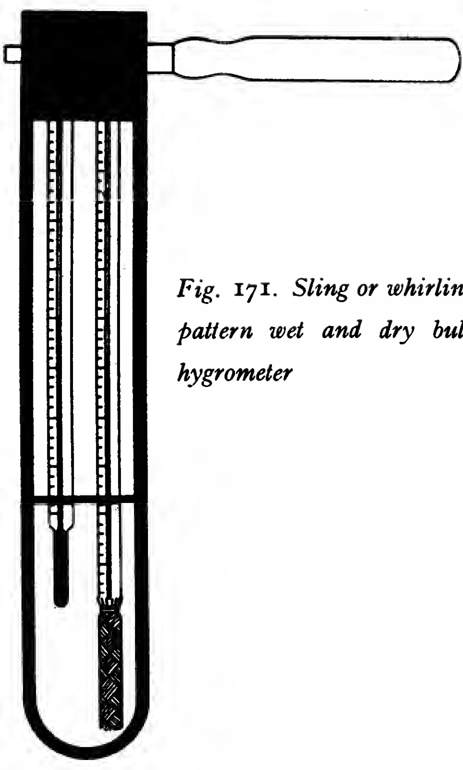


Fig. 171. Sling or whirling pattern wet and dry bulb hygrometer

Sling Hygrometer or Whirling Hygrometer

Two mercury-in-glass thermometers are securely fastened in a frame pivoted on a handle (*Fig.* 171). The wet bulb assembly with its wick is so constructed that it cannot spill during operation. The frame is rotated by hand, in a very similar fashion to a sports rattle, the motion of the thermometer bulbs through the air producing a relative velocity equal to or greater than the value necessary to keep A, in equation (205), constant. The thermometer readings are noted and the relative humidity must be found from the thermometer readings and tables.

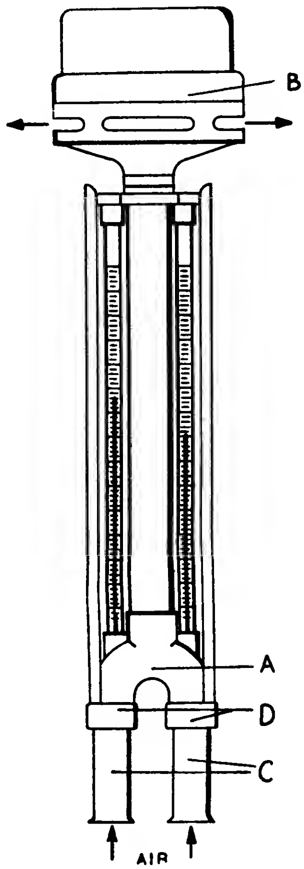


Fig. 172. Assmann pattern wet and dry bulb hygrometer

Assmann Hygrometer

In this design, the mercury-in-glass bulbs are situated in two small metal tunnels which form part of a small duct. At the top of the duct is a fan driven by a clockwork or electric motor. In operation, the fan draws air up through the small tunnels, past the bulbs, and exhausts at the top of the duct (see *Fig.* 172). Again, the two thermometer readings must be observed and the relative humidity found from tables.

Industrial Types

Mercury-in-steel Hygrometers

Wet and dry bulb hygrometers constructed from mercury-in-glass thermometers, as in *Figs.* 170, 171 or 172, have the advantage of simplicity but are not necessarily suitable for industrial plants where robustness is demanded. Neither are they of any use where a continuous record is required. For process applications, therefore, recourse must be made to one of the types of thermometers described in Chapters 8 and 9,

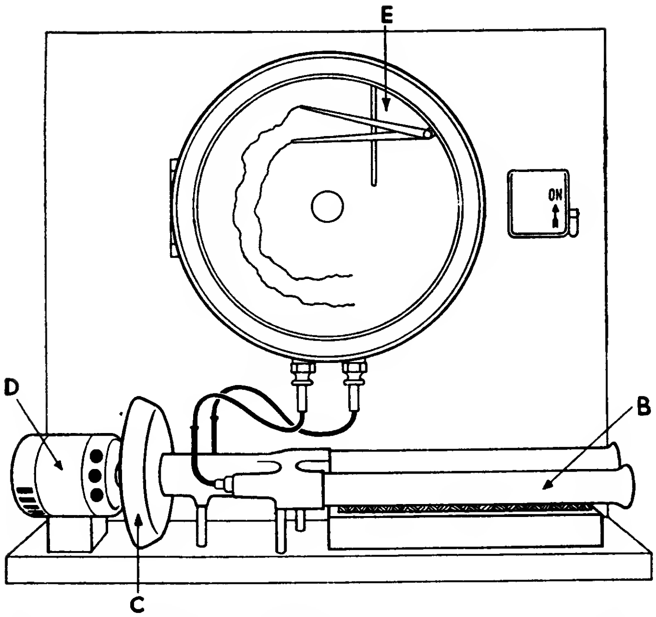


Fig. 173. *Mercury-in-steel pattern wet and dry bulb hygrometer*

and of these, mercury-in-steel thermometers have been widely used. The basic features of a design utilizing this type are indicated in *Fig.* 173. The two thermometers are situated in ducts *B*, through which air is drawn by fan *C* operated by motor *D*. The wet bulb is covered with a sleeve of material which dips down into a trough of water below the ducts. The water in the trough must, of course, be maintained at a sufficient level to keep the sleeve saturated. Observe that a two-pen recording instrument is shown, one for the dry bulb and the other for the wet bulb. As in the mercury-in-glass type of hygrometer, it is necessary to determine the relative humidity values by referring to tables. The last feature is inherent in this type of hygrometer and an instrument reading directly in percentage relative humidity is an advantage. For this purpose electrical resistance thermometers of the general pattern described in Chapter 9 have been used.

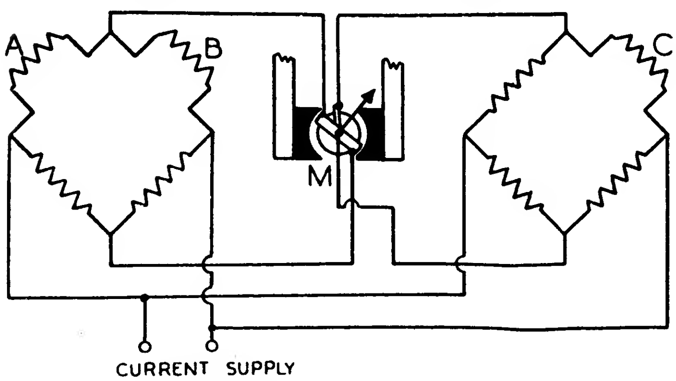


Fig. 174. *Electrical resistance pattern wet and dry bulb direct reading hygrometer*

Electrical Resistance Wet and Dry Bulb Hygrometer

A direct reading electrical resistance wet and dry bulb hygrometer is indicated in *Fig.* 174. It comprises two Wheatstone bridges. In one are a wet bulb thermometer *A* and a dry bulb thermometer *B* in opposite branches. The out-of-balance current is then a measure of the difference in their resistances and hence in their temperatures. A second dry bulb thermometer is connected in one arm of the other Wheatstone bridge, and the out-of-balance current here is a measure of the dry bulb temperature. The outputs of the bridges are fed to coils of a cross coil galvanometer as shown in *Fig.* 174. Since the deflection of the coil is proportional to the ratio of the currents in the two coils, the instrument scale may be calibrated directly in percentage relative humidity.

The accuracy of such an instrument may be of the order of 3%.

Hair Hygrometer

Human hair possesses the property of changing its dimensions, in particular the length, with the variation of moisture content in the air. The physical basis of the operation is beyond the scope of this course, but is explained in ref. (4).

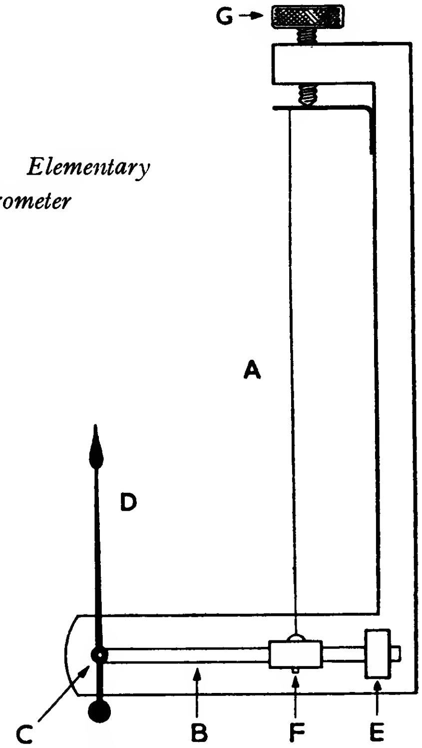


Fig. 175. *Elementary hair hygrometer*

It may be apposite to quote here one equation in that reference:

$$x = K.T.\log_{10}\frac{100}{H_R} \qquad (208)$$

where x = the change per unit initial length (saturated).
K = a constant.
T = the temperature of the atmosphere in °C. absolute.
H_R = the relative humidity.

The equation shows the change in length of the hair is dependent on the temperature of the air, and the relative humidity. The logarithmic relation is unfortunate in the sense that the instrument scale tends to become rather contracted at the higher humidity values. W. E. Knowles Middleton and A. F. Spilhaus[3] suggest that from 0 to 100% relative humidity the percentage change in length is $2\frac{1}{2}$. *Fig.* 175 shows a simple hair hygrometer movement. *A* is the hair element, *D* the pointer, *CBFE* a counterbalanced lever system and *G* a zero adjustment.

Whilst the instrument possesses the advantage of being a direct reading hygrometer, it has some drawbacks. It is not particularly accurate—a typical manufacturers' figure is 3 to 4 per cent. Rapid changes of humidity or temperature are likely to cause incorrect readings, and the element may take some hours to regain its normal condition. Temperatures above 160°F. are not recommended as the hair tends to become brittle, and below 15°F. a semi-permanent change in length takes place. The hygrometer is normally adjusted in the factory at a temperature of 60°F. to 65°F., and the recommended working temperature range is about 40°F. to 80°F. Outside these limits, errors are introduced; the instrument reads high at the low temperatures and low at the high temperatures.

In an effort to avoid the trouble caused by extreme humidities and temperatures (due, it is considered, to tension introduced into the hair element), one maker has fixed the hair to one end of a perforated tube and draws air over it by means of a small electrically driven fan. It is claimed that stable readings are produced in one-tenth of the time required in the normal type. An optical lever is used to produce a spot which travels up and down a translucent scale. With such an arrangement it is claimed that the accuracy is improved to about 1 per cent.

Gregory Hygrometer

Certain substances such as lithium chloride are hygroscopic. This means that they have the power to absorb or give up moisture to the surrounding atmosphere until a state of equilibrium is reached between the vapour pressure of the substance and the partial vapour pressure of the water vapour in the atmosphere. Consider a length of suitable fabric impregnated with lithium chloride. To the element so formed a low a.c. voltage is applied by means of two electrodes. It is found that the electrical resistance or conductivity varies in proportion to the amount of moisture absorbed or given up to the surrounding atmosphere and becomes effectively a means for measuring directly the relative humidity. The behaviour is analogous to that of an electrolyte in which the resistance or conductivity varies with the concentration of the salt in water.

The derivation of an equation connecting the various factors involved is not easy and reference (4) should be consulted for full theoretical treatment. It is sufficient to say that the resistance R of the element is a function of the relative humidity H_R and temperature T, i.e.

$$R = f(H_R T) \qquad \qquad (209)$$

A hygrometer based on the electrolyte principle involves normally threads of fibreglass, plastic or similar material impregnated with lithium chloride and wound on an assembly of platinum electrodes as in *Fig.* 176. A relatively large surface is exposed to the medium whose relative humidity is to be measured and there is a high speed of response.

The measuring circuit can take the simple form of *Fig.* 177 where the element acts as a variable resistance in series with a milliammeter. The voltage supply must of course be stabilized otherwise variation can affect the reading. Since the element is virtually a variable resistance it can also be used as one arm of a resistance bridge circuit. The bridge is balanced at one basic value of relative humidity and the out-of-balance current used as a measurement at all other values of relative humidity. In either circuit the indicating instrument (or recorder) can be calibrated directly in humidity values.

Equation (209) indicates that the hygrometer is temperature dependent and if the instrument is used at any other temperature than that for which it is calibrated corrections may be necessary.

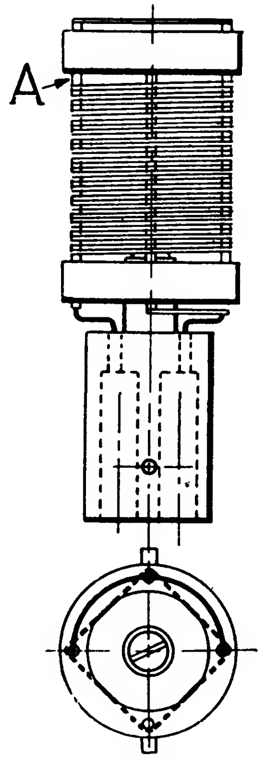


Fig. 176. *Gregory pattern hygrometer*

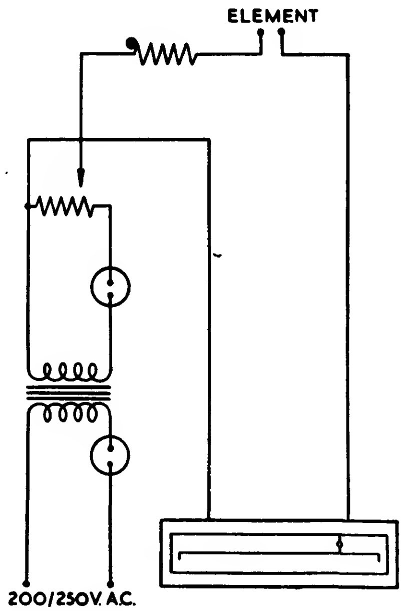


Fig. 177. *Simple circuit for Gregory pattern hygrometer with barretters for stabilization*

Typical ranges are 10% to 60% relative humidity and 15% to 100% or intermediately. If the work by H. Spencer-Gregory and E. Rourke[4] is consulted, it is suggested that if a hygroscopic solution can be produced in a state of tension, the saturation vapour pressure would be very much lower than the normal value at the same temperature. The property of electrolytic conduction or resistance would be retained. It is claimed that, if this could be achieved, values of less than 1% relative humidity could be measured.

ABSOLUTE HUMIDITY

This is defined as the weight of water vapour present in unit volume of moist air or gas at a given temperature.

Methods of Measuring Absolute Humidity

Thermal Conductivity Hygrometer

The heat lost per second from a heated wire surrounded by gas in a chamber or cell can take place by conduction, convection and radiation. If one could arrange by design that the loss is wholly by conduction, then the rate of heat loss would vary solely with the thermal conductivity of the gas. The conductivity is greatly influenced by the concentration of moisture in the gas, and this suggests a method of measuring absolute humidity.

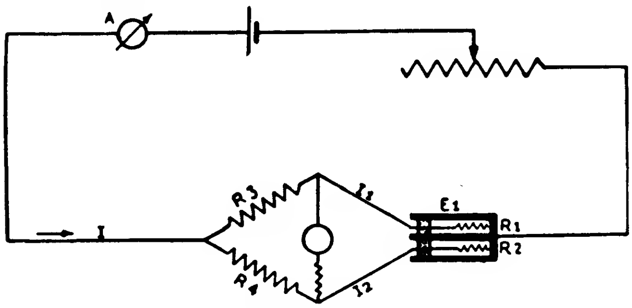


Fig. 178. *Thermal conductivity pattern hygrometer*

Consider *Fig.* 178, R_3 and R_4 are resistances and form two arms of a Wheatstone bridge. R_1 and R_2 are loops or spirals of a wire of identical resistances, such as platinum, contained in similar cells. In one is the gas whose humidity is to be measured (the measuring cell) and in the other is the same gas in a saturated or dry condition (the reference cell). A difference in moisture content of the gases in the two cells leads to a difference in temperatures and in resistances of the loops or spirals. The bridge is balanced at a datum value of moisture content and the out-of-balance current is, therefore, a measure of the moisture content at any other value.

In practice, the gas is passed through the measuring cell in the moist condition and is then thoroughly dried by a suitable absorbent. It is next passed through the reference cell in the dried condition. The sampling must be carefully regulated in view of the necessity to avoid convection losses and to ensure that the flow rate is the same through each cell.

Infra-red Hygrometer

Whilst the infra-red instrument here described is a general analytical instrument it has a particular application in the detection of moisture content in a gas.

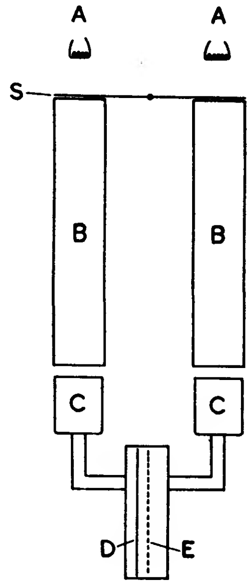


Fig. 179. *Infra-red pattern of comparator suitable for moisture content measurement*

If reference is made to *Fig.* 179, it will be seen that two identical tubes *BB* are involved. In the left-hand one is the reference gas and in the right-hand one the gas whose humidity is to be determined. The radiation from two identical heaters *AA* passes through the tubes to a differential detector *CC*. Obviously, if the reference tube contains the gas in a dry condition, the intensity of radiation reaching the detector via this tube is different from that reaching the detector via the tube containing the gas in a moist condition since some radiation is absorbed by the latter. If a suitable detector is used an electrical signal may be obtained and the electronic null balance measuring system incorporated. The indicator or recorder can be calibrated directly in terms of absolute humidity. The instrument is suitable for continuous sampling since the gases can be arranged to flow continuously through the tubes. It is possible to allow the moist gas to flow through one tube, dry it thoroughly and then pass it through the reference tube to obtain a differential signal.

DEW POINT INSTRUMENTS

Dew Point

Dew point is defined as the temperature at which the saturation vapour pressure equals the partial pressure of water vapour in the atmosphere or other gas. Alternatively the dew point can be regarded as the temperature to which the medium must be reduced for water to commence condensing from it. The meaning will become clearer in the following descriptions of dew point instruments. If the dew point is below 32°F. (0°C) it is termed the hoar frost point.

"Dewcel" Hygrometer

The Foxboro-Yoxall "Dewcel" hygrometer comprises a tube on which is wound a glass tape impregnated with lithium chloride. Over the glass tape is wound, in turn, a pair of gold wires parallel to each other but not touching. A low voltage a.c. supply is connected to the wires. Current can pass between the wires but only through the lithium chloride solution in the tape. Imagine the solution is initially in the saturated condition and in equilibrium with the surrounding

atmosphere. Consider an increase in relative humidity. It must be accompanied by an increase in the partial pressure of the water vapour in the atmosphere and this becomes greater than the vapour pressure of the lithium chloride solution. The trend then is for the water vapour to condense into the solution, diluting it. The resistance is lowered and an increased current flows between the wires. This produces an increased heating effect, raising the temperature of the element. The action continues until the vapour pressure of the latter equals the new partial pressure of the water vapour in the atmosphere. Any further heating would produce a super-saturated condition of the lithium chloride with corresponding high electrical resistance. But with a high resistance the current is correspondingly small with a consequent drop in the heating effect. Hence, a state of equilibrium between the lithium

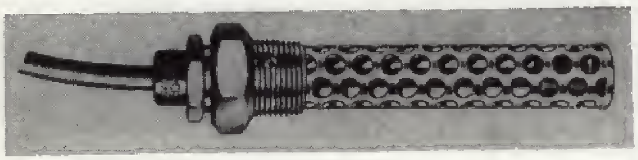


Fig. 180. *"Dewcel" hygrometer*

chloride and the water vapour in the atmosphere is reached. To illustrate the sort of vapour pressure and temperature values involved, with air at 70°F. and 40% relative humidity, the partial pressure of water vapour is 0·1474lb/in². This is equivalent to a dew point of 45°F. The vapour pressure of lithium chloride in solution at 70°F. is only 0·0434lb/in². To achieve equilibrium the solution must be heated to 112°F. when its vapour pressure is 0·1474lb/in².

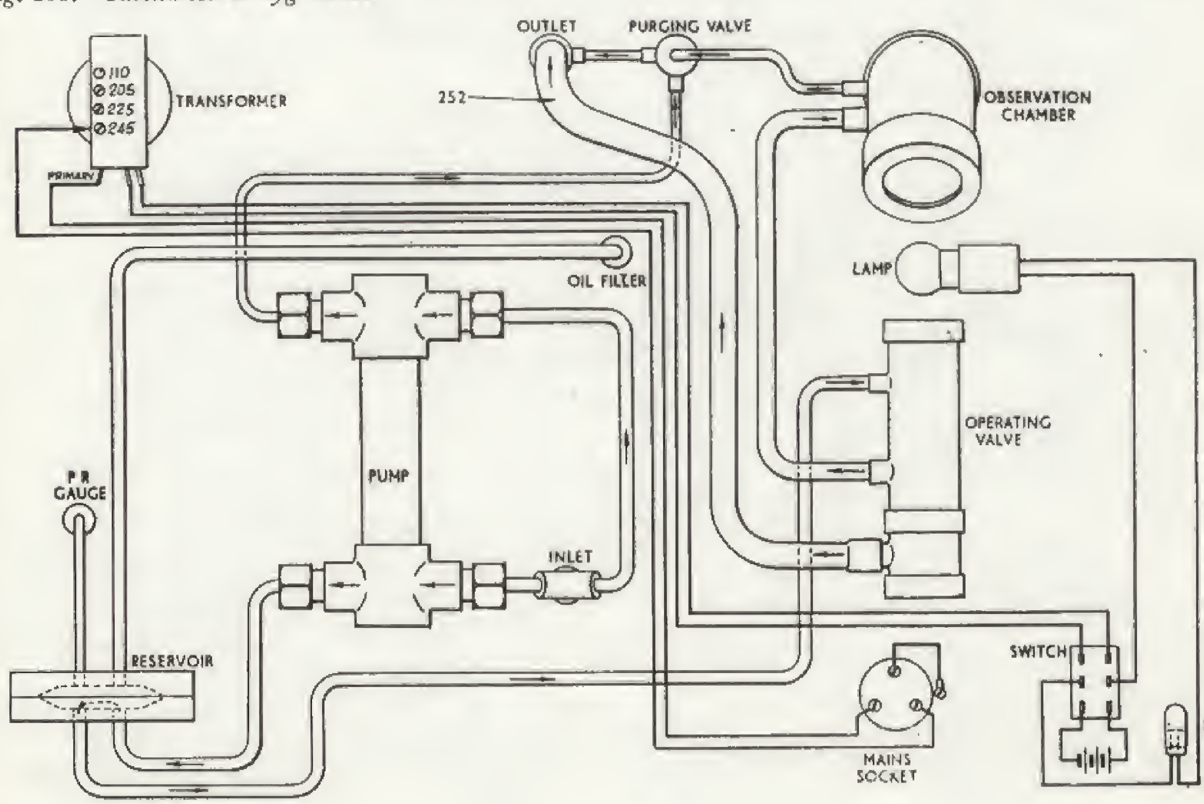
Inside the tube on which the tape is wound is a thermometer—the resistance pattern can be used with advantage. The thermometer measures the temperatures at which equilibrium of the "Dewcel" takes place and these, as can be seen from the foregoing description, are above those of the surrounding atmosphere.

If the relative humidity of the atmosphere decreases, the partial pressure of the water vapour also decreases and this is now lower than the vapour pressure of the lithium chloride. The reverse action must now take place with water from the lithium chloride tending to evaporate into the atmosphere. The element becomes relatively drier and its resistance increases. The current is smaller and is accompanied by a corresponding drop in heating effect. The operation continues until a state of equilibrium is reached between the element and the atmosphere at the lower vapour pressure. The temperature of the element has been lowered in the process, but is still above that of the surrounding atmosphere.

A general impression of the "Dewcel" can be obtained from *Fig.* 180. To obtain the dew point of the atmosphere a conversion is necessary, and the conversion is based on the vapour pressure characteristics of saturated lithium chloride solutions at various temperatures. The actual effective range for any particular set of conditions depends on the temperature of the medium. For air at 70°F., the minimum dew point that can be measured is 15°F. since this value corresponds to the vapour pressure of saturated lithium chloride at 70°F. Lower readings may be obtained with cooling equipment. In general, the instrument may be used between −50°F. and +140°F. dew points.

For relative humidity determination, it is necessary to add a dry bulb thermometer in the neighbourhood of the "Dewcel". A typical accuracy is ±3·5% relative humidity at 70°F. dry bulb temperature and 50° F. dewpoint. It is suitable for measurement of

Fig. 181. *Casella-Alnor hygrometer*



high humidities and is claimed to be satisfactory up to 100% relative humidity.

One precaution must be observed. The velocity of the medium across the "Dewcel" must be limited to 50ft/min to prevent excessive heat being carried away in the air stream, and suitable baffling may have to be installed to ensure this.

BCURA Dewpointmeter

The British Coal Utilization Research Association (BCURA) developed a dewpointmeter which had a particular application in the measurement of the dew point of flue gases. This was based on the change in conductivity between two electrodes on a cooled surface when a film of moisture condenses on the surface at the dew point[6].

Casella Alnor Hygrometer

If a gas is compressed from a particular pressure value and at a specific temperature and subsequently the pressure is released to another value, the gas expands and is cooled. The expansion should be adiabatic. If the pressure ratio is at the appropriate value, the condensation of moisture is affected.

In the Casella Alnor hygrometer, a basic diagram of which is shown in *Fig.* 181, samples are drawn into the hygrometer and are subjected to progressively higher and higher pressure ratios until on operation of the release valve a cloud of moisture is seen to form in the observation chamber. Tables are available by which the relative humidity or dew point may be determined from the temperature at which the operation was commenced and the pressure ratios.

The compression is carried out by a small hand pump shown to the left of *Fig.* 181 and the expansion is carried out by the operating valve on the right-hand side of the figure. The pressure gauge by which the pressure ratios may be observed is at the extreme left. The volume utilized in each sample is very small and to avoid errors it is necessary to purge the instrument before taking a series of measurements.

Dew point measurements down to −80°F. are possible at normal room temperature.

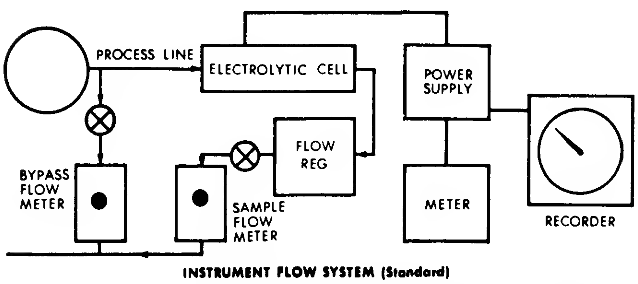


Fig. 182. *Block diagram of Beckman standard design of electrolytic hygrometer*

Electrolytic Hygrometer

This pattern of hygrometer involves an electrolytic cell. In one design the cell consists of a tube with a pair of closely spaced platinum wires wound in a double helix on its inner surface. In the space between the wires is a film of phosphorous pentoxide. A d.c. voltage is applied to the cell so formed.

If a sample of gas with a moisture content is passed through the cell the moisture is absorbed by the phosphorous pentoxide and is electrolized into oxygen and hydrogen ions. An electrolysis current is established and the value of this is proportional to the mass rate of flow of moisture into the instrument. The rate of sample flow must, therefore, be kept constant and this is achieved by incorporating a flow controller in the sample line after the cell. A normal flow rate is 100cc/min.

If high rates of sampling are necessary, a portion of the sample is diverted through a bypass system (*Fig.* 182). This speeds up the response without overloading the electrolytic cell.

Such an instrument is normally calibrated in parts of water per million parts of gas and is, strictly speaking, a means of absolute humidity measurement rather than relative humidity.

In a typical hygrometer (Beckman Instruments) ranges between 0-10p.p.m. and 0-1000p.p.m. are available with ±5% full scale accuracy.

Books, etc., for further reading

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British Standard Specifications Relating to Humidity Measurement

B.S. 1051 Moisture in Relation to Textile Materials.

B.S. 1339 Humidity of the Air: Definition, Formulae, and Constants.

B.S. 2841 General Purpose Wet and Dry Bulb Hygrometers.

Questions

1. *If the partial pressure of water vapour in the atmosphere is* 12·5*mm of mercury at* 20° *C., and the saturation pressure is* 17·5*mm of mercury, what is the relative humidity?*

   If the weight of vapour at saturation conditions is 0·00188 *lb/ft*^{3}*, what is the corresponding weight at the relative humidity value?* (*Ans.* 71·5 *per cent and* 0·00134 *lb/ft*^{3})
2. *Using formula* (207), *if* $A = 0{\cdot}000656$, $P = 760$*mm of mercury*, $p_S = 17{\cdot}5$*mm of mercury*, $T_D = 20$°*C. and* $T_W = 15$°*C., what is the relative humidity?* (*Ans.* 85·8 *per cent*)
3. *Using formula* (208) *find the ratio of the values of* x *at* 50 *per cent relative humidity and* 80 *per cent relative humidity at the same temperature* T. (*Ans.* 3·11)

Chapter 12

pH MEASUREMENT

IONIC DISSOCIATION

THE phenomena of dissociation is experienced in solutions of acids, alkalis and pure water. It can be explained as the tendency of some of the molecules to dissociate or split up into atoms or groups of atoms. The dissociated atoms or groups are termed ions and each carries a positive or negative charge of electricity. The action is reversible and a state of equilibrium is reached where the number of molecules in process of dissociation is balanced by the number of molecules being formed by recombined ions. This leads to a constant number of molecules in the dissociated state for a given concentration and temperature. To explain further the operation let us first consider the example of pure water:

$$(H_2O) \rightleftharpoons (H)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (210)$$

The relation indicates that a molecule (H_2O) of water can split up into a hydrogen ion $(H)^+$ and a hydroxyl ion $(OH)^-$ or that an ion of hydrogen and a hydroxyl ion can recombine to form a molecule. The hydrogen ion carries a positive charge and the hydroxyl ion a negative charge. These charges are equal and the molecule itself is electrically neutral.

Since the number of ions is constant at any one temperature their degree of dissociation, K, must also be a constant at the same temperature. This can be represented symbolically as:

$$K = \frac{(H)^+ \times (OH)^-}{(H_2O)} \quad \ldots\ldots\ldots\ldots (211)$$

For pure water, the concentration of ions is extremely small, and the concentration of undissociated molecules (H_2O) may be regarded as constant. Equation (211) can then be written:

$$K_w = (H)^+ \times (OH)^- \quad \ldots\ldots\ldots\ldots (212)$$

K_w is termed the Dissociation Constant for Water. At 22°C it has been found to have the value of 10^{-14} gram equivalents per litre. Since the number of hydrogen and hydroxyl ions are equal, the hydrogen ion concentration is 10^{-7} gram equivalents per litre. The value of K_w applies not only to water but to aqueous solutions.

Now let us consider a typical acid, sulphuric acid. The dissociation relation can be represented by:

$$(H_2SO_4) \rightleftharpoons 2(H)^+ + (SO_4)^{--} \quad \ldots\ldots\ldots (213)$$

(SO_4) is termed the acid ion carrying two negative charges to balance the two hydrogen ions. In general, acids may be represented by (HA) where H represents the hydrogen radical and A the acid radical:

$$(HA) \rightleftharpoons (H)^+ + (A)^- \quad \ldots\ldots\ldots\ldots (214)$$

Acids may be classified as weak or strong. In the case of weak acids few ions are produced, and the hydrogen ion concentration is small. For strong acids, however, a relatively large concentration of hydrogen ions (and acid ions) is achieved. For weak acids a similar relation to (211) exists.

$$K = \frac{(H)^+ \times (A)^-}{(HA)} \quad \ldots\ldots\ldots\ldots (215)$$

Now what happens in the case of an acid solution? As an example consider the addition of sulphuric acid to water or vice versa. We have two dissociation equations.

$$(H_2O) \rightleftharpoons (H)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (216)$$

$$(H_2SO_4) \rightleftharpoons 2(H)^+ + (SO_4)^{--} \quad \ldots\ldots\ldots (217)$$

It is clear that there is a greater concentration of hydrogen ions now than with water alone. But, at the same time, equation (212), which holds for aqueous solutions, must be satisfied. An increase in hydrogen ions infers a decrease in hydroxyl ions if their product is to remain at a constant value K_w. Some of the dissociated water molecules recombine, therefore, to effect the necessary reduction.

Turning to alkalis, we have substances which can be given a general symbolical representation $(B.OH)$ where B represents the base radical and OH the hydroxyl radical. The dissociation is:

$$(B\,OH) \rightleftharpoons (B)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (218)$$

Taking a common example, sodium hydroxide or caustic soda:

$$(Na\,OH) \rightleftharpoons (Na)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (219)$$

We next see the effect of adding an alkali to water. Keeping the same example we have

$$(Na\,OH) \rightleftharpoons (NA)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (220)$$

$$(H_2O) \rightleftharpoons (H)^+ + (OH)^- \quad \ldots\ldots\ldots\ldots (221)$$

The opposite phenomena occurs to that of acids. The hydroxyl ion is present in greater concentration than with water alone. This leads to a recombination of dissociated water molecules to reduce the number of hydrogen ions in solution, so that (212) may be obeyed. The hydrogen ions in concentration are then relatively small in number and are less than with pure water.

A similar relation to the dissociation constant for weak acids may be obtained for weak alkalis:

$$K = \frac{(B)^+(OH)^-}{(B\,OH)} \quad \ldots\ldots\ldots\ldots (222)$$

THE ORIGINAL pH DEFINITION AND SCALE

Some idea of the range of hydrogen ion concentration encountered in practice may be shown by one acid and one alkali. A Normal solution of hydrochloric acid has a hydrogen ion concentration of 10^{-1} or 1 gram equivalent per litre and a Normal solution of sodium hydroxide 10^{-14} gram equivalents per litre. With pure water the value is 10^{-7} gram equivalents per litre as we have seen. These figures suggest that a scale could be formed ranging from extreme acidity through water as a neutral value to extreme alkalinity.

The large range means that any scale adopted must be logarithmic in character and one which has been extensively used owes its origin to Sorensen, a Danish biochemist. Denoting hydrogen ion concentration by C_H,

$$C_H = 10^{-x} \qquad (223)$$

On a logarithmic basis,

$$x = -\log_{10} C_H \qquad (224)$$

$$\text{or } x = \log_{10} \frac{1}{C_H} \qquad (225)$$

Instead of x, we write pH,

$$\text{so that } pH = \log_{10}\frac{1}{C_H} \qquad (226)$$

Substituting the values given previously,
For a Normal solution of hydrochloric acid,

$$pH = \log_{10}\frac{1}{1} = 0$$

For pure water, $pH = \log_{10} \frac{1}{10^{-7}} = 7$

For a Normal solution of sodium hydroxide,

$$pH = \log_{10}\frac{1}{10^{-14}} = 14$$

In such a scale the hydrogen ion concentration is a maximum at 0 and a minimum at 14. All solutions with pH from 0-7 possess a higher concentration of hydrogen ions than pure water. Values 0-7 pH, therefore, represent acid solutions. Solutions with pH values 7-14 have a hydrogen ion concentration less than pure water and represent alkaline solutions. Each change of 1 pH is equivalent to a ten-fold change in hydrogen ion concentration. Note that it is possible to achieve a negative pH value if the hydrogen ion concentration is greater than 1 gram equivalent.

THE BRITISH STANDARD pH DEFINITION AND SCALE

Whilst the original pH definition and scale was not an ideal one it served for a large number of years. Further thought on the subject led to a British Standard Specification No. 1647 and the changed ideas may be best expressed by quoting from the specification.

"Originally, pH was intended to denote $-\log_{10}C_H$ where C_H is the hydrogen ion concentration. It is now realized that, if thus defined, pH is not measurable. In fact if pH is to be a measurable quantity it cannot have any exact simple fundamental significance. The definition adopted for pH is accordingly an operational one, and, to some extent, an arbitrary one."

A concession is made by B.S.1647 for aqueous solutions which are neither strongly acid nor strongly alkaline, i.e., corresponding to pH values between 2 and 12. If the concentration of such solutions does not exceed $\frac{1}{10}$ molar ($\frac{1}{10}$ gram equivalent) the pH relation may be expressed as:

$$pH = -\log_{10} C_H f_1 \pm 0{\cdot}02 \qquad (227)$$

where C_H = the hydrogen ion concentration
f_1 = the mean activity coefficient of a typical uni-univalent electrolyte in the solution.

This is similar to the relation (226) except for the activity coefficient f_1.

But the practical definition and basis of measurement proposed by B.S.1647 depends on the *difference* in pH values between a primary standard solution and the solution being measured. If the respective pH values are $pH(s)$ and $pH(x)$ the relation between the two is given by:

$$pH(x) - pH(s) = \frac{Ex - Es}{2{\cdot}3026\, RT/F} \qquad (228)$$

$$\text{or } pH(x) = pH(s) + \frac{Ex - Es}{2{\cdot}3026\, RT/F} \qquad (229)$$

where R = the gas constant
T = the absolute temperature °C
F = the Faraday constant.

Values of $2{\cdot}3026\, RT/F$ for various temperatures can be found in B.S.1647. E_x and E_s require some explanation. E_x is the e.m.f. of the cell formed as follows:

Pt, H_2/Solution x/3·5 M KCl/Reference electrode (230)

Es is similarly the e.m.f. of the cell:

Pt, H_2/Solution s/3·5 M KCl/Reference electrode (231)

The measurement of pH values requires the use of two electrodes: a measuring and a reference electrode, and it will be convenient at this stage if pH electrodes are discussed as the significance of the cell expressions above will then become clearer.

ELECTRODES

Consider initially what happens when a metallic electrode is placed in a solution. There is a tendency for positive ions of the metal to enter the solution resulting in the solution becoming positively charged, leaving the metal electrode negatively charged. But if the solution already contains ions of the metal there is a tendency for ions to be deposited on the electrode, giving it a positive charge. The electrode ultimately reaches an equilibrium potential relative to the solution. The value and sign of the potential depend on the concentration of metallic ions in the solution.

Two common examples are zinc, which assumes a negative potential relative to solutions of its salts, and copper which reaches a positive potential relative to solutions of its salts. In passing it may be noted that these two electrodes and solutions of their salts form the basis of a Daniell cell.

Hydrogen, although a gas, can be made to form the basis of an electrode.

Hydrogen Electrode

The hydrogen electrode involves normally a platinum wire (or plate) covered with platinum black. This is contained in a glass envelope which is either left open at the end or contains holes to allow contact with the liquid whose pH is to be measured (*Fig.* 183). Hydrogen is bubbled over the electrode wire or plate and is absorbed into its surface—the unit then functions as a hydrogen electrode.

There are several disadvantages to the use of such

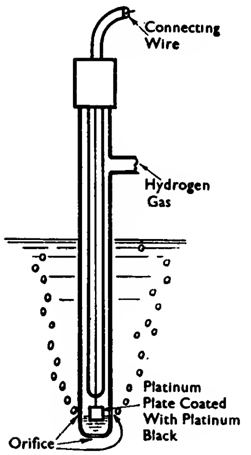


Fig. 183. *Hydrogen electrode*

an electrode in industrial processes and the main ones are:

1. It requires a continuous supply of pure hydrogen gas
2. It is susceptible to "poisoning" by many substances
3. It is difficult to maintain.

It does, however, form a useful standard to use as a basis for calibration and for this purpose the electrode is in contact with a 1·228 Normal solution of hydrochloric acid. The effective concentration of hydrogen ions is then taken as unity. The potential of a hydrogen electrode is related to the *pH* value of a solution by the following relation

$$E = E_0 - 0{\cdot}0001984T\,pH \text{ volts} \quad \ldots\ldots \quad (232)$$

where T = the temperature in °C absolute

E_0 = A constant potential equivalent to the potential of the electrode in a solution whose *pH* is zero.

For general industrial and laboratory applications the glass electrode has found a wide acceptance as a measuring electrode.

Glass Electrode

The glass electrode is covered by British Standard Specification No. 2586.

It comprises a thin glass bulb *a* fused to the end of a hollow glass stem *b* (*Fig.* 184). The unit so formed contains a solution of hydrochloric acid of known concentration. Into the solution and extending down into the bulb is a silver wire *d*, coated with silver chloride. A half cell is thus formed of the type silver/silver chloride/hydrochloric acid. The *pH* value of the half cell can be accurately known at any particular temperature. When the glass electrode is inserted in the solution whose *pH* value is to be determined, a potential difference, the value of which is proportional to the *pH* of the solution, is established across the glass membrane.

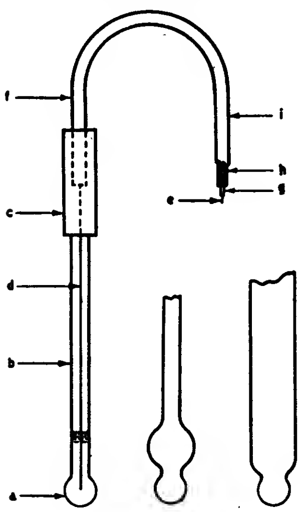


Fig. 184. *British Standard glass electrode. Reproduced by permission of the British Standards Institution from B.S.*2586

The exact process of the establishment of the potential difference can be explained in an elementary fashion by the transport of a small number of hydrogen ions through the glass membrane from the solution with the higher hydrogen ion concentration to the solution with the lower hydrogen ion concentration. The action can be considered to continue until the established potential difference balances the electrical forces opposing motion of the ions. This is an over-simplification and the explanation given in most text-books on *pH* measurement[1, 2], involving the introduction of potential energy levels, should be studied.

The electrical resistance of the glass membrane is of some importance and, of course, varies with the thickness of the glass. For example, a membrane of thickness 0·05 mm may have a d.c. resistance of the order of 5 megohms. The choice of glass also influences the resistance value and considerable investigation has gone on in recent years to develop glasses which may be used for relatively robust membranes and yet not result in excessive resistance. Note that the resistance of the glass may be temperature dependent.

Not every glass is suitable for the determination of *pH* values of solutions of high alkalinity, e.g., above about 10 *pH*. It is found that the positive base ions begin to influence the reading, causing it to indicate a lower value than the true one. Particularly is this pronounced with sodium ions and this has led to the effect being called the sodium ion error. The glass electrode could be calibrated with an alkaline buffer solution containing a suitable concentration of the appropriate salt, but much thought has been given to the production of glasses which substantially reduce, if they do not completely eliminate, the effect. Taking into account this and other factors, B.S.2586 has grouped glass electrodes as follows:

(a) General-purpose electrodes for *pH* 1 to 10
(b) High *pH* electrodes for *pH* 9-13
(c) Wide range electrodes for *pH* 1 to 13.

The maximum d.c. resistances allowable at 25°C for ordinary size general-purpose electrodes is 200 megohms, and for ordinary size high *pH* and wide range *pH* electrodes, 500 megohms.

Quoting at random from one manufacturer's catalogue, the general-purpose glass electrodes are suitable from 5° to 80°C in acids and alkalis and for *pH* values up to 10. The high *pH* electrodes are suitable for measurements at temperatures between 15°C and 80°C and another grade of electrodes with a low sodium ion error termed "amber glass electrodes" are suitable for determinations of *pH* in solutions up to a temperature of 100°C.

The asymmetry potential difference effect must be noted. This potential difference occurs even when the half cells or solutions on both sides of the membrane are identical. It is temperature dependent but allowance can be made for it in the design of the measuring circuit and temperature correction units.

Other Measuring Electrodes

It is not proposed to include detailed accounts of measuring electrodes which are not now in wide use in industrial applications. But mention must be made of the antimony electrode comprising a button or rod of pure antimony in an insulated holder, and the quinhydrone electrode involving a platinum wire or plate to which a small quantity of quinhydrone has been added. Descriptions of these can be found in references (1) and (3). These references give, in addition, the reasons for their supersession by other electrodes described in this chapter.

Calomel Electrode

The calomel electrode or half cell is the widest used of the reference electrodes. The construction of a typical pattern is indicated in *Fig.* 185. An inner tube contains calomel (mercurous chloride), potassium chloride paste and mercury. For electrical connection purposes, a platinum wire extends down the tube and makes contact with the mercury. An outer tube contains a saturated solution of potassium chloride and terminates in a porous plug. The latter can be of glass, ceramic material or asbestos fibre. The potassium chloride acts as a "salt bridge" and seeps or diffuses through the porous plug to make an effective liquid junction with the solution whose *pH* is to be determined.

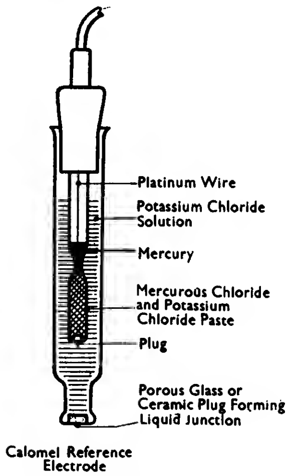


Fig. 185. *A typical calomel reference electrode*

Alternative designs have a small hole near the base of the outer tube and this is covered with a ground glass sleeve or a silicone rubber sleeve. These sleeves permit a faster rate of diffusion than with the asbestos fibre plug pattern generally. On a broad basis, the sleeve pattern is suitable for solutions of high viscosity or turbidity and with slurries, oil emulsions and soap solutions. It may be necessary after each measurement to loosen the sleeve and flush the tip of the electrode.

The choice of potassium chloride solution in the electrode is made for several reasons, but the main one is that the potential between this substance and many other solutions is extremely small and can be made very stable.

An expression deduced for the electrode in terms of the chloride ion concentration at 25°C is:

$$E = E_0 - \frac{0{\cdot}0591}{2} \log \frac{1}{(Cl^-)^2} \quad \ldots\ldots\ldots \quad (233)$$

A reference electrode must be substantially stable with regard to E.M.F. At room temperature, the calomel electrode is reasonably satisfactory in this direction.

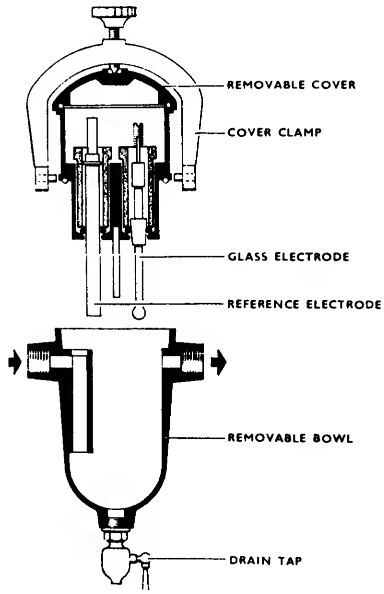


Fig. 186. *Cambridge - E.I.L. flow type pH electrode assembly*

CELL CONCEPT

It is now possible to see the *pH* measuring system as an e.m.f. cell. If we consider a combination of a glass measuring electrode and a calomel reference electrode, both contain half cells. The cell system is completed by the solution whose *pH* is being determined. The expressions (230) and (231) become clearer. Expression (230), for example, comprises a hydrogen electrode (P_t, H_2), the process solution x, a salt bridge 3·5 M KCl and the reference electrode.

ELECTRODE SYSTEMS FOR INDUSTRIAL PROCESSES

It will be realized that the electrodes shown in *Figs.* 184 and 185 in their simple state may not be suitable for industrial processes without further protection and it is proposed to give brief details of a few typical industrial electrode systems.

Fig. 186, for example, is a continuous flow design. In it the measuring glass electrode and the reference electrode are fixed in a removable cover assembly, but dip down into the lower bowl when in use. The process

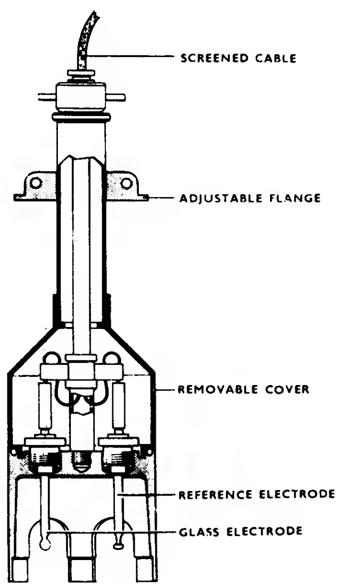


Fig. 187. *Cambridge - E.I.L. dip type pH electrode assembly. Reproduced by permission of the Cambridge Instrument Co. Ltd.*

liquid flows from left to right round the electrodes. This particular model is suitable for pressures up to 15lb/in^2.

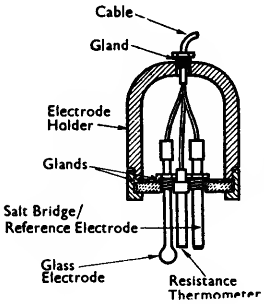


*Fig.*188. *Dip type pH electrode assembly with compensating temperature thermometer*

The model in *Fig.* 187, on the other hand, is designed for dipping or total immersion in a tank and can be used for installations up to 10lb/in^2.

A variation of the dip type electrode system is seen in *Fig.* 188 and includes a resistance thermometer for temperature compensation.

MEASURING CIRCUITS

The e.m.f. from the *pH* electrode system is d.c. and the measuring instruments fall into two broad classes:

1. Those which amplify directly the electrode e.m.f. so that the output can operate a standard indicator, recorder and controller. This class involves a d.c. amplifier with very stable characteristics.
2. Those which compare the electrode e.m.f. with a known standard e.m.f. The self-balancing potentiometer is the obvious instrument here, but it may involve the interposition of some form of d.c.-a.c. converter unit which transposes the d.c. e.m.f. signals to corresponding a.c. ones.

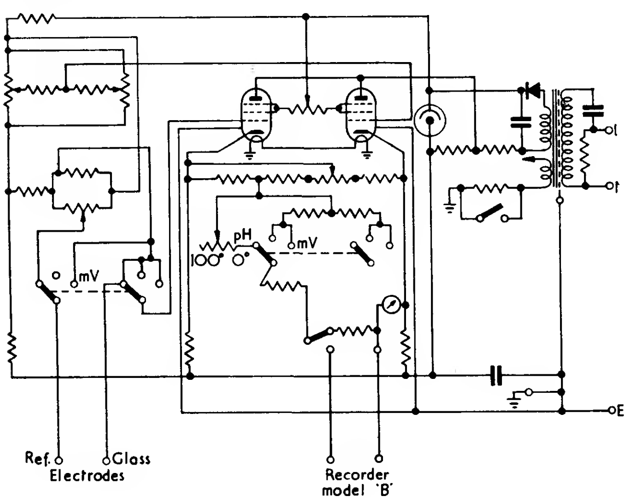
Whichever class is used, the following general requirements should be met:

1. The instrument to be substantially free from errors caused by supply voltage or frequency variations.
2. The zero reading to be stable over long periods.
3. The current taken from the electrode system to be extremely small to avoid polarization effects.
4. The design to include compensation for variations in characteristics of components or be such that the output is independent of such variations.
5. Temperature compensation to be included, either manual or automatic.
6. Facilities to be included for easy checking of calibrations.

With low resistance electrode systems a relatively simple potentiometer circuit could be used. In the low resistance category come the hydrogen-calomel, quinhydrone-calomel or antimony-calomel electrode combination.

The glass-calomel electrode system, however, may involve resistances of some hundreds of megohms, and the requirement of low current drain must be met. Any pattern of valve used for direct connection to the electrodes must satisfy the conditions of low grid current, stable anode current and maximum mutual conductance. The electrometer valve has been used for the initial amplification stage since it can meet these requirements, but at least one company has employed pentode valves as amplifiers. *Fig.* 189 shows the basic circuit of a Cambridge-E.I.L. industrial pattern *pH* meter. The amplifier uses two pentodes in a differential arrangement, the outputs of the two valves being balanced during zero adjustment. Application of the *pH* e.m.f. to the grid of one of the valves unbalances the circuit and causes a reading on the meter shown in the cathode circuit of the right-hand pentode. The meter is calibrated directly in *pH* values. Note that provision is made for a recorder to be connected to the amplifier. This particular design is stabilized by negative feedback so that wide changes in gain can occur without affecting the output current and, hence, the *pH* readings. Accuracy is stated to be 0·1 *pH* and zero drift is less than 0·1 *pH* in 24 hours. The grid current drain is only 10^{-12} amps. Temperature compensation in this case is manual, but ranges from 10-100°C. The instrument is safeguarded against mains variation by a constant voltage transformer and a neon stabilizer.

Fig. 189. *pH meter with d.c. amplification system. Reproduced by permission of the Cambridge Instrument Co. Ltd.*



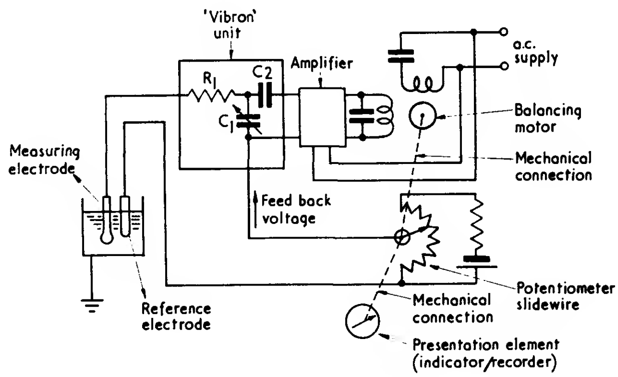


Fig. 190. *pH meter utilizing a vibrating capacitor unit for conversion of d.c. pH electrode potentials*

In the requirements for *pH* instruments the possibility of using a d.c.-a.c. converter was mentioned. This has been carried out in two systems—units using a capacitor as the basic component. Consider one example in *Fig.* 190. This incorporates the Cambridge-E.I.L. Vibron unit. Here, the capacitor involves one fixed plate, and one movable plate oscillated by an electromagnetic drive. The d.c. potential difference from the

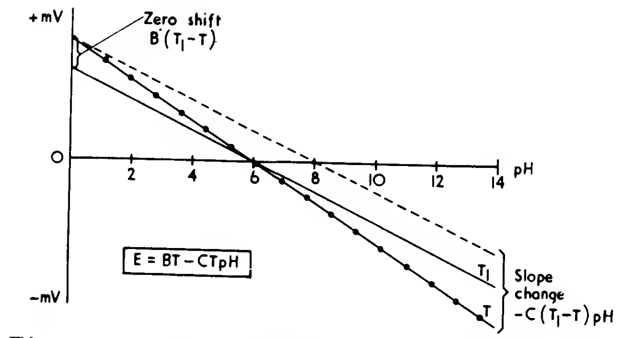


Fig. 191. *Effect of temperature on electrode potential*

electrodes is applied to the capacitor system via resistance R_1. From the vibrating capacitor C_1 an alternating voltage is produced, the amplitude depending on the d.c. potential difference of the electrodes. The blocking capacitor C_2 prevents any d.c. potential from reaching the input circuit of the a.c. amplifier. A potentiometer supplies a feedback voltage and this opposes the electrode potential difference. If the *pH* value is stable, the two are equal and there is no input to the capacitor unit. If the *pH* changes, however, the electrode potential difference becomes greater or less than the feedback voltage depending on the direction of the change. An a.c. unbalance signal is fed into the amplifier, the servo motor is energized and the potentiometer slider is adjusted in such a direction as to rebalance the circuit. In addition to the potentiometer slider, the pointer of the indicating instrument or the pen of a recording instrument is coupled to the shaft of the servo motor. By this means the presentation instrument reads the new value of *pH*.

TEMPERATURE RELATIONS AND COMPENSATION

The popular combination of glass electrode and calomel electrode gives rise to an e.m.f. which can be represented by an equation of the form:

$$E = A + BT - CT\,pH \qquad \ldots\ldots\ldots (234)$$

where $E =$ the e.m.f.

$T =$ the temperature in °C or °C absolute.

A, B, and C are constants.

Constant A, it will be observed, is not associated with either temperature T or pH and can be off-set by a suitable bias in the measuring circuit. This reduces the equation to

$$E = BT - CT\,pH \qquad \ldots\ldots\ldots\ldots (235)$$

The first term is proportional to temperature only but the second is proportional to both temperature and *pH*. Suppose now that the temperature changes from T to T_1. Then,

$$E_1 = BT_1 - CT_1\,pH \qquad \ldots\ldots\ldots\ldots (236)$$

and $E_1 - E = B\,(T_1 - T) - C\,(T_1 - T)\,pH$ . . (237)

Fig. 191 shows the e.m.f. plotted against *pH* values for equation (237).

We see that the *BT* term results in a zero change, but the *CT pH* term involves a change in slope. The former term produces a zero shift of the scale whilst the latter term increases the scale length.

Complete temperature compensation must take account of both factors and automatic methods are desirable.

The electrical resistance thermometer has been extensively used for automatic compensation purposes.

Space does not permit detailed descriptions of the various arrangements, but briefly, the methods depend on whether partial or complete automatic compensation is desired. The change in slope corresponding to the *CT pH* term can be compensated by using a resistance thermometer to vary the gain of the main amplifier. For complete compensation, i.e., including the zero shift, it is necessary to introduce a compensating voltage into the circuit. This has been carried out in the case of one manufacturer by using a resistance thermometer with double windings. One winding is connected in the feedback circuit of the main amplifier and serves to correct a change in slope. The other winding is connected to one arm of a resistance bridge. The bridge output voltage is then used to compensate for zero shift. The voltage is made adjustable by a suitable manual control so that it can be made to satisfy the characteristics of any electrode combination.

Buffer Solutions

In the calibration of pH equipment it is necessary to have standard solutions, the pH values of which are known accurately. These solutions undergo relatively little change of pH value on the addition of acid or alkali or with a large change of concentration. They usually consist of a mixture of a weak acid and its salt, of a strong alkali or of a weak base and its salt with a strong acid.

B.S.1647 contains fuller details of such solutions which are known as buffer solutions.

Questions

No questions are appended to this chapter.

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British Standard Specifications Relating to pH Measurement

B.S.1647 pH Scale.
B.S.2586 Glass Electrodes.
B.S.3145 Laboratory Potentiometric pH Meters.
B.S.3422 Laboratory Deflection pH Meters.

Chapter 13

FEEDBACK AND AUTOMATIC CONTROL

INTRODUCTION

In the preceding chapters the measurement of the principal physical conditions such as pressure, temperature and flow have been described. Consideration must now be given to methods of automatic control of such conditions. In view of the fundamental part feedback plays in control systems, this subject will be given detailed treatment first.

The normal symbols are used in the explanatory diagrams which follow. A circle represents a junction point or meeting place of two or more signals where summation or subtraction of the signals may take place. A rectangular block indicates a unit in a system which could be, for example, an amplifier. Connections are shown as single lines, but it should be realized that in practical arrangements these are, in effect, the normal two-line electrical circuit, a pipeline carrying pneumatic signals, a pipeline carrying hydraulic signals or even a mechanical linkage.

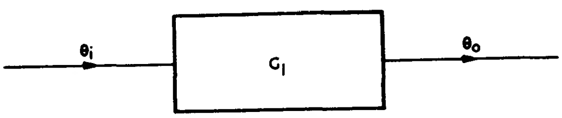


Fig. 192. *Simple control unit*

Generally speaking, feedback occurs when the output or part of the output from a system or unit is applied to the input of the system or unit to produce a modifying action. Let *Fig.* 192 represent a control unit, the exact nature of which it is unnecessary to specify at the moment. The output θ_o is related to the input θ_i by the following expression:

$$\theta_o = G_1 \theta_i \qquad \ldots\ldots\ldots\ldots (238)$$

$$\frac{\theta_o}{\theta_i} = G_1 \qquad \ldots\ldots\ldots\ldots (239)$$

To the ratio of the output θ_o to the input θ_i of a system or unit the term *transfer function* is applied.

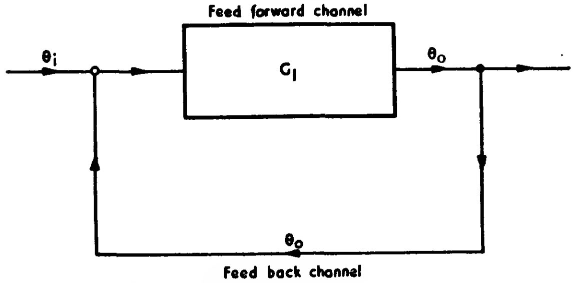


Fig. 193. *Simple feedback loop*

In a very simple fashion *Fig.* 192 illustrates *open loop* action. The meaning of the term will be clearer if comparison is made with *Fig.* 193 where the output of the unit of *Fig.* 192 is connected back or fed back to the input. The arrangement forms a *closed loop* and

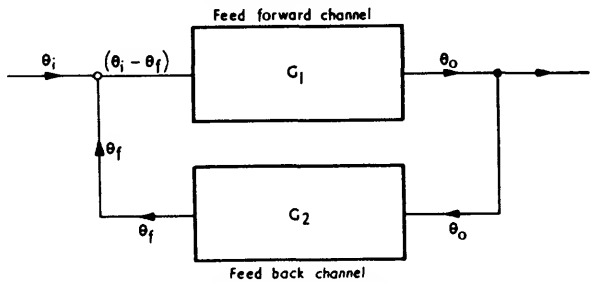


Fig. 194. *Simple feedback loop with feedback unit*

there is now a feed forward channel through the unit and a feedback channel from output to input. To carry the feedback conception one stage farther we include in the feedback channel a feedback unit with a transfer function G_2 (*Fig.* 194). If θ_f is the output of this unit

$$\frac{\theta_f}{\theta_o} = G_2 \qquad \ldots\ldots\ldots\ldots (240)$$

The feedback θ_f can assist or oppose the input θ_i and leads to a state of conditions where the actual input to the control unit is either $(\theta_i + \theta_f)$ or $(\theta_i - \theta_f)$. For the present we will consider the latter condition $(\theta_i - \theta_f)$ to which the term *negative feedback* is given. An alternative term sometimes used is degenerative feedback.

NEGATIVE FEEDBACK

Now $(\theta_i - \theta_f)$ is the actual input to the unit so that

$$\theta_o = G_1 (\theta_i - \theta_f) \qquad \ldots\ldots\ldots (241)$$

But since $\theta_f = G_2 \theta_o \qquad \ldots\ldots\ldots\ldots (242)$

$$\theta_o = G_1 (\theta_i - G_2\theta_o) \qquad \ldots\ldots\ldots (243)$$

Rearranging,

$$T = \frac{\theta_o}{\theta_i} = \frac{G_1}{1 + G_1G_2} \qquad \ldots\ldots\ldots (244)$$

The ratio θ_o/θ_i represents the transfer function T of the system of *Fig.* 194.

Examples

Some simple examples may assist at this stage. *Fig.* 195 represents an electronic d.c. amplifier with $G_1 = 500$. A potentiometer acts as a feedback device. Suppose the tapping ratio is 0·9, i.e., $G_2 = 0{\cdot}9$. Substituting these values in equation (244).

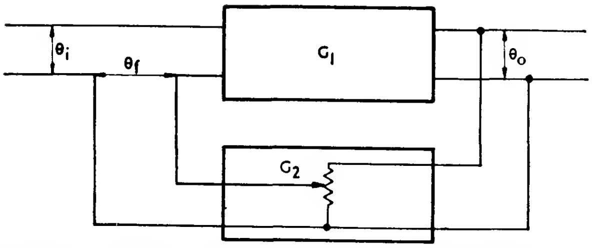


Fig. 195. *Simple feedback loop with potentiometer as feedback element*

$$\frac{\theta_o}{\theta_i} = \frac{500}{1 + 0{\cdot}9 \times 500} = 1{\cdot}1 \text{ approximately.}$$

Compare this with *Fig.* 192 where

$$\frac{\theta_o}{\theta_i} = G_1 = 500$$

Next consider the reduction of the tapping ratio to 0·5, i.e., $G_2 = 0{\cdot}5$,

$$\frac{\theta_o}{\theta_i} = \frac{500}{1 + 0{\cdot}5 \times 500} = 1{\cdot}94$$

In both examples, there is a striking diminution in the value of $\frac{\theta_o}{\theta_i}$ compared with the open loop unit.

The obvious question is what are the advantages of introducing feedback? To answer this question let us examine the effect of the variation parameter G_1.

Variation in G_1

Differentiating equation (244) with respect to G_1,

$$\frac{\delta\ \theta_o}{\delta\ G_1} = \left[\frac{1}{(1 + G_1G_2)^2}\right]\theta_i \quad \ldots\ldots \quad (245)$$

$$\text{But, } \frac{\theta_o}{\theta_i} = \left[\frac{G_1}{(1 + G_1G_2)}\right] \quad \ldots\ldots\ldots \quad (246)$$

$$\text{So that, } \frac{\delta\theta_o}{\delta G_1} = \frac{\theta_o}{\theta_i\, G_1}\left[\frac{1}{1 + G_1\, G_2}\right]\theta_i \quad \ldots\ldots \quad (247)$$

Rearranging,

$$\frac{\delta\ \theta_o}{\theta_o} = \frac{\delta\ G_1}{G_1}\left[\frac{1}{1 + G_1G_2}\right] \quad \ldots\ldots\ldots \quad (248)$$

Example

This equation can be better realized if values are assigned: Let G_1 be 500 as before and let it decrease to 400 making $\delta\ G_1 = 100$. G_2 is fixed at 0·9.

$$\text{Then } \frac{\delta\ \theta_o}{\theta_o} = \frac{100}{500}\left[\frac{1}{1 + 500 \times 0{\cdot}9}\right]$$

$$\frac{\delta\ \theta_o}{\theta_o} = \frac{1}{2280}$$

Thus for a 20% change in G_1, the change in output is of the order of 0·04% only. This effect can be seen in another manner from equation (244).

If G_1 is very large, the equation reduces with very little error to

$$\frac{\theta_o}{\theta_i} = \frac{1}{G_2} \quad \ldots\ldots\ldots\ldots \quad (249)$$

If the conditions of equation (249) are realized, $\frac{\theta_o}{\theta_i}$ is substantially independent of changes in G_1. This is particularly useful in an electronic amplifier where variations in the characteristics of the components can affect the amplifier gain and where stability is a vital requirement. Another change we must examine is that in G_2.

Variation in G_2

Differentiating equation (244) with respect to G_2,

$$\frac{\delta\ \theta_o}{\delta\ G_2} = -\frac{G_1{}^2\ \theta_i}{(1 + G_1\, G_2)^2} \quad \ldots\ldots\ldots \quad (250)$$

$$\text{But since } \theta_o = \left[\frac{G_1}{1 + G_1\, G_2}\right]\theta_i \quad \ldots\ldots \quad (251)$$

$$\frac{\delta\ \theta_o}{\theta_o} = -\frac{G_1\, G_2}{1 + G_1\, G_2} \cdot \frac{\delta\ G_2}{G_2} \quad \ldots\ldots \quad (252)$$

If the condition that G_1 is large still obtains, and the product $G_1\, G_2$ is extremely larger than 1, equation (252) reduces to

$$\frac{\delta\ \theta_o}{\theta_o} = -\frac{\delta\ G_2}{G_2} \quad \ldots\ldots\ldots\ldots \quad (253)$$

Ignoring the negative sign, this means that a change in feedback gain G_2 is accompanied by a proportional change in output θ_o.

Example

If G_2 varies from 0·9 to 0·5 making δG_2 0·4 and G_1 is, in this example, 1000

$$\frac{\delta\ \theta}{\theta_o} = \frac{0{\cdot}9 \times 1000}{1 + 0{\cdot}9 \times 1000} \cdot \frac{0{\cdot}4}{0{\cdot}9}$$

$$\frac{\delta\ \theta_o}{\theta} = \frac{900}{900{\cdot}9} \cdot \frac{0{\cdot}4}{0{\cdot}9}$$

This is practically

$$\frac{\delta\ \theta_o}{\theta_o} = \frac{0{\cdot}4}{0{\cdot}9}$$

A percentage change in G_2, therefore, is accompanied by a proportional percentage change in the output. The significance of this relation will appear later.

Series and Parallel Arrangements of Units

In a feedback system, the component units may exist in the form of series and parallel arrangements. In *Fig.* 196, three units with respective gains of G_1, G_2 and G_3 are arranged in series. From *Fig.* 196

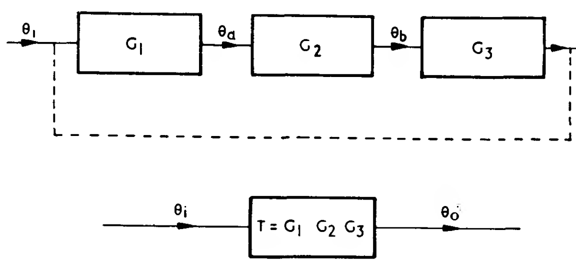
$$\theta_a = G_1\ \theta_i \quad \ldots\ldots\ldots\ldots \quad (254)$$

$$\theta_b = G_2\ \theta_a = G_1\, G_2\ \theta_i \quad \ldots\ldots\ldots \quad (255)$$

$$\theta_o = G_3\ \theta_b = G_1\, G_2\, G_3\ \theta_i \quad \ldots\ldots \quad (256)$$

The system is then equivalent to the single unit shown in the lower part of *Fig.* 195 with an overall transfer

Fig. 196. *Control units in series*



function T equal to the product of the three separate functions:

$$\frac{\theta_o}{\theta_i} = T = G_1 G_2 G_3 \qquad \dots\dots\dots (257)$$

If the feedback loop is completed as shown by the dotted line in *Fig.* 196 the transfer function for the system becomes:

$$\frac{\theta_o}{\theta_i} = T = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3} \qquad \dots\dots (258)$$

In the parallel scheme of *Fig.* 197, one must consider that the input θ_i is applied to each of the three units. The outputs are then $G_1\,\theta_i$, $G_2\,\theta_i$ and $G_3\,\theta_i$. These are added at the output junction and form the output θ_o. The following relation then holds:

$$\theta_o = G_1\,\theta_i + G_2\,\theta_i + G_3\,\theta_i = \theta_i\,(G_1 + G_2 + G_3) \quad . . (259)$$

The overall transfer function is given by:

$$\frac{\theta_o}{\theta_i} = T = G_1 + G_2 + G_3 \qquad \dots\dots (260)$$

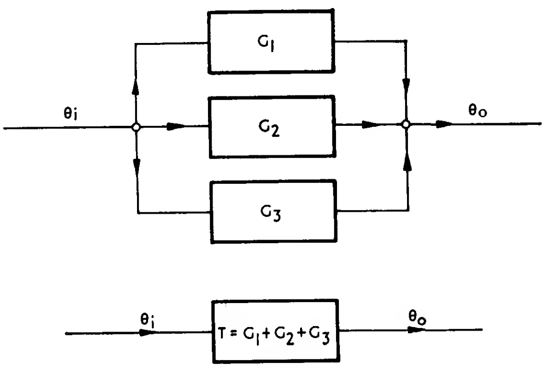


Fig. 197. *Control units in parallel*

Finally let us consider a series-parallel arrangement as in *Fig.* 198. In the upper path the transfer function is $G_1\,G_2$, and in the lower path the transfer function is $G_3\,G_4$. The addition of the two functions gives the overall value of:

$$T = G_1 G_2 + G_3 G_4 \qquad \dots\dots\dots\dots (261)$$

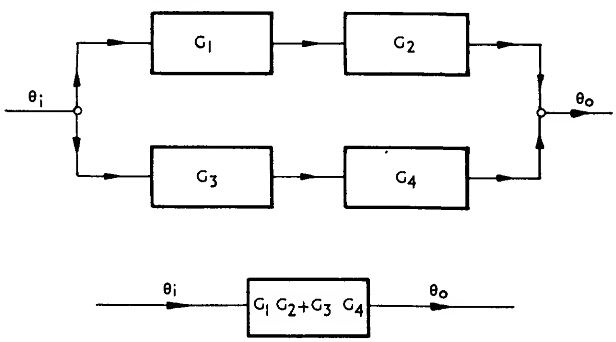
It is convenient at this point to consider some simple negative feedback loops used in automatic control systems to illustrate the foregoing theoretical deductions.

PRACTICAL EXAMPLES OF SIMPLE NEGATIVE FEEDBACK LOOPS

Nozzle and Flapper Unit

This unit is utilized in the majority of pneumatic

Fig. 198. *Control units in series-parallel arrangement*



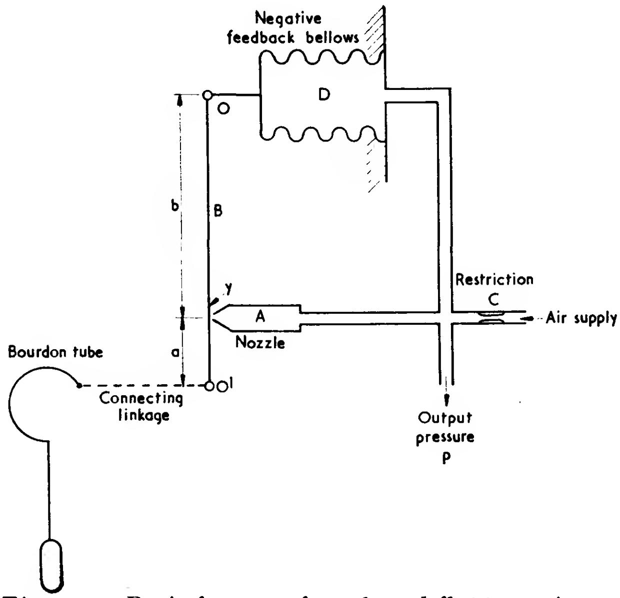


Fig. 199. *Basic features of nozzle and flapper units*

controlling instruments. *Fig.* 199 shows the basic features of the device.

It comprises a fine bore nozzle A, the diameter of which normally ranges between 0·02in. and 0·04in. A flapper or baffle B is pivoted so that it can move towards and away from the mouth of the nozzle. The distance of travel is minute and can be as small as 0·0003in. In any case it never exceeds a few thousandths of an inch.

Air is supplied at a specific pressure to the nozzle via a restriction C which is $\frac{1}{4}$ to $\frac{1}{2}$ the nozzle diameter. The air flows out through the small gap between the flapper and the mouth of the nozzle. A pressure is set up inside

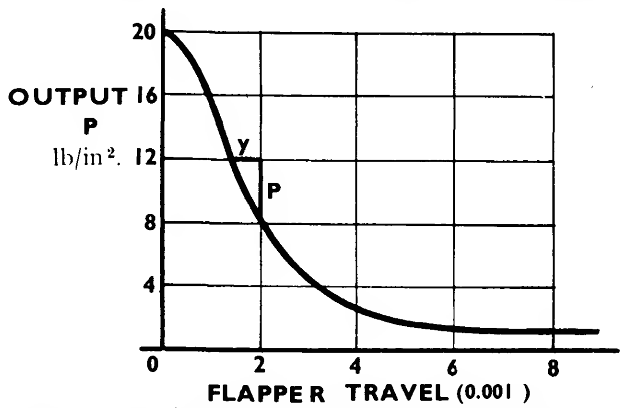


Fig. 200. *Relation between flapper movement and pressure output in a nozzle and flapper unit. The total flapper movement is* 0·001 *in.*

the nozzle, the value of which depends on the position of the flapper in its travel. When the flapper is closest to the nozzle the pressure value is a maximum, and when the flapper is farthest away the pressure is at a minimum. A typical relation is shown in *Fig.* 200. It is not linear, but an approximation to a straight line can be obtained if the operating pressure range is restricted to 3 to 15lb/in².

The flapper is normally coupled by suitable linkage to a measuring element, and purely for the purposes of explanation this is shown in *Fig.* 199 as the Bourdon tube of a temperature measuring system.

Consider a temperature change which deflects the Bourdon tube and positions the flapper nearer to the nozzle, using O as the pivot. The pressure increases and the increase is communicated to the feedback bellows D. D extends and tends to move the flapper away from the nozzle, this time with O' as pivot. This action tends to reduce the pressure in the nozzle and a position of equilibrium is eventually reached where the nozzle pressure and hence the output pressure p is a measure of the new temperature value.

If y is the net flapper movement at the nozzle,

$$y = \left(\frac{b}{a+b}\right)\theta - \left(\frac{a}{a+b}\right)Sp \qquad \ldots (262)$$

Where θ = the displacement due to measuring element, inches

S = the "rate" of the bellows unit in in/lb/in^2

p = the output pressure lb/in^2.

The second term in equation (262) is due to the negative feedback of the bellows.

What has negative feedback contributed to this unit?

It must be remembered that the total travel of the flapper is minute. Without the feedback bellows, backlash and similar phenomena could cause errors in positioning and it is essential that the flapper be accurately positioned. Negative feedback in the form of the bellows enables this to be achieved.

Observe, too, the effect of the supply pressure variations. Suppose this drops in value, the pressure in the nozzle falls likewise. But this pressure is also that existing in the bellows. A decrease in pressure here means that the bellows contracts. But, in contracting, the bellows moves the flapper nearer to the nozzle, increasing the pressure therein. Thus, one effect tends to counteract the other, minimizing the influence of a change in the supply pressure.

The role of the restriction C requires some explanation. Without C present and with the flapper in its extreme position of travel away from the nozzle, the pressure within the nozzle is very little different from that existing when the flapper is in its closest position. The fitting of a restriction between supply and nozzle creates a pressure drop. The diameter of the restriction is always less than that of the nozzle and the ratio of restriction diameter to nozzle diameter is normally between $\frac{1}{4}$ and $\frac{1}{2}$. At the extreme of the flapper travel from the nozzle, the restriction has a relatively large pressure drop leaving a small pressure for the nozzle. Some figures may help in the explanation. Suppose that the supply pressure is 20lb/in^2. The pressure drop across the restriction may reach 17lb/in^2 with the flapper farthest away. This leaves 3lb/in^2 for the nozzle. When the flapper is closest to the nozzle, the pressure drop may fall to 5lb/in^2 leaving 15lb/in^2 for the nozzle. At intermediate positions, the pressure drop across the restriction and the nozzle pressure vary in proportion to the flapper travel.

The minute value of the nozzle diameter leads to a small consumption of air and also reduces the back pressure on the flapper to a minimum.

A block diagram of the unit is shown in *Fig.* 201. The nozzle and flapper unit as shown is a proportional controller and its operation as such is described in Chapter 14.

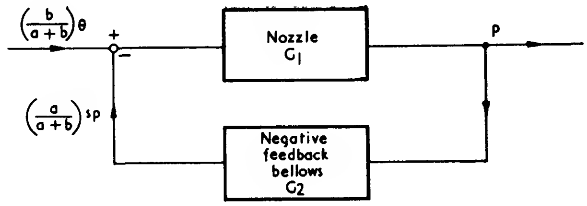


Fig. 201. *Block diagram nozzle and flapper unit*

Hydro-mechanical System

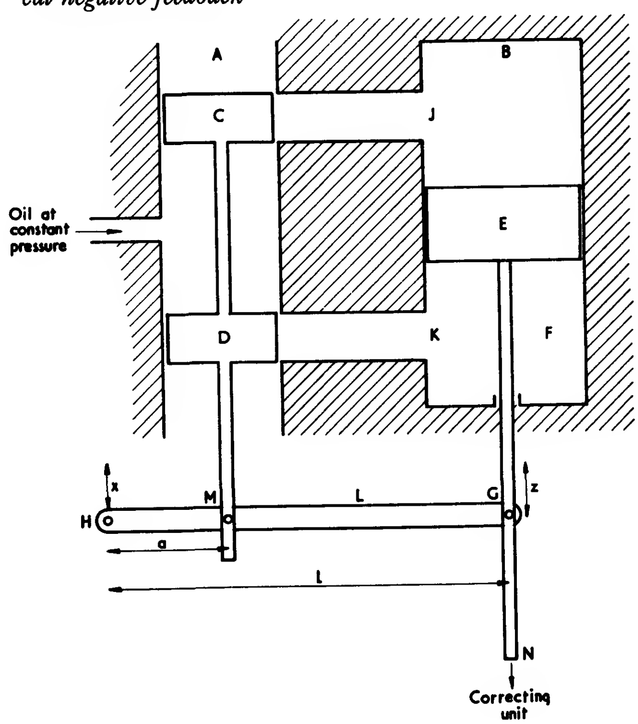
A simple hydraulic system with mechanical feedback is illustrated in *Fig.* 202. A is a pilot valve whose stem has two piston heads C and D. Oil at constant pressure is supplied via inlet K. The pilot valve communicates with a hydraulic actuator B by two inlet passages J and K. The actuator has piston E, the rod F of which is pivoted to lever L at G. The pilot valve piston stem is coupled to the lever at M.

To the end H of the lever a movement is imparted by a measuring element. Consider a movement in the downward direction using G as a pivot. The stem of the pilot valve is carried down at the same time and the lower piston head D begins to uncover inlet K. Oil at constant pressure then enters the valve and flows through K to the underside of piston E. E commences to rise and starts to carry the stem of A upwards with it. Thus inlet K tends to be covered again by D and operation continues until equilibrium is reached. If a correcting unit is coupled to the end N of the rod F correcting action may be initiated.

If x is a displacement given to end H of lever L, and z is the motion of G, the deflection of the pilot valve piston is given by:

$$y = \frac{l-a}{l} \cdot x - \frac{a.z}{l} \qquad \ldots\ldots\ldots (263)$$

Fig. 202. *Simple hydraulic control system with mechanical negative feedback*



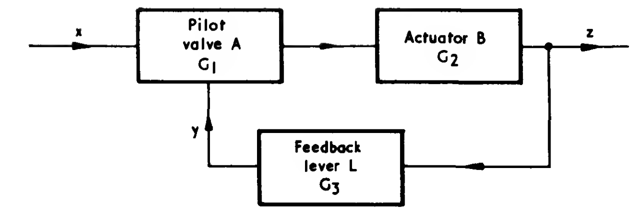


Fig. 203. *Block diagram of hydraulic system shown in Fig.* 202

The second term in the expression represents negative feedback and bears a similarity to the feedback term in equation (262) for the nozzle and flapper unit.

The use of this arrangement as an integral control system will be described in Chapter 14. A block diagram is shown in *Fig.* 203.

Electronic System

Fig. 191 of Chapter 12 is an example of variable negative feedback being used for null balancing purposes. A block diagram of the system is shown in *Fig.* 204.

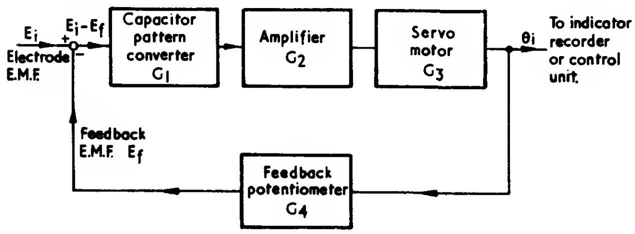


Fig. 204. *Block diagram of pH measuring system with negative feedback*

POSITIVE FEEDBACK

At the beginning of the chapter it was stated that the feedback signal θ_f can oppose or assist the input signal θ_i. We have discussed negative feedback where the actual input to a unit is $(\theta_i - \theta_f)$. The case of positive or regenerative feedback must now be considered. Here,

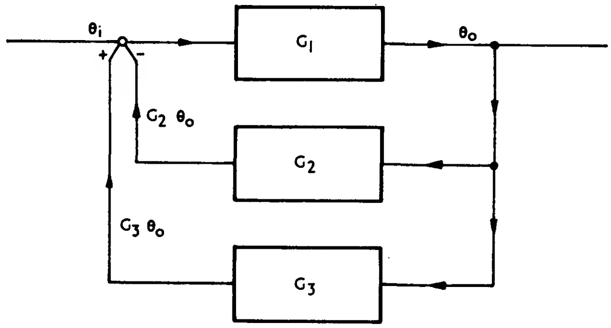
$$\theta_o = G_1 (\theta_i + \theta_f) \quad \ldots\ldots\ldots \quad (264)$$

A similar equation to (244) may be deduced:

$$T = \frac{\theta_o}{\theta_i} = \frac{G_1}{1 - G_1 G_2} \quad \ldots\ldots\ldots \quad (265)$$

Positive feedback is incorporated in control systems, possibly more in association with negative feedback than by itself. *Fig.* 205 shows in an elementary fashion the employment of negative and positive feedbacks in a

Fig. 205. *Block diagram of system with negative and positive feedback*



control unit. The transfer function is then:

$$\frac{\theta_o}{\theta_i} = \frac{G_1}{1 + G_1 G_2 - G_1 G_3} \quad \ldots\ldots \quad (266)$$

An example may show the influence of positive feedback in such an arrangement.

Example

Let $G_1 = 10$

$G_2 = 0{\cdot}5$

$G_3 = 0{\cdot}56$

$$\frac{\theta_o}{\theta_i} = \frac{10}{1 + 10 \times 0{\cdot}5 - 10 \times 0{\cdot}56}$$

$$\frac{\theta_o}{\theta_i} = 25$$

With negative feedback alone the transfer function would be

$$\frac{\theta_o}{\theta_i} = \frac{G_1}{1 + G_1 G_2}$$

$$\frac{\theta_o}{\theta_i} = \frac{10}{1 + 10 \times 0{\cdot}50}$$

$$\frac{\theta_o}{\theta_i} = 1{\cdot}66$$

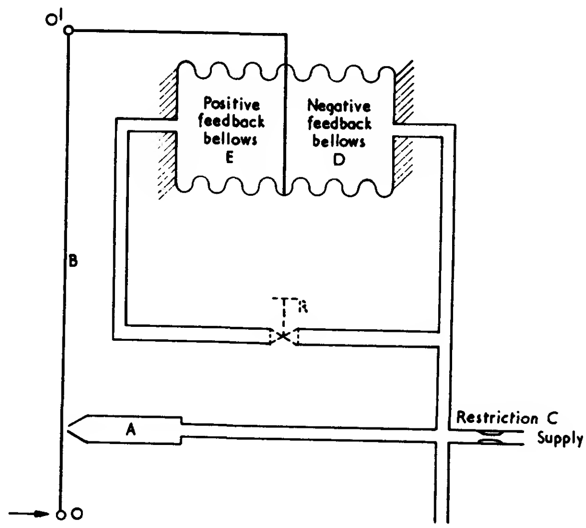


Fig. 206. *Positive feedback in a simple control unit*

Practical positive feedback in a pneumatic control unit is illustrated in *Fig.* 206. An opposing *E* bellows is added to the unit of *Fig.* 199 and is connected to the nozzle chamber via restriction *R*. *E* tends to adjust the flapper in the opposite direction to the negative feedback bellows *D*. The arrangement is actually a simple version of a proportional plus integral controller. Its operation will be described later. It is introduced here for explanatory purposes only.

One final point in connection with positive feedback should be noted. In equation (265) if G_1G_2 becomes equal to 1 the transfer function $\frac{\theta_o}{\theta_i}$ becomes infinite, i.e., we obtain an infinite output for a finite input leading to a state of instability.

THE AUTOMATIC PROCESS CONTROL CLOSED LOOP SYSTEM

Introduction

In all the preceding material, the closed loop has been considered in a general fashion and the examples of negative feedback have been related to local closed loops in instruments rather than to a process. The automatic control closed loop system applied to industrial processes must now be examined. *Fig.* 207 shows a block diagram of such a closed loop control.

British Standard Specification No. 1523, Section 2, contains a list of definitions of automatic control terms. These have been carefully drawn up by the relevant committee, and represent a standard to which it is desirable to adhere. All terminology in this chapter is based on this specification unless otherwise stated.

An automatic control system is one in which, without human intervention, the value of a controlled condition such as temperature, flow or pressure is compared with a set value and corrective action taken dependent on the difference or deviation between the two values. The set value, it should be explained, is the value to which the automatic control mechanism is set. It is, in fact, the value to which we adjust the control index on the scale of the control instrument.

With the aid of *Fig.* 207 let us amplify the foregoing definitions by examining the function of each member of the closed loop.

The detecting element responds directly to the value of the physical condition and is, therefore, the first element in the loop to detect a change in that value. The element is inserted at a convenient point in the process and passes its signal to the measuring element. This element, as its name implies, measures the value of the controlled condition. The following table gives examples of some of the more common controlled conditions and the relevant detecting and measuring elements.

Controlled Condition	Detecting Element	Measuring Element
Temperature	Liquid and gas expansion thermometers, vapour pressure thermometer.	Bourdon tube, bellows unit
	Resistance bulb.	Wheatstone bridge.
	Thermocouple.	Millivoltmeter or potentiometer.
Flow	Orifice plate, Venturi tube, nozzle, weir	Manometer, U-tube with float and lever mechanism, bellows unit.
Humidity	Wet and dry bulb expansion type thermometers.	Bourdon tubes.
Pressure	None.	Bourdon tubes, bellows unit
Liquid Level	Float, standpipe.	Bellows unit U-tube with float and lever mechanism
pH	Glass electrode. Calomel electrode.	Amplifier and electrical circuits.
Conductivity	Conductivity cell.	Wheatstone bridge or other electrical circuit.

From the measuring element a signal is transmitted to the comparing element in which the set value unit is contained. The measured value is compared with the set value here and, if the two are different, a deviation signal is generated. Comparing elements differ considerably in design as one would expect when pneumatic, electrical, electronic and hydraulic systems are used in practice. They will be dealt with when the modes of control and controlling elements are described. One may perhaps instance a single simple example in passing. A d.c. millivolt signal from a thermocouple could be compared with a known value of d.c. millivolts, corresponding to the set value, on a potentiometer. Any difference between the two would result in a d.c. signal being passed to the controlling element.

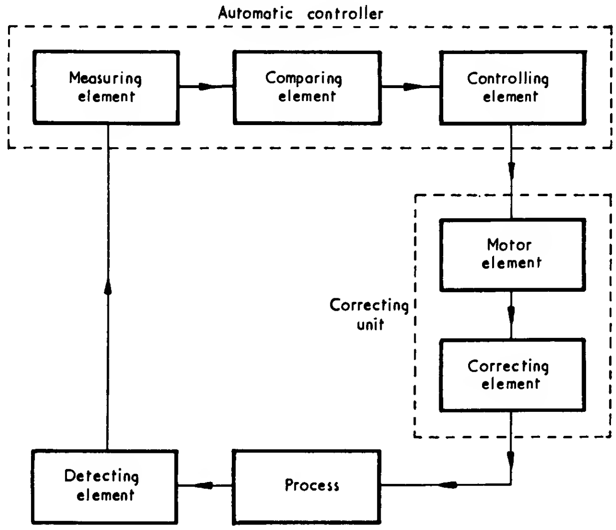


Fig. 207. *Automatic process control loop*

The controlling element is responsible for transmitting a signal, dependent on the deviation, to the correcting unit. In so doing it may produce a signal which is related to one or more of the modes of control. One controlling element has already been mentioned—the nozzle and flapper unit under "Practical Examples of Simple Negative Feedback Loops". Both the control modes and the corresponding controlling elements receive treatment in Chapter 14.

The controlling signal is received by the correcting unit. It is shown in *Fig.* 207 as existing in two parts. This is purely for convenience in examining the operation of a control loop as the two parts, the motor element and the correcting element, are normally in one assembly. Correcting units are described in detail in Chapter 14, but it will assist in understanding the control loop if one of the most popular, the control valve, is given a little attention at this point. A diagram of the essential parts of a control valve are indicated in *Fig.* 208. It should be noted that the term "motor" is applied to pneumatic and hydraulic actuating devices in the automatic control field and is not, therefore, restricted to electrical designs.

In the valve shown in *Fig.* 208, the motor element consists of a diaphragm assembly *A* with a flexible diaphragm of rubber, fabric or other suitable material. Is is supported at the central portion by a metal plate which serves as a fixing for stem *D* and a means for holding the upper end of spring *C*. The correcting element proper is seen at the lower part of the unit. In essence it comprises a variable orifice whose area is determined by the contours of plugs *F* operating in

openings G. The plugs are part of the stem member. Fluid flows through the orifices so formed from left to right. A pneumatic correcting signal from the correcting element is applied to the upper side of diaphragm A via inlet B. The extent and mode of travel of the valve stem and plugs are governed by this signal. It is the fluid flowing through this valve which regulates the operation of the process and, hence, influences the value of the controlled condition.

To complete this initial section on the control loop, an elementary automatic control scheme is illustrated in *Fig.* 209. This system is for liquid level control. The detecting element is the standpipe which detects the liquid level as a varying head of pressure (see Chapter 3 for details of this method). The pressure is measured in the control instrument in the upper part of the figure by a metal capsule stack, a metal bellows or similar device forming the measuring element. The signal from the stack or bellows is applied to the comparing element, the deviation signal is generated and applied to the controlling element which supplies the pneumatic correcting signal to the diaphragm of the control valve. This operates to allow more or less liquid to pass through its orifices to maintain a constant level in the tank. The tank of liquid is, in this example, a process, if a very elementary one.

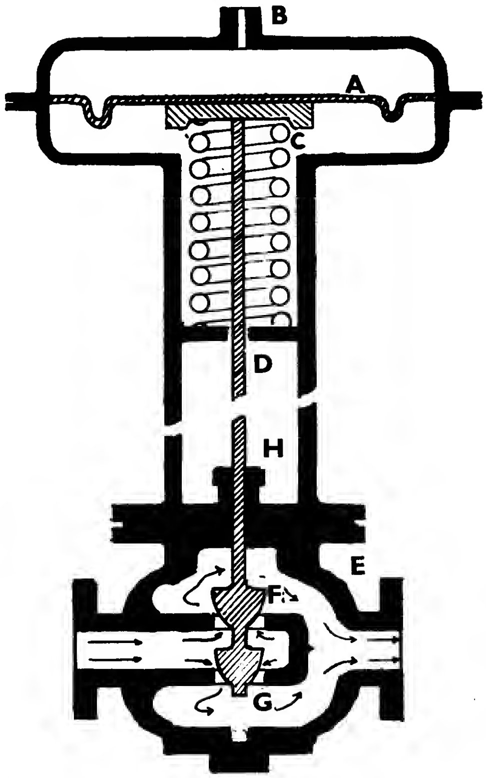


Fig. 208. *The essential features of a control valve*

Modes or Actions in Automatic Control

We have already mentioned that not only is the control signal dependent on the deviation but can be modified by the modes of control impressed on it by the controlling element. We must now consider the most important of these modes or actions.

Two-step Controller Action

When the measured value of the controlled condition

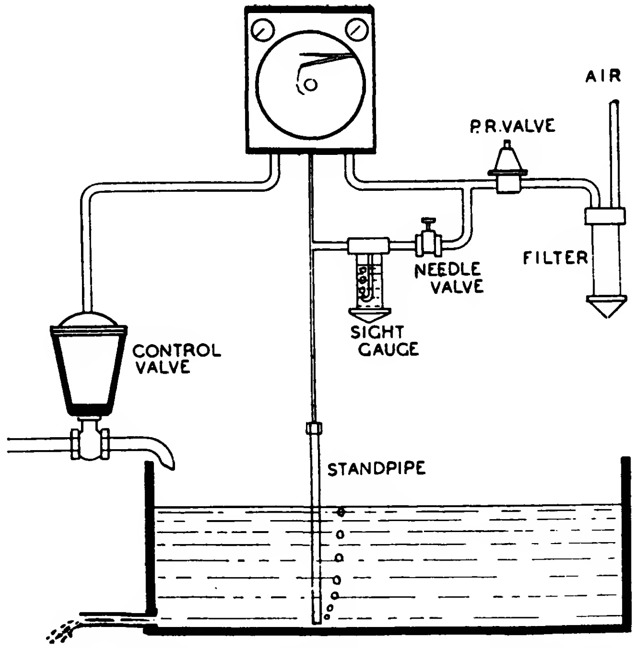
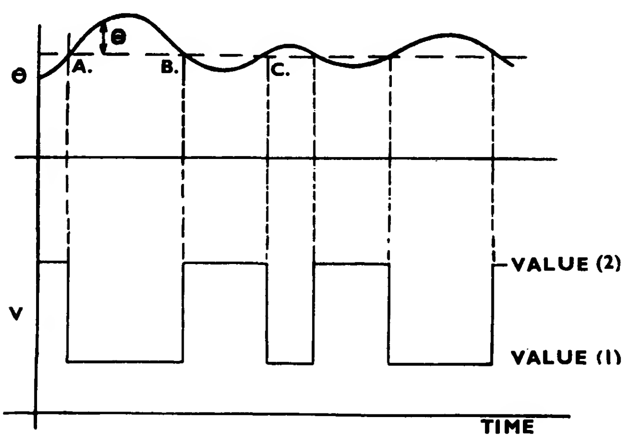


Fig. 209. *Simple automatic control scheme—liquid level control*

is above the set value the deviation is given a positive sign, and when the measured value is below the set value the deviation is given a negative sign. In two-step controller action the output signal from the controller changes sign from one predetermined value to another when the deviation changes sign. The action is shown in *Fig.* 210. To comment on this action it will be necessary to consider the operation of the correcting unit which for convenience will be taken to be a control valve of the general type shown in *Fig.* 208. We must assume that the control valve follows the control signal implicitly and that it moves from one extreme position to another in so doing. These are normally the fully open or the fully closed positions. The valve position curve would then be the replica of the control signal V in *Fig.* 210, with the valve fully open to correspond to value 2 and fully closed to correspond to value 1. It will be noticed that the deviation oscillates round the set value and we must now see the reason for this.

Fig. 210. *Two-step controller action*



Lags are present in any control system, resulting in a finite interval occurring between the application of corrective action and the equilibrium value of the controlled condition. Imagine that the measured value in *Fig.* 210 is below the set value initially. The correcting unit is fully opened, and the measured value increases until it reaches the set value at A. Because of lags, however, the system has not assimilated the full effect of corrective action and, in spite of the correcting unit now going to the fully closed position, the measured value still increases. It continues to do so until the full effect of the correction is felt. The measured value now begins to decrease, reaching the set value at B. The correcting unit changes to the fully opened position, but the lag causes the value to fall until the correction is fully realised. An increase then follows and the measured value attains the set value at C. The cycle is repeated *ad lib.*, and a sustained oscillation of the measured value is obtained.

The practical difficulties in arranging a sharp change-over have given rise to a two-step control with overlap. Here, there is a difference in the value at which the corrective action takes place in one direction and the other. An "overlap" exists, resulting in the frequency of oscillation being reduced, and, in consequence, the number of times the correcting unit is operated. On the other hand, the amplitude is increased, and a satisfactory compromise between the two depends on the application.

Two-step control has disadvantages in fluctuating between the extreme values, and multi-step control was introduced to improve the performance. In this action, the correcting unit can stop at predetermined intermediate positions.

Proportional Controller Action

In proportional controller action the controller output signal is proportional to the deviation:

$$V = -K_1 \theta \qquad \dots\dots\dots\dots \quad (267)$$

where V = the controller output signal
θ = the deviation
K_1 = the proportional action constant.

We will assume that the correcting unit is a control valve as in the case of two-step controller action, and that it follows closely the controller signal.

It is instructive to examine the effect of a sustained load or demand change on a system with proportional control. Refer again to *Fig.* 209, and imagine that the tank of liquid is being used to supply a number of outlets. The initial conditions are such that for the specific demand the control valve passes sufficient liquid to maintain a constant head in the tank, i.e., the rate of inflow just balances the rate of outflow. Consider that the demand now increases and is sustained. The level falls and a deviation occurs. The control signal sent to the valve motor causes the valve to open wider to allow an increased rate of liquid to flow into the tank. The level rises and in so doing reduces the deviation. But the valve now tends to close, obeying the proportional action. The sustained demand, on the other hand, requires that it remain in the increased open position. The conflicting requirements result in the controlled condition, in this example liquid level, assuming a final equilibrium value, but remaining offset from the set value.

A second example is provided by a furnace whose temperature controller has a set point adjusted for a given load of steel. Fuel is being supplied to the furnace via a valve operated from the controller. Additional material is now added to the melt causing the furnace temperature to fall. A deviation is produced causing the valve to open to admit more fuel for a greater heat input. The temperature tends to rise, the deviation to become less, and the valve closes, cutting down the supply of fuel. But the new furnace conditions demand an increased supply of fuel and the next tendency would be for the valve to open again. A position is ultimately reached at which the temperature is offset from the previous value and, of course, from the set value.

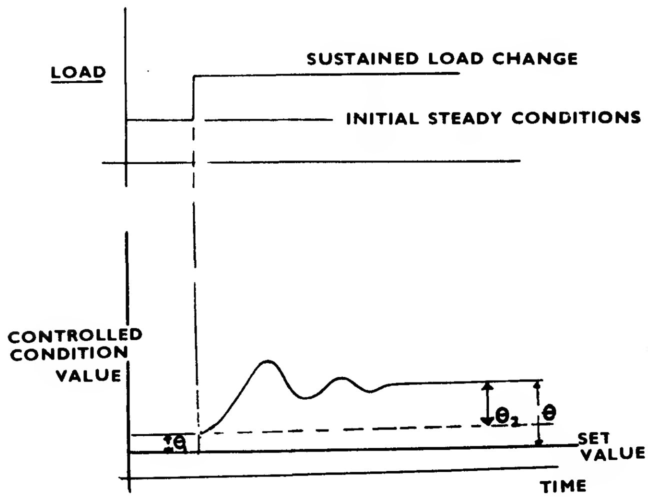


Fig. 211. *Effect of sustained load change with proportional controller action*

Fig. 211 shows the effect of a sustained load or demand change and the corresponding change in the measured value of the controlled condition. Observe the oscillatory effect before the value settles down to the final steady offset θ, and observe that there is an initial offset θ_1 which must be present since offset is inherent in proportional action or control.

Whilst in some processes a moderate offset can be tolerated in others it is a disadvantage and the amount may be serious in some applications. It is desirable to reduce this to reasonable proportions without, at the moment, bringing in another type of control. Now, if we could make an adjustment in the controlling unit so that we could vary a range of values for which the correcting unit would be fully open or fully closed, we should have a means of adjusting the corrective effect for a given deviation. For example, a small deviation might cause a comparatively large fuel supply change in the previous example, and reduce the offset value.

This is equivalent in equation (267) to varying K_1, which we have hitherto regarded as a constant. What we are desirous of doing is, in effect, to change the sensitivity of the controlling element. For example, for the same signal, V if we halved K_1, θ would have to be doubled for the equation to hold. Looking at this problem from another aspect, however, for the full operating range of the controller output signal, by adjusting K_1, we can obtain a corresponding range of deviation values. To this range the term Proportional Band is applied, and it can be expressed as a percentage of the range of values of the controlled condition which the measuring element of the controller is designed to measure (i.e., the instrument range). The relation can

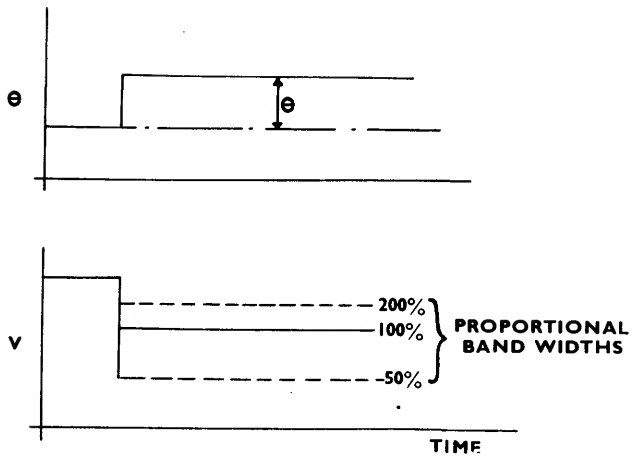


Fig. 212. *Effect of change in proportional band on controller output signal for the same deviation*

be seen in *Fig.* 212 in which the deviation θ has a sustained value. The corresponding controller output signal is indicated for 50%, 100% and 200% bandwidths.

In effect K_1, the proportional action factor, is proportional to the reciprocal of the bandwidth B_1 or 100/percentage proportional band B_2.

$$K_1 \propto \frac{1}{B_1} \qquad \ldots\ldots\ldots\ldots\ldots \quad (268)$$

$$K_1 \propto \frac{100}{B_2} \qquad \ldots\ldots\ldots\ldots\ldots \quad (269)$$

From *Fig.* 212 it will be observed that the proportional band can be greater than the range on the instrument scale. Some examples may assist further in understanding the proportional band.

If the instrument scale range is 0-1000°C and the temperature must change from 100°C to 150°C to produce the full operating range of the controller output signal, the proportional band is

$$\frac{(150 - 100) \times 100}{1000} = \frac{50 \times 100}{1000} = 5\%$$

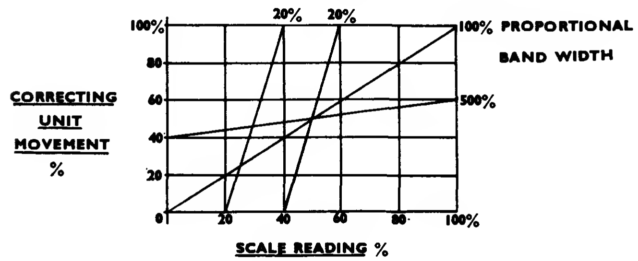
In a similar manner if it takes five times the range of the instrument scale to produce the full range of the controller output signal, the proportional band is

$$\frac{5000 \times 100}{1000} = 500\%$$

The proportional band is made adjustable to provide stable control under different process conditions.

If the correcting unit is assumed to follow the controller signal V in a linear manner the valve position S

Fig. 213. *Relations between instrument scale range, correcting unit movement and proportional band*



can be expressed as

$$S = K_x V = K_y \theta \qquad \ldots\ldots\ldots \quad (270)$$

where K_x and K_y are constants.

A series of straight line relations can then be drawn up connecting percentage instrument scale, percentage correcting unit movement and proportional bandwidth. These are shown in *Fig.* 213.

Thus, it seems possible by adjusting the proportional band to a small percentage to obtain a correspondingly small offset. But if this is carried to the extreme, hunting or oscillation will occur.

Summarizing, offset is inherent in proportional control. If different positions of the correcting element are necessary for the same value of the controlled condition due to sustained demands, offset will be produced. The value will depend on the extent of the load change in response to a demand and the proportional bandwidth. The allowable proportional bandwidth must depend entirely on the process. In some applications, the use of a very narrow band is permissible and the offset of no significance. One can approach a situation where the narrowness of the proportional band leads to a control mode which is equivalent to two-step.

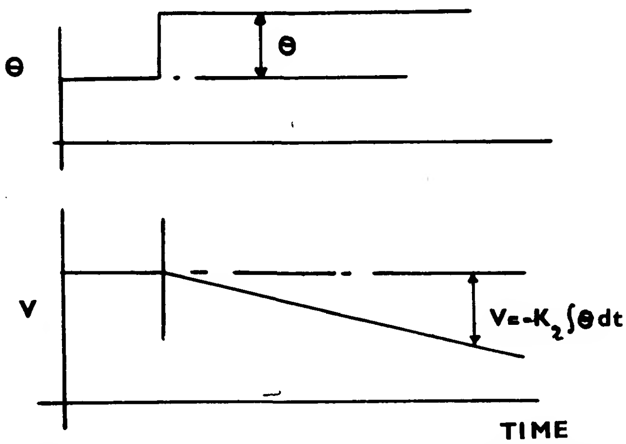


Fig. 214. *Integral controller action. This is shown without proportional controller action*

Integral Controller Action

Offset, as we have seen, is inherent in proportional control, and another type must be introduced to give a fully compensatory effect. Integral controller action can achieve this. It is defined as an action in which the controller output signal changes at a *rate* proportional to the deviation.

$$\frac{dV}{dt} = -K_2\,\theta \qquad \ldots\ldots\ldots\ldots \quad (271)$$

or

$$V = -\,K_2\int \theta.dt \qquad \ldots\ldots\ldots\ldots \quad (272)$$

K_2 is the Integral Action Factor.

Equation (272) gives the action its name.

If we adopt the same assumptions, as in proportional control, of a linear system, then the control valve functions as follows:

$$\frac{dS}{dt} \propto K_2\,\theta \qquad \ldots\ldots\ldots\ldots \quad (273)$$

The valve opens or closes at a speed proportional to the deviation, and will continue to do so as long as there is a deviation. Thus there will be no offset, but the

control by itself may take some time to restore the controlled condition to its set value. *Fig.* 214 shows the effect of integral action alone. This is amplified in Chapter 14.

There are one or two variations of integral control in the manner in which the speed of the correcting unit relative to the deviation is arranged. These are briefly described.

Single-speed Floating Controller Action and Control

The term "Floating" was one of the old descriptions of integral control. In this version, the valve moves at a constant rate *irrespective of the deviation*, i.e.,

$$\frac{dS}{dt} = k \qquad \ldots\ldots\ldots\ldots\ldots \quad (274)$$

where k is a coefficient.

The electrical motorized valve can be applied in this control.

Multi-speed Floating Controller Action and Control

In this mode, the control valve moves at a number of different speeds:

$$\frac{dS}{dt} = k_1, \frac{dS}{dt} = k_2, \frac{dS}{dt} = k_3 \qquad \ldots\ldots \quad (275)$$

where k_1, k_2, and k_3 are coefficients.

Hunting can occur in an integral control, the conditions depending on the particular characteristics of the control loop.

Derivative Controller Action

It might be thought that a combination of proportional and integral controls gives all that is necessary for normal processes. This is true to a large extent, but plants with large transfer lag or distance velocity lag (see Chapters 14 and 15) may be difficult to control. To avoid continuous hunting, the proportional band may have to be made excessively wide and the integral time large. When load changes occur, big deviations take place and recovery time is long. To counteract this, derivative control is used. It is defined as a type of controller action in which the controller output signal is proportional to the rate of change of the deviation.

$$V = -K_3\frac{d\theta}{dt} \qquad \ldots\ldots\ldots\ldots \quad (276)$$

where K_3 = the Derivative Action Factor.

Again assuming linearity, the valve opening S is given by

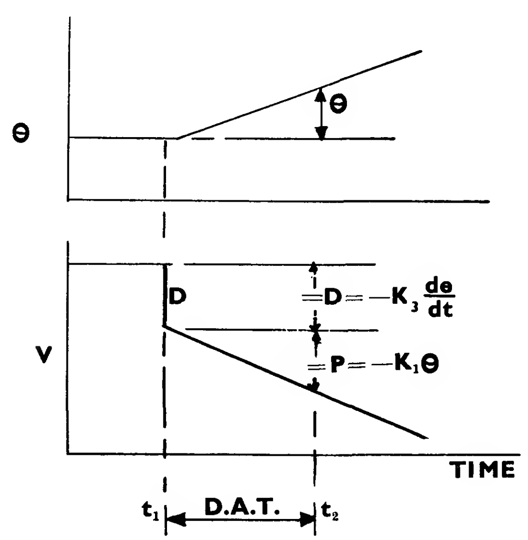
$$S \propto K_3\frac{d\theta}{dt} \qquad \ldots\ldots\ldots\ldots \quad (277)$$

Whereas proportional and, to a lesser extent, integral control can be used alone, derivative control must be used in association with one or both of the others. Alone, it has no real application since it does not depend on θ itself, but on $\frac{d\theta}{dt}$.

If θ is constant, $\frac{d\theta}{dt} = 0$.

Thus no valve movement can be initiated however large θ is, provided it is held at a constant value. *Fig.* 215 illustrates derivative plus proportional controller action. This is amplified in Chapter 14.

Fig. 215. *Derivative controller action. D.A.T. is deviative action time. D is the immediate change in signal due to derivative action. P is the change in signal value due to proportional action*



British Standard Specifications

B.S. 1523 Glossary of Terms used in Automatic Controlling and Regulating Systems. Section 2: Process Control.

In addition to Section 2, other sections of B.S. 1523 have an interest for general control purposes:

Section 3: Kinetic Control.

Section 5: Components of Servo-mechanisms.

Books for Further Reading

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Chapter 14

AUTOMATIC CONTROL SYSTEMS: CONTROLLER UNITS AND CORRECTING UNITS

IN chapter 13, the four main forms of controller actions are traced. The units producing the actual controller signals must now be considered.

PNEUMATIC CONTROLLER UNITS

Proportional Control Unit, Position Balance Type

Fig. 199 in chapter 13 illustrates how a metallic bellows, or similar device, is utilized to produce negative feedback with a nozzle and flapper. In the associated descriptive matter it is indicated that this arrangement can produce proportional controller action. The nozzle and flapper-bellows combination will now be examined in more detail as a proportional controller unit. It is shown again in *Fig.* 216. This is substantially the same unit as *Fig.* 199 of chapter 13, but has the measuring element omitted.

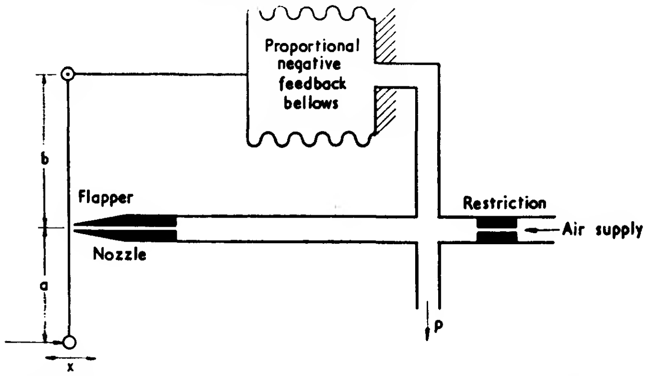


Fig. 216. *Proportional controller unit utilizing nozzle and flapper unit*

The net flapper movement facing the centre line of the nozzle is, based on chapter 13,

$$y = \left(\frac{b}{a+b}\right)x - \left(\frac{a}{a+b}\right)S.p \quad \ldots (278)$$

where x = the deflection of the flapper at the ower pivot due to the deviation θ between the measured and set values.

S = the bellows rating.

p = the nozzle pressure.

From *Fig.* 200 in chapter 13, the slope of the pressure-flapper movement characteristic is

$$G = -\frac{p}{y} \quad \ldots\ldots\ldots\ldots\ldots (279)$$

This is in the nature of an amplification factor or a gain. Its value may be calculated approximately, assuming that the curve is a straight line, over the pressure range 3–15lb/in². The base units, i.e., for flapper deflection, are in terms of 0·0002in., the total movement being 0·001in. For a 4lb/in² change in nozzle pressure, the change in flapper movement is 0·00005in. This makes G:

$$G = -\frac{4}{0{\cdot}00005}$$

$$G = -8 \times 10^4 \text{ lb/in}^2\text{/in.} \quad \ldots\ldots\ldots (280)$$

Substituting for y in equation (278),

$$-\frac{p}{G} = \left(\frac{b}{a+b}\right)x - \left(\frac{a}{a+b}\right)S.p \quad \ldots (281)$$

$$p\left[\left(\frac{aSG}{a+b}\right) - 1\right] = \left(\frac{bG}{a+b}\right)x \quad \ldots (282)$$

But G from (280) is very much greater than 1 so that $\left(\frac{aSG}{a+b}\right)$ is also very much greater than 1. Hence equation (282) reduces to

$$p = \left(\frac{b}{aS}\right)x \quad \ldots\ldots\ldots\ldots\ldots (283)$$

If we assume that x is directly proportional to the deviation θ then

$$x = \alpha\,\theta \quad \ldots\ldots\ldots\ldots\ldots (284)$$

where α = a constant.

From (283),

$$p = \left(\frac{b\alpha}{aS}\right)\theta \quad \ldots\ldots\ldots\ldots\ldots (285)$$

$$\text{or } p = -K_1\,\theta \quad \ldots\ldots\ldots\ldots\ldots (286)$$

where $K_1 = -\dfrac{b\alpha}{aS}$

Equation (286) is then of the same form as (267) in chapter 13 which defines proportional controller action.

If the travel l of the control valve stem can be related to the output pressure p of the unit by a relation of the form:

$$l = -k_1 p \quad \ldots\ldots\ldots\ldots\ldots (287)$$

then $l = -A\,\theta \quad \ldots\ldots\ldots\ldots\ldots (288)$

where A = a constant.

Proportional Band Adjustment

The influence of proportional band adjustment has already been described. It is equivalent to varying K_1, the proportional action constant. From equation (285) this could be achieved by altering the ratio b/a. There are a vast number of methods of adjusting b/a

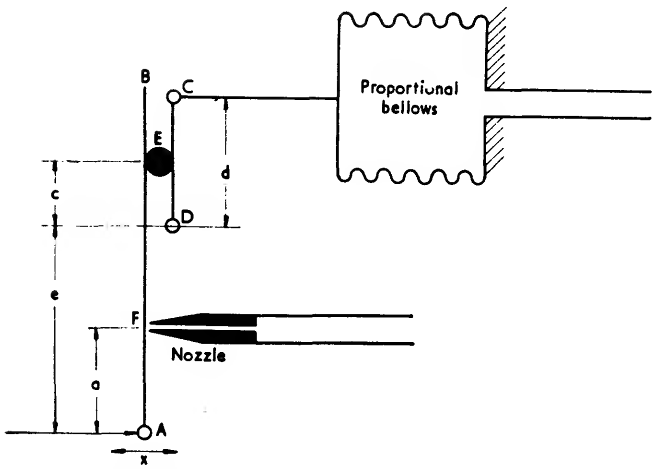


Fig. 217. "Scissors" double lever proportional band adjustment

and only one or two typical ones will be dealt with. One relatively simple example is the "scissors" double lever movement. An explanatory diagram is given in *Fig.* 217. The two levers are AB and CD. A pin E slides between the two, forming an adjustable pivot. If the feedback bellows deflects a distance z for the full operating range of the controller signal (i.e., 3–15lb/in²) the pin moves a distance $\frac{zc}{d}$. The point F on the flapper opposite the centre line of the nozzle moves, by simple proportion, $\frac{zca}{d\,(e+c)}$. At A the flapper is actuated a distance x by the deviation θ. Consider the movement necessary to restore the flapper to its original position before deflection by the bellows. This is given by $\frac{x\,(e+c-a)}{e+c}$. Then,

$$\frac{x\,(e+c-a)}{e+c} = \frac{zca}{d\,(e+c)} \qquad \ldots (289)$$

Since the full operating range 3–15lb/in² acting on the bellows is also the full output signal range, x must be the movement of the flapper opposite the nozzle to produce this range. But this is dependent on the deviation θ; hence we can establish a relation between the value of deviation necessary to produce the full signal range and the range itself.

From equation (289),

$$x = \frac{zca}{d(e+c-a)} \qquad \ldots\ldots\ldots\ldots (290)$$

z, a and e are fixed, but c is variable by adjusting the position of pin E. This, in turn, alters the value of x and the deviation θ, therefore, necessary to produce the full output range of the controller signal.

Another well-known proportional band adjustment is the arc lever. If *Fig.* 218 is examined, the flapper is connected by link A to the arc-shaped lever B, in such a manner that it can slide along B. The deviation θ acts on end C of the arc-shaped lever, and using the lower end D of the link as fulcrum adjusts the position of the flapper to or away from the nozzle. The pressure inside the nozzle changes and is transmitted to the proportioning bellows E via a relay F. The movement of the bellows, again using D as fulcrum, adjusts the flapper in the opposite direction to the initial deflection, hence exerting a feedback action, until balance is achieved. The flapper travel is extremely small, so that D is practically a fixed fulcrum. It can be seen that the position of A along the arc of B determines the relative effect of the deviation and the bellows, and so adjusts the proportioning band by altering the leverage ratio.

The nozzle and flapper unit in the form shown in *Fig.* 216 is a position balance device in that the control pressure is determined by the position of the flapper. The force balance type must now be considered. There are two general versions: the beam balance and the "stack".

Proportional Control Unit, Force Balance Type

The basic features of the beam balance are shown in *Fig.* 219. It can be seen that the beam is acted upon by four forces, three due to bellows and one due to a spring. The set value is produced in the form of a pressure proportional to the required value θ_s and this pressure p_s acts on bellows A. In a similar manner the measured value θ_m is produced as a pressure p_m and this acts on bellows B. The forces produced by bellows A and B oppose each other on the beam and any deflection of the beam due to these two forces is a measure of the deviation θ. At the opposite end of the beam is the proportional bellows C which is opposed by a spring D. The operation of the associated nozzle and flapper unit is similar to that already described. It is assumed that each bellows unit has the same effective area A. If F is the force due to the spring, and P the initial pressure in the proportional bellows, p the change in pressure due to a deviation, the equation for balance is:

$$(P+p)Aa + p_s A\,b = p_m\,A\,b + F\,a \qquad \ldots\ldots (291)$$

$$\text{From which, } P + p = \frac{b}{a}\,(p_m - p_s) + \frac{F}{A} \qquad \ldots\ldots (292)$$

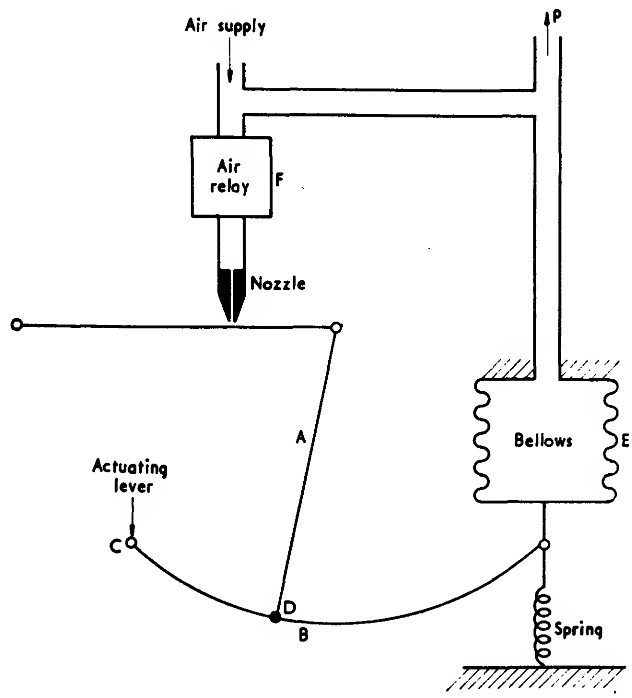


Fig. 218. Arc lever proportional band adjustment

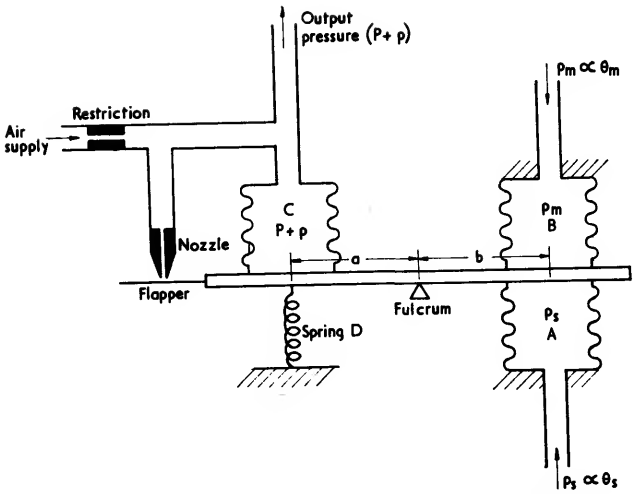


Fig. 219. *Force balance proportional controller unit*

$$p = \frac{b}{a}(p_m - p_s) + \left(\frac{F}{A} - P\right) \quad (293)$$

$$\text{or} \quad V = -K_1\theta + \beta \quad \ldots\ldots \quad (294)$$

$$\text{where} \quad V = P + p$$

$$K_1 = \frac{b}{a}$$

$$\theta = (p_m - p_s)$$

$$\frac{F}{A} = \beta, \text{ a constant.}$$

Note that when the deviation is zero, $(p_m - p_s)$ is zero and p is zero. Then the initial pressure P is given by

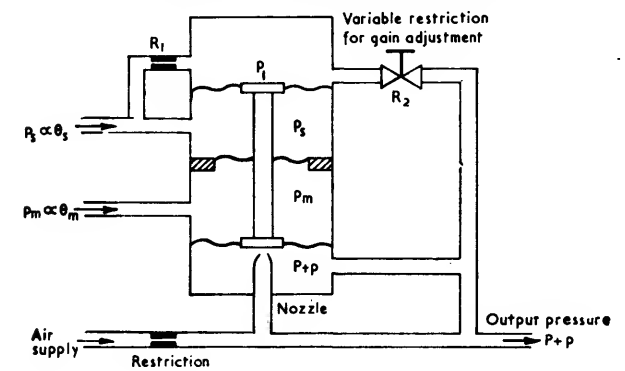
$$P = \frac{F}{A} \quad \ldots\ldots\ldots\ldots \quad (295)$$

The proportional band can be adjusted by varying the ratio b/a. References 1 and 2 should be consulted for methods of doing this.

Proportional Control Unit, "Stack" Type

The elements of a "stack" type proportional control unit are shown in *Fig.* 220. The unit involves three diaphragms in a kind of "stack" formation. The upper and lower diaphragms are normally of the same area, but are larger than the central one. The diaphragms are fixed at their inner edges to a balancing column, the lower end of which acts in the manner of a flapper or a baffle over the mouth of a nozzle. As in the beam balance type the set value θ_s and the measured value

Fig. 220. *"Stack" proportional controller unit*



θ_m are produced as corresponding pressures p_s and p_m. The initial output pressure when the unit is in equilibrium is P.

There are two features of special interest. The proportional band is adjusted by the variable restriction R_2 which in effect supplies a positive feedback pressure p_1 to the upper diaphragm. Fixed restriction R_1 and variable restriction R_2 form a kind of pneumatic pressure divider as indicated in *Fig.* 221.

Consider an increase in measured value θ_m. There is a deviation θ given by $\theta_m - \theta_s = p_m - p_s$. In a large number of units of this type it is customary to make the upper and lower diaphragms twice the area of the central one and it will simplify matters if this is considered to be the case in the present example.

The respective areas are then $2A$ and A. p is the increase in pressure in the nozzle chamber due to the deviation θ.

For equilibrium all forces present must balance.

$$2Ap_1 + Ap_s + 2Ap_m = 2A(P + p) + Ap_m + 2Ap_s \quad \ldots\ldots\ldots \quad (296)$$

This reduces to

$$p_1 + \tfrac{1}{2}(p_m - p_s) = P + p \quad \ldots\ldots\ldots \quad (297)$$

Now refer to *Fig.* 221, where the flow must be considered stream line or laminar. If q is the rate of flow,

$$q = \frac{p_s - p_1}{R_1} = \frac{p_1 - (P + p)}{R_2} \quad \ldots\ldots \quad (298)$$

$$\text{From which } p_1 = \frac{(P + p)R_1}{R_1 + R_2} + \frac{p_s R_2}{R_1 + R_2} \quad (299)$$

Substituting for p_1 in (297),

$$P + p = \left(\frac{R_1 + R_2}{2R_2}\right)(p_m - p_s) + p_s \quad \ldots\ldots \quad (300)$$

$$p = \left(\frac{R_1 + R_2}{2R_2}\right)(p_m - p_s) + p_s - P \quad \ldots \quad (301)$$

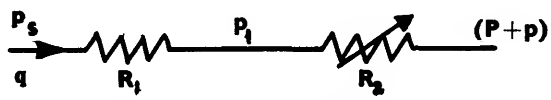


Fig. 221. *Pneumatic pressure divider*

This can be rearranged in the form

$$V = -K_1\theta + \gamma \quad \ldots\ldots\ldots\ldots\ldots \quad (302)$$

$$\text{where} \quad V = (P + p)$$

$$K_1 = -\left(\frac{R_1 + R_2}{2R_2}\right)$$

$$\theta = (p_m - p_s)$$

$$p_s = \gamma, \text{ a constant.}$$

Reverting to the equation (301) when the deviation θ is zero $(p_m - p_s)$ is zero and P the initial pressure is given by

$$P = p_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (303)$$

Proportional Plus Integral Control Unit, Position Balance Type

Since integral controller action is invariably associated with proportional controller action the two are considered in combination. The controller unit involves a proportional bellows and an integral bellows acting in opposition in the manner of *Fig.* 222. This figure is virtually the same as *Fig.* 206 of chapter 13 in which the integral bellows is shown acting as a positive feedback device. This latter term is justified since the

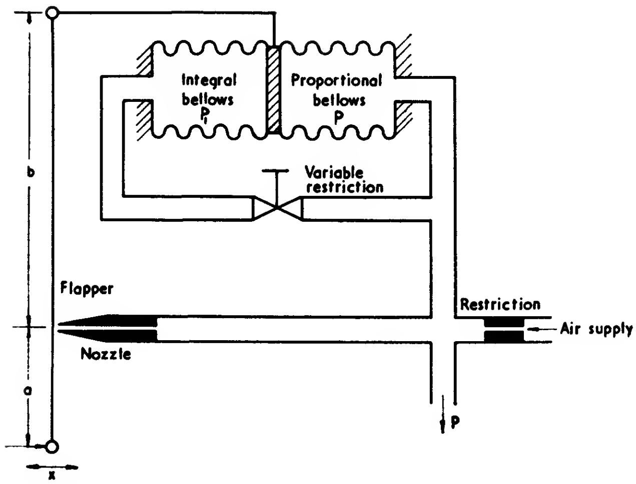


Fig. 222. Proportional plus integral controller unit

integral bellows opposes the negative feedback action of the proportional bellows on the nozzle flapper.

Consider first a laminar flow of gas through a restriction into a closed chamber as in *Fig.* 223. The pressure p on the upstream side of the restriction is higher than p_1 on the downstream side and p_1 is the pressure existing inside the closed chamber. The flow will continue until $p_1 = p$. But at any instant, the rate of change of p_1 is given by

$$\frac{dp_1}{dt} = \frac{(p-p_1)}{e} \qquad \ldots\ldots\ldots\ldots (304)$$

where e = a constant.

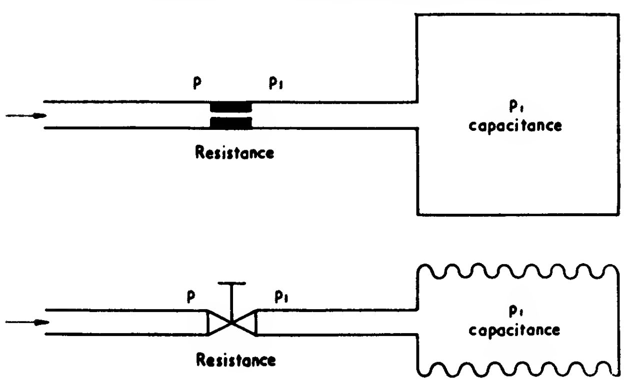
The conditions of *Fig.* 223 correspond to those of the integral restriction and bellows after a deviation has occurred and before equilibrium is re-established.

Suppose a sudden or step change occurs in the measured value and a deviation θ is produced. There is a displacement x of the flapper at the lower pivot. Let us take x as being proportional to θ or $x = \alpha\theta$, where α is a constant.

The change is such that the proportional bellows receives a pressure $(P + p)$ where P is the initial pressure in both bellows in the original state of equilibrium. Due to the restriction, the pressure in the integral bellows increases at a slower rate. At any instant, let the value be $(P + p_1)$. The relation between pressure and deviation may be expressed as:

$$(P + p) - (P + p_1) = \frac{b\alpha}{aS}\theta \qquad \ldots\ldots\ldots (305)$$

Fig. 223. Pneumatic resistance-capacitance network



or
$$p-p_1 = \frac{b\alpha}{aS}\theta \qquad \ldots\ldots\ldots (306)$$

But,
$$p-p_1 = e\frac{dp_1}{dt} \qquad \ldots\ldots\ldots (307)$$

so that, integrating,

$$e\frac{dp_1}{dt} = \left(\frac{b\alpha}{aS}\right)\theta \qquad \ldots\ldots\ldots (308)$$

$$p_1 = \left(\frac{b\alpha}{eaS}\right)\int \theta\, dt \qquad \ldots\ldots (309)$$

Substituting in (306)

$$p - \left(\frac{b\alpha}{eaS}\right)\int \theta\, dt = \left(\frac{b\alpha}{aS}\right)\theta \qquad \ldots\ldots\ldots (310)$$

$$p = \left(\frac{b\alpha}{aS}\right)\theta + \frac{b\alpha}{eaS}\int \theta\, dt \quad \ldots (311)$$

or
$$V = -K_1\theta - K_2\int \theta\, dt \quad \ldots (312)$$

where
$$p = V$$
$$K_1 = -\left(\frac{b\alpha}{aS}\right)$$
$$K_2 = -\left(\frac{b\alpha}{eaS}\right)$$

Before leaving the subject of proportional plus integral control one more aspect of the fluid resistance —capacitance combination must be mentioned.

Equation (307) can be written:

$$\frac{p-p_1}{R_1} = C_1\frac{dp_1}{dt} \qquad \ldots\ldots\ldots (313)$$

where R_1 = fluid resistance of the integral restriction.

C_1 = fluid capacitance of the integral bellows.

From (313),

$$p-p_1 = R_1 C_1 \frac{dp_1}{dt} \qquad \ldots\ldots (314)$$

Comparing with (304),

$$e = R_1C_1 \qquad \ldots\ldots\ldots (315)$$

The product R_1C_1 has the dimensions of time and in fact is equivalent to the integral time constant. Its value can be varied by adjusting the value of R_1. The basic equation for proportional plus integral controller action can be re-written from (311) as

$$p = \frac{b\alpha}{aS}\left(\theta + \frac{1}{R_1C_1}\int\theta\, dt\right) \quad . (316)$$

Now let us consider *Fig* 224 which represents the relation between controller output signal V and time. If the deviation θ remains constant after the initial step change, the contribution of the proportional action alone remains constant at $-K_1\theta$. The integral action, however, increases from zero initially to $-K_2\int_{t_1}^{t_2}\theta\, dt$ after a time interval (t_2-t_1). At this value of the time interval, the contributions of the two actions are equal, i.e.

$$-K_1\theta = -K_2\int_{t_1}^{t_2}\theta\, dt \qquad \ldots\ldots (317)$$

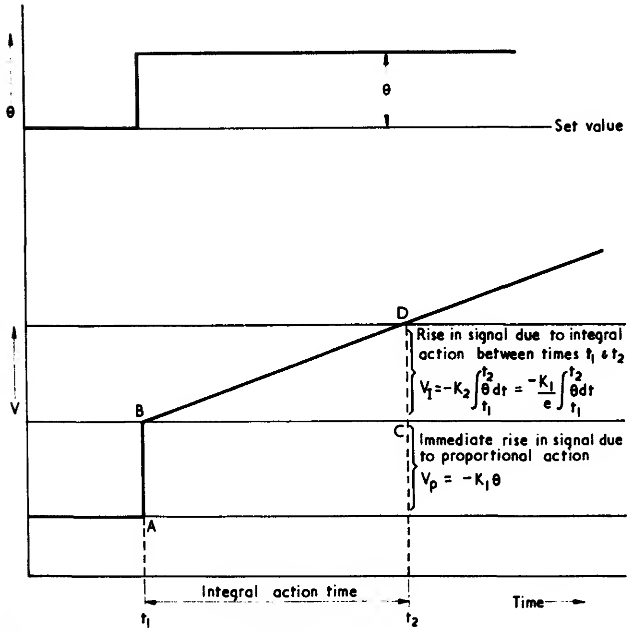


Fig. 224. *Integral action time*

Since θ is now constant,

$$-K_1\theta = -K_2\theta\,(t_2-t_1) \quad \ldots\ldots (318)$$

Calling $(t_2-t_1) = t_I$

$$t_I = \frac{K_1}{K_2} \quad \ldots\ldots\ldots\ldots (319)$$

i.e.

$$t_I = \frac{\dfrac{b\alpha}{aS}}{\dfrac{b\alpha}{eaS}} = e \quad \ldots\ldots\ldots (320)$$

i.e.

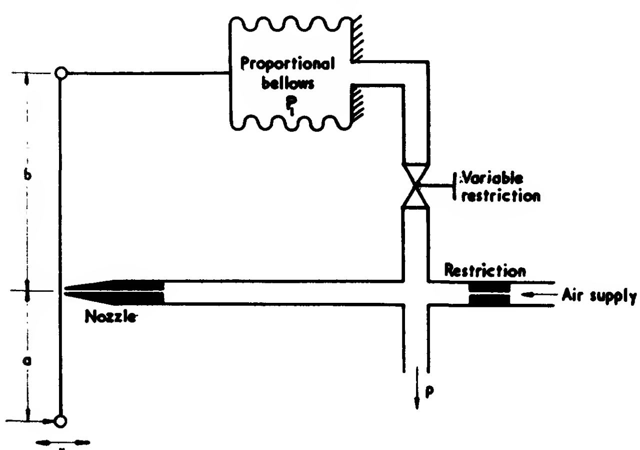
$$t_I = R_1C_1 \quad \ldots\ldots\ldots (321)$$

t_I is by definition from B.S. 1523 integral action time, and, as we have seen, can be varied by adjusting the value of R_1.

Proportional Plus Derivative Control Unit, Position Balance Type

A proportional plus derivative control unit is shown in *Fig.* 225. For this combined action, a derivative restriction is placed between the nozzle and proportional bellows.

Fig. 225. *Proportional plus derivative controller unit*



Again we have an air stream flowing through a restriction into a closed volume, but the effect is vastly different to that of the integral unit.

If a sudden or step change occurs in the measured value, the nozzle pressure will be higher for a time than the pressure p_1 in the proportional bellows. The following relation holds:

$$p-p_1 = f\,\frac{dp_1}{dt} \quad \ldots\ldots\ldots (322)$$

But since the bellows itself contributes to a proportional action control,

$$p_1 = \left(\frac{b\alpha}{aS}\right)\theta \quad \ldots\ldots\ldots (323)$$

From which

$$\frac{dp_1}{dt} = \left(\frac{b\alpha}{aS}\right)\frac{d\theta}{dt} \quad \ldots\ldots (324)$$

Substituting for $\frac{dp_1}{dt}$ in (324),

$$p - \left(\frac{b\alpha}{aS}\right)\theta = f\left(\frac{b\alpha}{aS}\right)\frac{d\theta}{dt} \quad \ldots\ldots (325)$$

$$p = \left(\frac{b\alpha}{aS}\right)\theta + \left(\frac{fb\alpha}{aS}\right)\frac{d\theta}{dt} \quad . (326)$$

or

$$V = -K_1\theta - K_3\frac{d\theta}{dt} \quad \ldots (327)$$

where

$$V = p$$

$$K_1 = -\left(\frac{b\alpha}{aS}\right)$$

$$K_3 = -\left(\frac{fb\alpha}{aS}\right)$$

For the derivative restriction and proportional bellows it can be shown that

$$f = R_2\,C_2 \quad \ldots\ldots\ldots (328)$$

where R_2 = fluid resistance of the derivative restriction

C_2 = fluid capacitance of the proportional bellows

Equation (326) may then be re-written:

$$p = \frac{b\alpha}{aS}\left(\theta + R_2\,C_2\,\frac{d\theta}{dt}\right) \quad . (329)$$

Fig. 226 illustrates the definition of derivative action time. This, from B.S. 1523, is the time interval in which that part of the controller output signal due to proportional action increases by an amount equal to the part of the signal due to derivative action. The deviation is considered to be changing at a constant rate.

The deviation θ may be represented by:

$$\theta = \int_{t_1}^{t_2}\frac{d\theta}{dt}\,dt \quad \ldots\ldots (330)$$

Since $\frac{d\theta}{dt}$ is constant,

$$\theta = \frac{d\theta}{dt}\int_{t_1}^{t_2}dt = \frac{d\theta}{dt}\,(t_2-t_1) \quad (331)$$

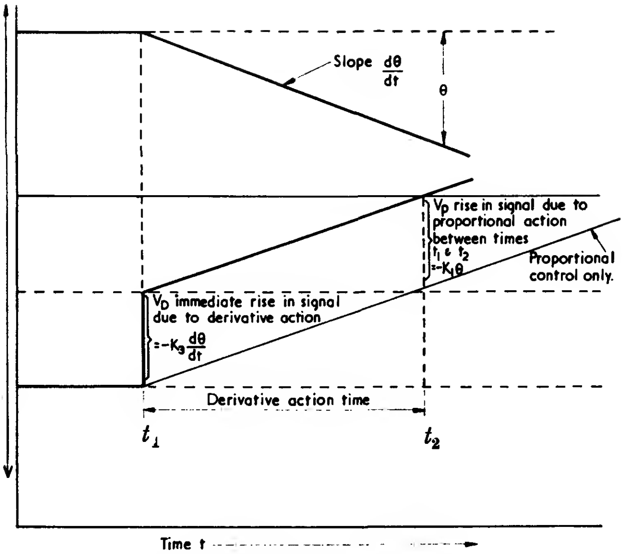


Fig. 226. Derivative action time

The proportional action contribution to the controller output signal is

$$-K_1\,\theta = -K_1\frac{d\theta}{dt}(t_2 - t_1) \quad \dots (332)$$

The derivative action contribution to the controller output signal is $-K_3\dfrac{d\theta}{dt}$

Equating the two values

$$-K_1\frac{d\theta}{dt}(t_2 - t_1) = -K_3\frac{d\theta}{dt} \quad \dots\dots (333)$$

So that

$$(t_2 - t_1) = \frac{K_3}{K_1} \quad \dots\dots\dots (334)$$

Thus $(t_2 - t_1)$ is the time interval as defined by B.S. 1523 for derivative action time. Designating $(t_2 - t_1)$ by t_D,

$$t_D = \frac{K_3}{K_1} \quad \dots\dots\dots (335)$$

$$t_D = \frac{\dfrac{bf\alpha}{aS}}{\dfrac{b\alpha}{aS}} = f \quad \dots\dots\dots (336)$$

i.e.

$$t_D = R_2\,C_2 \quad \dots\dots\dots (337)$$

Proportional Plus Integral Plus Derivative Control Unit, Position Balance Type

From the two term control units so far described, it is possible to construct a three term control unit to provide proportional plus integral plus derivative action. An ideal controller embracing the three actions would be given by:

$$V = -K_1\theta - K_2\int\theta dt - K_3\frac{d\theta}{dt} \quad \dots (338)$$

$$V = -K_1\left[\theta + \frac{K_2}{K_1}\int\theta dt + \frac{K_3}{K_1}\frac{d\theta}{dt}\right] \quad \dots (339)$$

But from (319) and (335),

$\dfrac{K_2}{K_1} = \dfrac{1}{t_I}$ where t_I is the integral action time

$\dfrac{K_3}{K_1} = t_D$ where t_D is the derivative action time

so that

$$V = -K_1\left[\theta + \frac{1}{t_I}\int\theta dt + t_D\frac{d\theta}{dt}\right] \quad \dots (340)$$

Unfortunately in constructing a three term control unit as shown in *Fig.* 227 from *Figs.* 222 and 225 the ideal equation is not realized. It is found that *Fig.* 227 produces a relation of the following kind:

$$V = -K_1 I\left[\theta + \frac{1}{I\,t_I}\int\theta dt + \frac{t_D}{I}\frac{d\theta}{dt}\right] \quad . (341)$$

I is known as the interaction factor and for *Fig.* 227 has the value:

$$I = \left(1 + \frac{2f}{e}\right) \quad \dots\dots\dots\dots (342)$$

The full equation for *Fig.* 227 is, indeed,

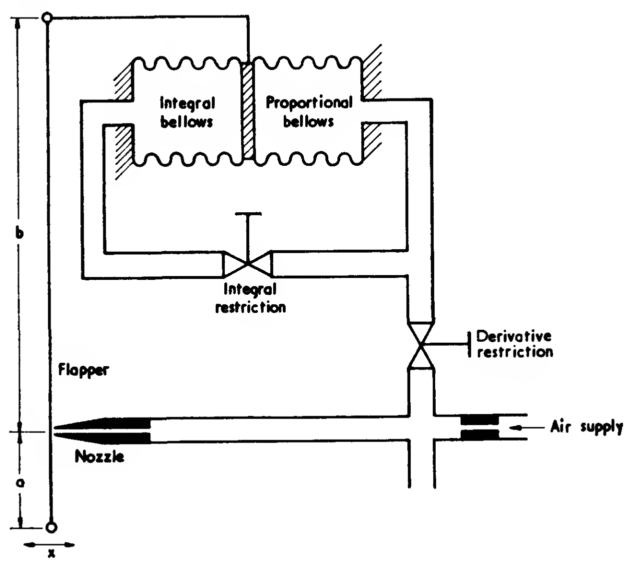
$$p = P - K_1\left(1 + \frac{2f}{e}\right)\left[\theta + \frac{1}{e\left(1 + \frac{2f}{e}\right)}\int\theta dt + \left(\frac{f}{1 + \frac{2f}{e}}\right)\frac{d\theta}{dt}\right] \quad \dots (343)$$

Replacing *e* by t_I and *f* by t_D

$$p - P = -K_1\left(1 + \frac{2t_D}{t_I}\right)\left[\theta + \frac{1}{t_I\left(1 + \frac{2t_D}{t_I}\right)}\int\theta dt + \left(\frac{t_D}{1 + \frac{2t_D}{t_I}}\right)\frac{d\theta}{dt}\right] \quad . (344)$$

The full implications can now be seen. The proportional factor is now greater than K_1 by $\left(1 + \dfrac{2t_D}{t_I}\right)$.

Fig. 227. Proportional plus integral plus derivative controller unit



This, in turn, reflects on the proportional bandwidth which will be narrower than the controller dial setting for proportional action alone. The integral action time is increased by $\left(1 + \frac{2t_D}{t_I}\right)$ but the derivative action time is decreased by the reciprocal of the same factor. Thus any adjustment in the setting of one action time will affect the others.

How serious is the effect of this interaction?

This has been discussed by a number of authorities, particularly by A. J. Young[2]. Obviously one cannot be dogmatic since so much depends on the nature of the process and one may perhaps quote from the same authority: "The limitation which is set by interaction between the action units to the phase advance obtainable from a 3-action controller is not in general a serious disadvantage in present practice, although there are applications in which some controllers have proved unsuitable on this account." In this quotation the term phase advance is used and this may be a convenient place to consider the significance of frequency response.

FREQUENCY RESPONSE

In making simple mathematical analyses of control units or systems three types of variation in measured value are considered:

1. The sudden or step change. This has already been used in examining the various forms of controller actions and units.
2. The constant rate change, where the measured value varies at a constant rate. This is described in Chapter 15 under Detecting Element Lags.
3. The sinusoidal change where a sine wave signal is applied to the input of the unit or system and the output signal is examined in the light of phase change and gain or attenuation. This method has several advantages, particularly as it can be carried out at different frequencies.

None of the changes mentioned are necessarily typical of actual variations occurring in an industrial process, but the sinusoidal approach can lead to valuable data on the behaviour of a unit or a complete control system including the process. Particularly is this true for assessing the controllability of a process.

$\theta_a \sin \omega t$ is now used instead of θ as the deviation signal. The controller unit is normally considered to possess a gain G. Let us see the effect of substituting $\theta_a \sin \omega t$ for θ.

Proportional Controller Action

The basic equation for proportional controller action is:

$$V = -K_1 \theta \qquad (345)$$

Substituting $\theta_a \sin \omega t$ for θ,

$$V = -K_1 \theta_a \sin \omega t \qquad (346)$$

The amplitudes of the input and output signals are both θ_a so that

$$G = K_1 \frac{\theta_a}{\theta_a} = K_1 \qquad (347)$$

Proportional Plus Integral Controller Action

For proportional plus integral controller action,

$$V = -K_1 \left[\theta + \frac{1}{t_I}\int \theta dt\right] \qquad (348)$$

Substituting $\theta_a \sin \omega t$ for θ,

$$V = -K_1 \left[\theta_a \sin \omega t + \frac{1}{t_I}\int \theta_a \sin\omega t \; dt\right] \qquad (349)$$

The solution of this is:

$$V = -K_1 \theta_a \left(\sqrt{1 + \frac{1}{(\omega t_I)^2}}\right) \sin (\omega t - \phi) \qquad (350)$$

The amplitude is now given by $\theta_a \sqrt{1 + \frac{1}{(\omega t_I)^2}}$ and the gain of the controller is then:

$$G = \frac{K_1 \theta_a \sqrt{1 + \frac{1}{(\omega t_I)^2}}}{\theta_a}$$

$$= K_1 \sqrt{1 + \frac{1}{(\omega t_I)^2}} \qquad (351)$$

This means that the controller gain has been increased by a factor $\sqrt{1 + \frac{1}{(\omega t_I)^2}}$.

There is also a phase lag ϕ now given by the following relation:

$$\tan \phi = -\frac{1}{\omega t_I} \text{ or } \phi = -\tan^{-1} \frac{1}{\omega t_I} \qquad (352)$$

As the value of $\frac{1}{\omega t_I}$ approaches infinity the phase lag angle ϕ reaches a limiting value of $-\pi/2$ or $-90°$. When $\frac{1}{\omega t_I}$ equals 1, the phase angle ϕ is $-9°$. The period T tends to be used more than the frequency f so that, using T, the phase angle would become $-\tan^{-1} \frac{T}{2\pi t_I}$ instead of $-\tan^{-1} \frac{1}{\omega t_I}$.

Proportional Plus Derivative Controller Action

The basic equation for proportional plus derivative controller action is:

$$V = -K_1 \left[\theta + t_D \frac{d\theta}{dt}\right] \qquad (353)$$

Substituting $\theta_a \sin \omega t$ for θ, the final equation can be shown to be:

$$V = -K_1 \theta_a \left[\sqrt{1 + (\omega t_D)^2}\left(\sin \omega t + \phi\right)\right] \qquad (354)$$

Thus the controller gain is:

$$G = K_1 \sqrt{1 + (\omega t_D)^2} \qquad (355)$$

As distinct from the proportional plus integral mode, there is a phase advance now, given by:

$$\tan \phi = \omega t_D \text{ or } \phi = \tan^{-1} \omega t_D \qquad (356)$$

Using the period T instead of frequency f,

$$\tan \phi = \frac{2\pi}{T} t_D \text{ or } \phi = \tan^{-1} \frac{2\pi}{T} t_D$$

As $\frac{T}{t_D}$ approaches zero the phase angle approaches a limiting value of $+\frac{\pi}{2}$ or $+90°$.

Proportional Plus Integral Plus Derivative Controller Action

For the three term controller action, the gain G can be shown to be:

$$G = K_1 \sqrt{\left(\omega t_D - \frac{1}{\omega t_I}\right)^2 + 1} \quad \ldots\ldots (357)$$

and the phase angle ϕ by

$$\tan \phi = \left(\omega t_D - \frac{1}{\omega t_I}\right) \quad \ldots\ldots\ldots\ldots (358)$$

Two things emerge from these equations: the relation $\frac{G}{K_1}$ is always greater than 1 and the phase angle may be a lead or a lag depending on the relative values of t_D and t_I.

The foregoing frequency response equations relate to ideal controller actions. It is necessary to examine the three term controller of *Fig.* 227 in the light of frequency response.

Let us call the *effective* integral action time S and the *effective* derivative time R.

Then from equation (344),

$$S = t_I\left(1 + \frac{2t_D}{t_I}\right) \quad \ldots\ldots\ldots\ldots (359)$$

$$R = \left(\frac{t_D}{1 + \frac{2t_D}{t_I}}\right) \quad \ldots\ldots\ldots\ldots (360)$$

The phase angle ϕ increases with the ratio $\frac{R}{S}$ and if $\frac{R}{S}$ reaches a maximum value then so does ϕ.

For the controller of *Fig.* 227,

$$\frac{R}{S} = \frac{\frac{t_D}{t_I}}{\left(1 + \frac{2t_D}{t_I}\right)^2} \quad \ldots\ldots\ldots\ldots (361)$$

This has a maximum value $\frac{1}{8}$ when $\frac{t_D}{t_I}$ is $\frac{1}{2}$. In a commercial controller based on *Fig.* 227, the maximum phase angle is $+37°$ when the proportional band is 100% and about 34° at 10% proportional band.

PARALLEL TYPE CONTROLLERS

In the three term controller of *Fig.* 227, the derivative restriction lies between the integral restriction and the output, i.e. only the derivative restriction is connected directly to the output. This forms a sort of "series" arrangement. In another general class of controllers, both the integral and the derivative restrictions are connected to the output giving a "parallel" arrangement. The same form of reasoning can be applied to this class as for the series and equivalent equations can be produced.

Frequency response is considered again in Chapters 15 and 16 with reference to complete control systems and electronic controllers respectively.

CORRECTING UNITS

The correcting unit is the final link in the control chain, and since it is responsible for controlling the medium applying the correcting action to the process its design is of considerable importance.

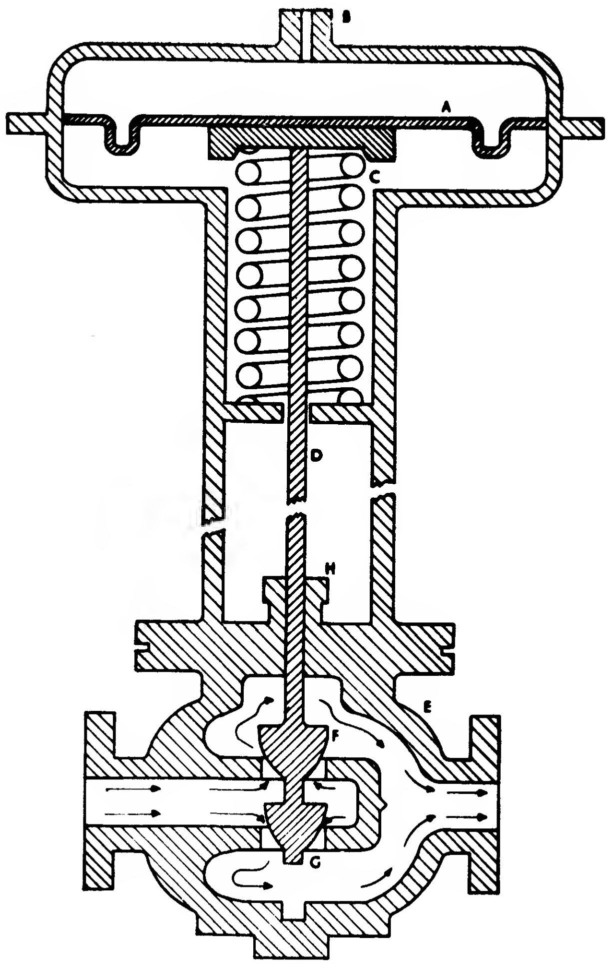


Fig. 228. *Basic features of a pneumatic control valve*

With the operation of the majority of controlling units by compressed air, even with electronic controllers, it follows that the greater percentage of correcting units are actuated pneumatically. A smaller number are electrically driven, either by motor or solenoid, and some depend on water or oil for motive power.

Pneumatic Correcting Units

Fig. 228 illustrates, diagrammatically, the essential features of a pneumatic correcting unit or control valve. At the head of the unit is a chamber containing a flexible diaphragm A of rubber, fabric, or similar substance. The diaphragm virtually seals the chamber into two parts, the upper section receiving the pneumatic signals from the controlling unit, via inlet B. This section of the assembly is often known as the diaphragm motor or valve motor. The signals deflect the diaphragm which, it will be observed, is fixed to a solid centre member from which shaft D extends downwards into the body E of the valve. The deflection is opposed by spring C, whose rating determines the extent of travel of the shaft for a given pressure range and effective diaphragm area.

At the lower end of the shaft are two plugs F working in a vertical direction in the orifices or seats G, and, by virtue of their contour, varying the orifice area as they are moved up and down by the shaft motion. The latter is dependent on the diaphragm deflection,

which is related to the pneumatic signals received. Hence, we have a means of automatically adjusting the orifice area in response to the action from the controlling unit, and thereby altering the flow of the medium through the valve.

H is a gland, or stuffing box, which allows the shaft to move vertically, but prevents leakage of the fluid flowing through the valve body. This is particularly necessary at large working pressures. It is packed into preformed rings of material such as asbestos, p.t.f.e., or even metals. The choice depends on such factors as operating temperatures, etc. A refinement is to provide the stuffing box with fins for cooling purposes at high temperatures. On one type of finned control valve, it is estimated that the stuffing box temperature is 300°F lower than the body of the valve when very high temperature fluids are flowing. It should be noted that, with temperatures below freezing, the fins exert the opposite effect and may prove beneficial at low values.

The valves are not alway double seated, a considerable number in use possessing only one seat. Some advantages of double seated patterns will be seen later when forces acting on the valve are considered.

With air failure in some types, the diaphragm rises to its maximum setting and withdraws the plug or plugs out from the orifices to the fully open position. Maximum flow takes place and the valve is said to open with air failure. On the other hand, other types tend to open with increasing air pressure and close with air failure. Both have their applications. If a valve were controlling the flow of a cooling fluid, for example, in a high power thermionic transmitter, it would be very desirable to continue the flow in event of air failure and use an open-with-air-failure valve. With a heating process, on the other hand, it may be desirable to use a close-with-air-failure valve, and so prevent possible damage to the process by the full application of heat which would result.

It is necessary to consider what forces are acting on the valve.

Forces Acting on a Pneumatic Control Valve

Basic Forces

If A is the effective area of the diaphragm, and $(p_1 - p_2)$ the pressure range to operate the valve between the fully open and fully closed positions, F the rate of the spring, and x the total travel of the shaft between extreme positions,

$$A\,(p_1 - p_2) = F\,x \qquad \ldots\ldots\ldots\ (362)$$

This is the basic equation of the valve, but, as we shall see, other forces are present to modify it.

Pressure Load

The orifice portion of the valve can be regarded as being roughly of the annular pattern. With a pressure drop existing across it, the pressure on the lower surface of the plug is greater than that on the upper surface. A set of conditions exists similar to those in the variable area flow meter (see Chapter 5). The result is that there is a certain thrust, and this is always present in a single seated valve. In a double seated type the thrusts may be made to act in opposite directions, and to cancel each other to some extent. An alternative name given to the double seated valve is the "balanced valve".

The pressure load F_P can be represented approximately by the expression:

$$F_P = k\,A\,\triangle p \qquad \ldots\ldots\ (363)$$

where k = a constant

A = the area of the valve plug

$\triangle p$ = pressure drop across the orifice.

The above equation shows a difficulty in positioning a valve plug, caused by the variable pressure drop across the valve with changing flow rates. This imposes a fluctuating load on the valve motor.

Weight Load

If the valve possesses any unbalanced weight in the moving parts, then this constitutes a load to be supported by the motor.

Designating this load by F_W,

$$F_W = W_1 \qquad \ldots\ldots\ldots\ (364)$$

where W_1 = the weight of the unbalanced moving parts.

Inertia Load

The valve involves a moving assembly comprising the central member of the diaphragm, the shaft and the plugs. If any acceleration or deceleration occurs, inertia will effect an influence. The force or load F_1 produced is:

$$F_1 = \frac{W}{g}\frac{d^2x}{dt^2} \qquad \ldots\ldots\ldots\ (365)$$

where W = weight of moving parts

g = gravitational acceleration

$\frac{d^2x}{dt^2}$ = the acceleration or deceleration of the moving parts.

Friction Load

There are two species of this type of load: dynamic and static.

Dynamic Friction

Referring to *Fig.* 208, the presence of the stuffing box or gland, through which the shaft travels, may occasion a dynamic friction load, particularly if a viscous fluid is present. The simplest law relating the resistance to motion through a viscous fluid is that of Stokes:

$$F_D = k\,V\,l\,\eta \qquad \ldots\ldots\ (366)$$

where k = a constant

V = the velocity of the shaft

l = a dimension equivalent to a length

η = the absolute viscosity of the fluid.

Static Friction

If moving parts are in contact, static friction forces are set up tending to oppose any travel. The force is:

$$F_S = \mu F_N \qquad \ldots\ldots\ (367)$$

where F_N = the normal force applied at the surfaces in contact.

μ = the coefficient of friction.

The additive effect of the above forces can be serious enough to influence accuracy of control, particularly in proportional plus integral control where the valve must be positioned so that there is no

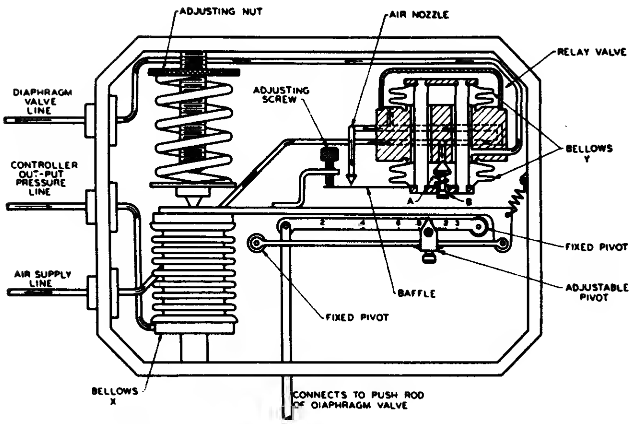


Fig. 229. *Valve positioner*

offset. An example may serve to illustrate the errors likely to be set up.

In a well designed valve, the total error force may be as low as a few ounces, but it can rise to be a few pounds. A typical operating pressure range for a control valve is from 3 to 15lb/in², i.e. 12lb/in². Let us consider a temperature controller, instrument range 0°C to 200°C, operating with a proportional band of 120 per cent. For a 1°C change, the following pressure change is required:

$$\frac{100 \times 1 \times 12}{120 \times 200} = 0{\cdot}05\text{lb/in}^2$$

If the effective diaphragm area is 50in², corresponding approximately to an 8in. effective diameter, the total force available to produce the necessary valve change is $50 \times 0{\cdot}05 = 2{\cdot}5$lb.

An arbitrary ruling is that the hysteresis should not exceed 1% of the full range of operating force. In the present example this would be 1% of 600lb = 6lb This is greater than the 2·5lb required to move the valve stem for 1°C temperature change. One could increase the diaphragm area to 100 or more in² which would decrease the relative effect, but this is not necessarily the solution. One statement by R. B. Werey[6] perhaps summarizes the situation, "When used for throttling control service, the conventional spring and diaphragm motor, as previously stated, can hardly be classified as a power unit. Actually, it is merely a set of balanced forces whose potential power diminishes practically to zero as the width of the proportional band adjustment in the control instrument is increased. Any attempt to improve this potential operating force by merely enlarging diaphragm areas is illogical and unsatisfactory.

"The answer to the problem lies in the use of a valve positioner and a complete re-appraisement of motor design."

A typical valve positioner is shown in *Fig.* 229. The operation is relatively simple. The output signal from the controller is applied to the bellows X. Imagine an increase in pressure. The bellows extends against the spring and moves the upper lever. This in turn moves the baffle nearer to the nozzle of the relay valve. The pressure in the nozzle increases. It is amplified by the relay and is transmitted to the diaphragm of the control valve. The amplified signal moves the stem of the valve downwards. The intermediate graduated lever is connected to the valve stem or push rod and is carried downwards. By virtue of the adjustable pivot and the lower lever, it carries with it the upper lever and withdraws the baffle away from the nozzle. A position of equilibrium is eventually reached where the valve opening corresponds to the value of the controller signal.

Valve positioners can produce gains of the order of 200/1. The general effect is illustrated in *Fig.* 230 where a 10lb total force without positioner can reach a value of 2000lb with positioner. There is ample force available then to overcome any friction effects, etc.

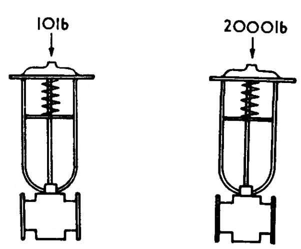


Fig. 230. *The effect of a valve positioner*

Rangeability

A valve possesses a minimum flow. Due to the clearances necessary to prevent binding and sticking, a complete shut-off is not necessarily achieved. The ratio of the maximum flow to the minimum flow is termed the rangeability of the valve. From the flow formulae, an expression connecting the valve rangeability, the flow rangeability, and the pressure drop rangeability may be produced.

$$R_V = R_F\sqrt{R_P} = \frac{V_{\max}}{V_{\min}}\sqrt{\frac{\Delta p_{\max}}{\Delta p_{\min}}} \quad \ldots \quad (368)$$

where R_P = pressure drop rangeability (maximum pressure drop/min pressure drop $\left.\dfrac{\Delta p_{\max}}{\Delta p_{\min}}\right)$

R_V = valve rangeability

R_F = flow rangeability (max flow/min flow $\left.\dfrac{V_{\max}}{V_{\min}}\right)$

The *normal* maximum flow should be distinguished from the maximum flow that the valve can carry. For example, the normal maximum flow may be very often about 70 per cent of the maximum, and the name *turndown* is given to the ratio of this to the minimum flow. If a flow rangeability of 20 is required between the normal maximum, and the minimum flow, the valve rangeability must be at least $\dfrac{100 \times 20}{70} = 29.$

These factors should be considered when determining valve diameters.

Effect of By-pass Valves

An installation as indicated in *Fig.* 231 is often included in process control, but the by-pass is normally closed. If the control valve has to be removed from service for any reason, the hand valves are shut, and the by-pass valve opened so that manual control may be substituted. If the by-pass valve is opened when the control valve is operating the results may be serious as the following figures show:

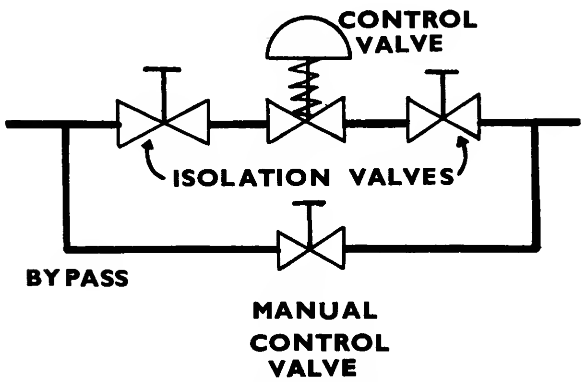


Fig. 231. *Control valve with by-pass*

Fraction of Flow in By-pass	Valve Rangeability	Turndown (70% Max. Flow)
0	40	28
$\frac{1}{10}$	14	9·8
$\frac{1}{4}$	4	2·8
$\frac{1}{2}$	2·5	1·75

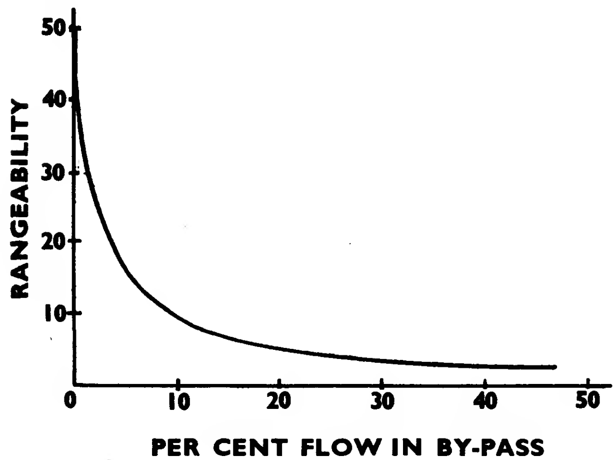
Not only are the rangeability and turndown vastly modified, but the actual flow characteristic is altered. *Fig.* 232 illustrates the general effect.

The isolating valves, normally fully open, must be the full line size and offer very low resistance to flow. The control valve is normally smaller than the line, and the by-pass valve should be of the same size as the control valve.

Effect of Line and Fitting Resistance

The pressure loss in the lines leading to and from the control valve must be taken into account in determining the drop through the valve. Knowing the full pressure differential available, the pressure drop in the lines should be known or calculated. The characteristic changes with the ratio of pressure drop across the valve to the total pressure drop available. A serious change in characteristic of the valve arises as the ratio becomes smaller. A rough working rule is for the valve pressure drop to be about 70 per cent of the available differential.

Fig. 232. *Effect of by-pass on rangeability*



Valve Characteristic Curves

The characteristic curve is usually presented as the percentage of total valve lift (i.e. shaft travel) plotted against the percentage of total or maximum flow. The orifice opening is a function of the lift, and the flow characteristic may be considerably varied by shaping the plunger to different contours or by making the plunger of the hollow ported pattern.

Linear Characteristic

In this, an effort is made to achieve equal increments in flow for equal increments in lift, at a given pressure drop. The curve is shown in *Fig.* 233. A dead straight line is difficult to obtain practically, but the normal characteristic is near enough for most control purposes. The plunger shape required is parabolic, or V-ported.

Equal Percentage Characteristic

For a linear characteristic, δQ, the increment in flow is proportional to an equal increment in lift, δl, i.e.

$$\delta Q = k_1 \delta l \qquad (369)$$

For equal percentage valves, however, equal *percentage* increments in flow are produced for equal increments of lift:

$$\frac{\delta Q}{Q} \times 100 = k_2 \delta l \qquad (370)$$

Integrating both sides of this equation, gives

$$\log_{10} Q = k_3 l \qquad (371)$$

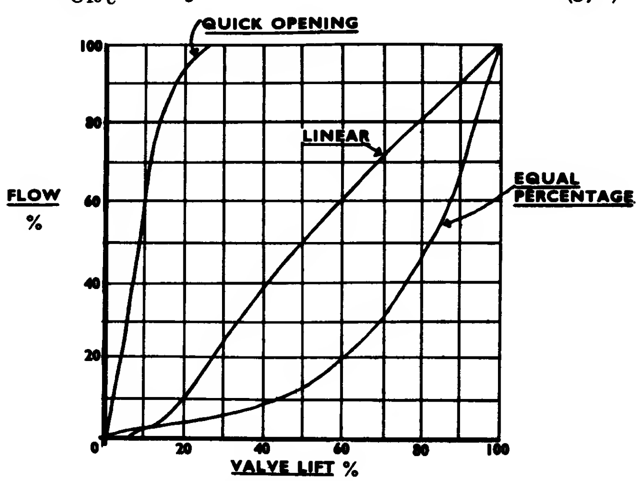


Fig. 233. *Valve characteristics*

Thus a semi-logarithmic characteristic is required. It is found that a plunger with a general parabolic contour is suitable for obtaining a flow-lift curve which approximates very closely to a logarithmic characteristic. V-ported plugs with specially shaped ports are also included in the design of equal percentage valves.

Figs. 234 and 235 show commercial valve designs with different plugs.

Quick Opening Valves

These valves employ bevel disc types of plugs and the flow-lift curve appears as in *Fig.* 233.

Weir Pattern Valve

This valve is rather different to those discussed above. It makes use of a non-metallic diaphragm

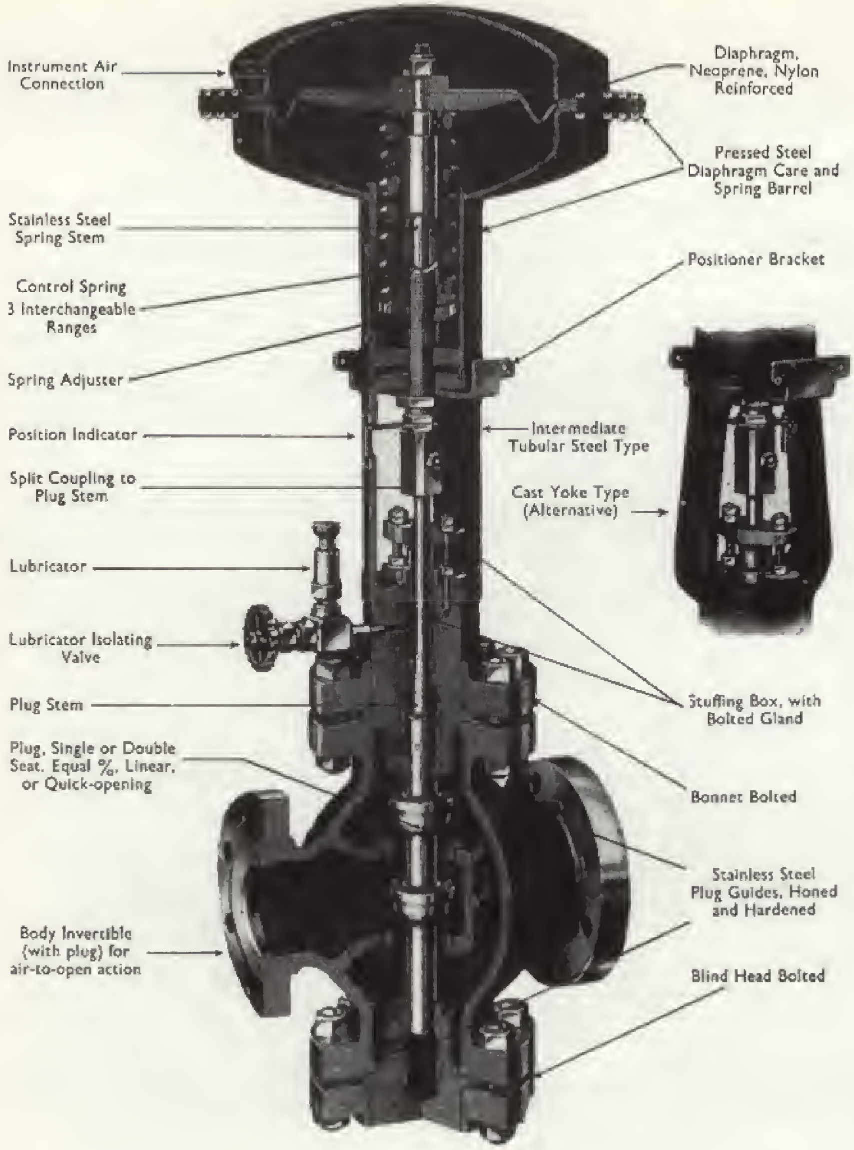


Fig. 234. The construction of a commercial double seated control valve. Reproduced by permission of J. Blakeborough and Sons Ltd.

instead of the normal plugs. It has considerable application in such industries as the manufacture of acids, alkalies, food, etc., where corrosion is likely to take place (*Fig.* 236).

The particular application of the foregoing valves in control work depends on the nature of the process and control demanded. In a very broad classification, quick opening valves would be used for two-step control, equal percentage types for one, two or three term control, and linear patterns for special plants.

Butterfly Valves

The diaphragm type of valve is manufactured in sizes ranging from ⅜in. diameter to about 18in. For larger diameters, it is desirable to utilize the butterfly model, with the proviso that low pressure drops only may be tolerated. If the construction is examined in *Fig.* 235, it will be seen that the valve consists of a disc rotating in the cylindrical bore of the valve, the axis of rotation being a diameter of the cylinder. It can be realized without much difficulty that as the disc rotates to close or open the valve, a large differential would set up high unbalanced forces.

Louvre Damper

The louvre is composed of a series of adjacent rectangular vanes, and is used for the control of air and gases. There are different models: in one their blades all rotate in the same direction, in another, the blades are equipped with guides to improve the flow characteristic, and in a third type, the blades may be made to operate in opposite directions.

Calculation of Control Valve Sizes

The control valve as an orifice device has a flow equation fundamentally the same as the standard orifices described in Chapter 4. A shorter form is used here.

For liquids,

$$L = C_L d^2 \sqrt{\frac{p_1 - p_2}{\rho}} \qquad \ldots\ldots (372)$$

For gases,

$$Q = C_G d^2 \sqrt{\frac{(p_1 - p_2)\, p_1}{\rho T}} \qquad \ldots\ldots (373)$$

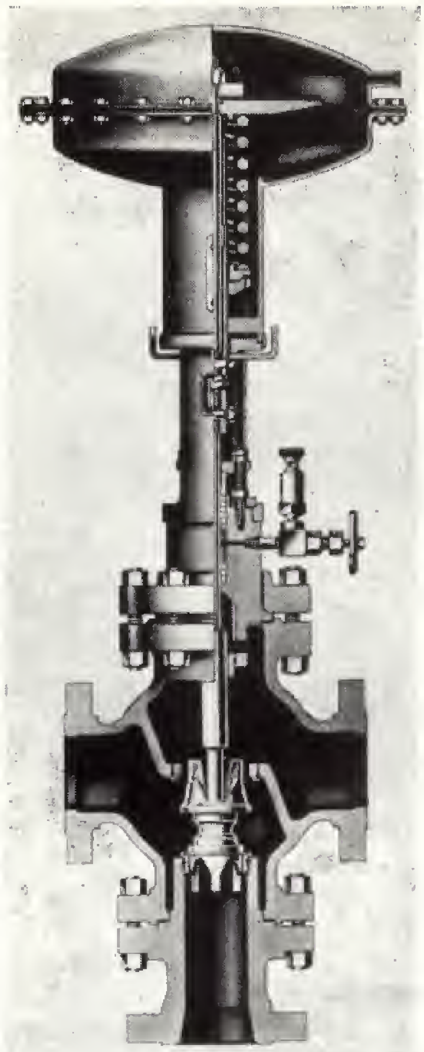


Fig. 235. The construction of a three way control valve with characterized ports. Reproduced by permission of J. Blakeborough and Sons Ltd.

For steam,

$$W = C_s d^2 \sqrt{\frac{(p_1 - p_2)}{V}} \qquad \ldots\ldots (374)$$

where L = liquid flow in gallons/minute or hour
C_L = discharge coefficient (liquids)
p_1 = upstream pressure lb/in^2
p_2 = downstream pressure lb/in^2
d = valve diameter inches
ρ = density lb/ft^3
Q = gas flow in c. ft/minute or hour
C_G = discharge coefficient (gases)
T = absolute temperature (°F)
W = flow of steam in lb/hour
C_s = discharge coefficient (steam)
V = specific volume in c. ft/lb.

C_v Values

An alternative method of calculating valve sizes makes use of a measured hydraulic valve coefficient, known as the "C_v value". This is the value obtained for the flow of water through a fully open valve, when the pressure drop is maintained at 1lb/in^2. Then a general expression for other liquids and pressure drops is:

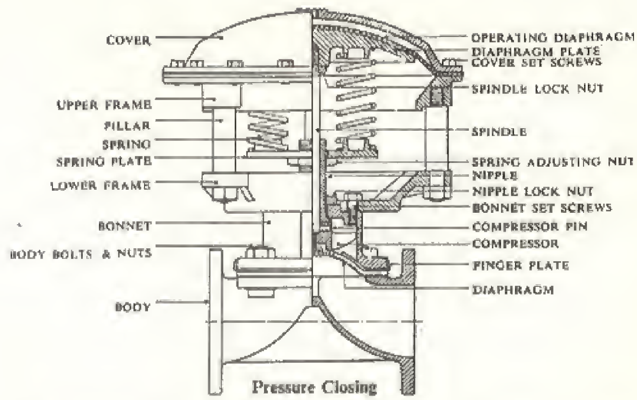


Fig. 236. Weir type of control valve (Saunders Valve Co. Ltd.)

$$V = C_v \sqrt{\frac{\triangle P}{SG}} \qquad \ldots\ldots\ldots\ldots (375)$$

or

$$C_v = V \sqrt{\frac{SG}{\triangle p}} \qquad \ldots\ldots\ldots\ldots (376)$$

where V = the flow rate in gals/min
SG = the specific gravity of the liquid
$\triangle p$ = the pressure drop in lb/in^2.

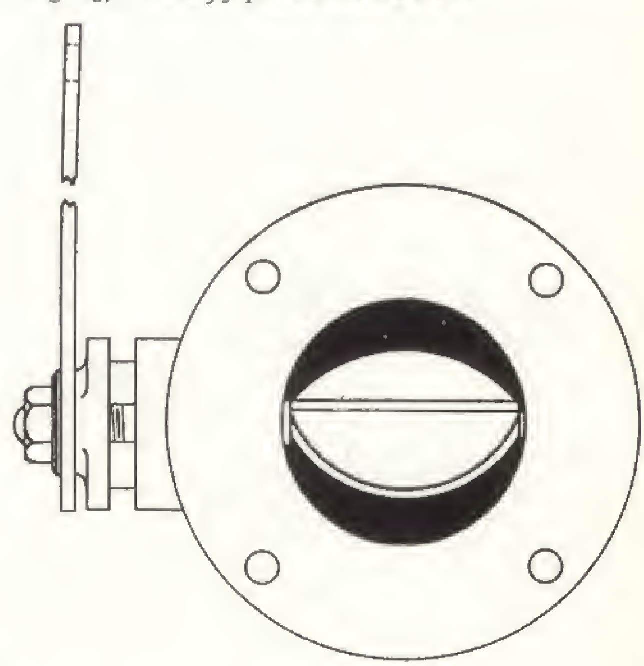
The flow is expressed in gallons per minute, and since the Imperial gallon differs from the U.S. gallon (1 imp. gal = 1·2 U.S. gal), *there is a difference in British and U.S. hydraulic valve coefficients for similar valves.*

For gases, it is customary to consider the rate of flow in terms of a normal pressure and temperature. This is usually 14·7lb/in^2 absolute and 60°F.

The formula can be expressed in a number of ways

$$Q = K_1 C_v \sqrt{\frac{\triangle p}{SG}\left(\frac{P_b T_f}{P_f T_b}\right)} \qquad \ldots\ldots (377)$$

Fig. 237. Butterfly pattern control valve



$$Q = K_2 C_v \sqrt{\frac{\triangle p}{SG}\left(\frac{P_f}{T_f}\right)} \quad \ldots\ldots (378)$$

$$Q = K_3 C_v \sqrt{\frac{\triangle p}{SG}\left(\frac{P_f T_b}{T_f}\right)} \quad \ldots\ldots (379)$$

where Q = the rate of flow
$\triangle p$ = the pressure drop
P_f = the pressure under operating conditions.
P_b = the basic pressure, i.e. 14·7lb/in²
T_f = the temperature under operating conditions
T_b = the basic temperature. i.e. 60°F or 512°F absolute
$S.G.$ = the specific gravity of the gas
K_1, K_2, K_3 = constants.

The correct value of P_f needs a little consideration. If the upstream pressure value is taken, the calculated flow is higher than the actual flow. If the downstream value is taken, the calculated value is too low. An approximation often used is the average of the upstream and downstream pressures, i.e. $P_f = \left(\frac{P_1 + P_2}{2}\right)$.

For vapours the following equations can be obtained, in a similar manner, expressing the rate of flow (W) in lb/hr and the specific volume V_s in cu ft/lb.

$$W = K_4 C_v \sqrt{\frac{\triangle p}{V_s}} \quad \ldots\ldots\ldots (380)$$

or

$$C_v = \frac{W}{K_5}\sqrt{\frac{1}{\triangle p\, W_v}} \quad \ldots\ldots\ldots (381)$$

where W_v = vapour density.
K_4, K_5 = constants.

Books, etc., for Further Reading

1. CHARNLEY, E. J. Section XVI, Instrument Manual. United Trade Press. 1960.
2. YOUNG, A. J. An Introduction to Process Control System Design. Longmans Green. 1955.
3. JONES, E. B. Instrument Technology. Vol. III. Butterworths. 1957.
4. ECKMAN, D. P. Automatic Process Control. John Wiley. 1958.
5. AIKMAN, A. R.; WALTON, H. R.; BALLS, B. W. A Symposium on Control Valves. *Trans. Soc. Inst. Tech.* Vol. 5, No. 1, March, 1953.
6. WEREY, R. B. Instruments. Vol. 23, No. 8, Aug., 1950, p. 784.

QUESTION

1. The true gas fluid capacitance C_1 of a closed vessel is given by:

$$C_1 = \frac{V}{n\,RT} \quad ft^2$$

where V = the volume in ft³.
n = a constant.
R = the gas constant 53·3ft/deg.
T = the absolute temperature Fahrenheit.

Calculate C_1 for a metal bellows with a mean diameter of 1in. and length 2½in. Take n as 1·1, R as 53·3ft/deg. and T as 60°F = 520°F absolute. ($C_1 = 0{\cdot}410 \times 10^{-8}$ ft²).

The true resistance R_1 of a restriction for laminar flow is given by:

$$R_1 = \frac{128\ \delta\ L}{g\ \pi\ D^4} \quad \text{sec/ft}^2$$

where δ = the kinematic viscosity of the gas flowing through the restriction ft²/sec.
L = the length of the restriction in ft.
D = the inside diameter of the restriction in ft.
g = 32·2ft/sec².

Calculate R_1 for a length 2in. of capillary tubing 0·01in. internal diameter. Take δ as $16{\cdot}8 \times 10^{-5}$ft²/sec. ($R_1 = 7{\cdot}35 \times 10^7$sec/ft²).

Hence find the value of the time constant $R_1 C_1$ (0·30sec).

British Standards

B.S.1523. Glossary of Terms used in Automatic Controlling and Regulating Systems.

Section 2. Process Control.

In addition to Section 2, other sections of B.S. 1523 have an interest for general control purposes:

Section 3: Kinetic Control.

Section 5: Components of Servo-mechanisms.

B.S. 3512. Method of Evaluating the Performance of Pneumatic Transmitters with 3-15lb/in² (Gauge) Output.

B.S. 2643. Terms relating to the Performance of Measuring Instruments.

Chapter 15

FIRST AND SECOND ORDER LAGS AND MISCELLANEOUS CONTROLS

TIME LAGS

THE controllability of a plant or process depends to a vital extent on the delay occurring between a change in conditions at one point in a plant and the manifestation of this change at another point. The important delays or time lags can be classified under two general headings: Distance Velocity Lags and Transfer Lags. Consider Distance Velocity Lags first.

Distance-velocity Lags

Distance-velocity lag is defined by B.S. 1523, Section 2, as "The time interval between an alteration in the value of a signal and its manifestation *unchanged* at a later part of the system, arising solely from the finite speed of propagation of the signal." The tendency is for the term to be restricted to cases where the medium is moving. An example is furnished by *Fig.* 238. The detecting element, a thermometer is situated at a distance L_1 from the plant in pipeline. A medium flows from the plant at a velocity Vft./second. The time for any signal, arising from a change in process conditions, to reach the thermometer is $\frac{V}{L_1}$ seconds.

If, for example, L_1 = 200ft., and V is 5ft./second, the distance-velocity lag is 40 seconds.

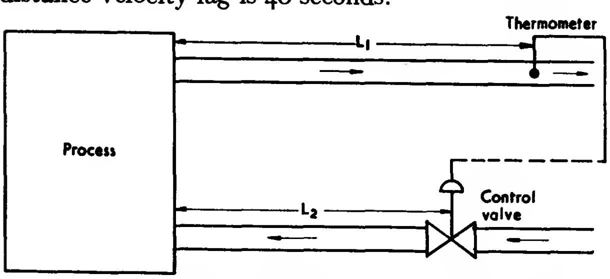


Fig. 238. *Distance-velocity lags in a control loop*

A distance-velocity lag can also occur when the correcting unit is at a large distance from the process. If the valve in *Fig.* 238 is away from the process to any great extent, a delay is experienced between the adjustment of the valve and the new flow value arriving at the plant.

Transfer Lags

In the case of transfer lags, these must be considered at more than one point in the control loop. Detecting elements are responsible for some significant transfer lags and since they respond directly to changes in the values of the controlled conditions we may usefully consider them before the plant or process lags. The meaning of first and second order lags will become apparent as the chapter proceeds.

First Order Lags: Detecting Element

Most serious detecting element lags can occur in a temperature control system. A transfer of heat is necessary from the medium whose temperature is being controlled, through the walls of a pocket or bulb to the sensitive element or filling. Any lag involved, therefore, is strictly speaking a transfer lag. The temperature detecting element is examined at some length because its lag can be detrimental from a control point of view. Consider the average element. If it is electrical—either thermocouple or resistance type—it is enclosed in a metal pocket with, very often, an air space between the element and the inner wall of the pocket. If it is a liquid or gas expansion, or

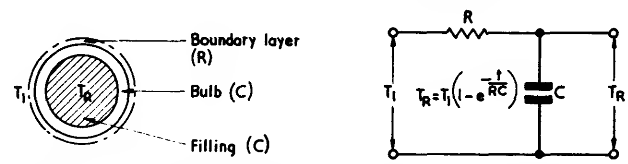


Fig. 239. *A mercury-in-steel bulb as a first order system*

vapour pressure type, it consists of a metal bulb with the appropriate filling and again may be inserted in a pocket with an intervening air annulus. An analogy may be traced with a simple electrical circuit consisting of resistance and capacity. In the case of a mercury-in-steel bulb, for example, there is resistance—thermal resistance—formed by the boundary layer of fluid on the outer surface of the bulb. Capacitance, in the form of thermal capacitance, exists in the bulb and liquid filling (*Fig.* 239). Such a bulb can be approximated to a resistance in series with a capacitance. If a sudden or step temperature change is applied to the bulb, following the electrical analogy, an equation may be established:

$$RC\frac{dT_R}{dt} + T_R = T_1 \quad \dots\dots\dots\dots \quad (382)$$

where

R = the thermal resistance of the boundary layer
C = the total thermal capacitance of the bulb and filling
T_R = the temperature at any time t after the application of the temperature change
T_1 = the final steady or equilibrium temperature

The solution of the equation is:

$$T_R = T_1(1 - e^{-\frac{t}{RC}}) \quad \dots\dots\dots \quad (383)$$

RC is known as the time constant of the bulb, and is actually the time for the temperature to reach 63·2% of its final value T_1. *Fig.* 240 shows the exponential type curve obtained by plotting T_R against

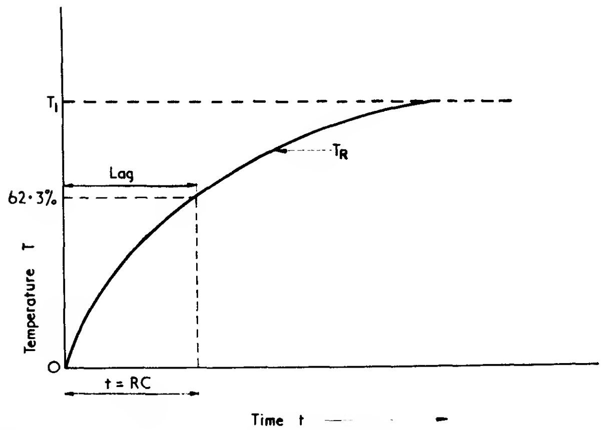


Fig. 240. *Response to a step change*

t, clearly showing the nature of the lag. The description "exponential lag" is frequently applied. Tests are very often carried out on simple temperature indicating systems by plunging the element into a bath at a higher temperature and observing the progress of the indicated temperature with time. The time for the temperature to reach the 63·2% mark is then taken as a measurement of the lag of the temperature element.

Alternatively suppose that the bulb is exposed to a medium whose temperature is changing, for simplicity, at a constant rate, i.e.

$$T_1 = At \quad \ldots\ldots\ldots\ldots \quad (384)$$

where A = a constant.

Substituting for T_1 in equation (382) and solving gives

$$T_R = A\left(t - RC(1 - e^{-\frac{t}{RC}})\right) \quad \ldots\ldots \quad (385)$$

But $T_1 = At$

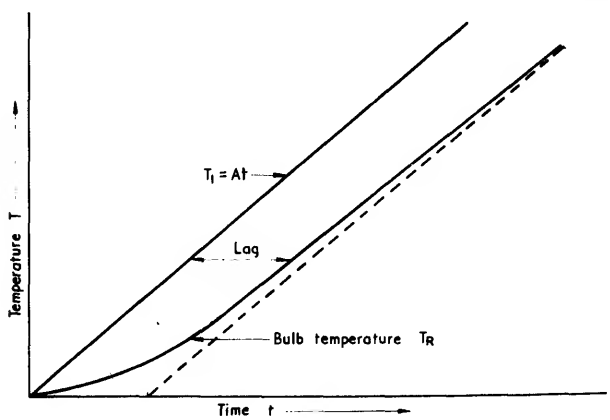
Hence

$$T_R = T_1 - ARC(1 - e^{-\frac{t}{RC}}) \quad \ldots\ldots \quad (386)$$

The lag is given by $(T_1 - T_R)$ and this ultimately equals a constant value RC. The curves of temperature and time appear in *Fig.* 241. $T_1 = At$ is a straight line. The bulb temperatures commence with a pronounced curve, and then tend to straighten out as the constant lag value is reached.

Then we must consider the behaviour of the element to a sinusoidal change in the temperature of the medium in which it is placed (see *Fig.* 242).

Fig. 241. *Response to a constant rate or ramp change*



Let the variation proceed according to the equation:

$$T = T_0 \sin \omega t \quad \ldots\ldots\ldots\ldots \quad (387)$$

where T_0 = the amplitude of the change
T = the temperature at any time t during the cycle
ω = the angular frequency = $2\pi f$, where f is the frequency

The basic equation is:

$$RC\frac{dT_R}{dt} + T_R = T_0 \sin \omega t \quad \ldots\ldots \quad (388)$$

It is not necessary to present the full treatment of the solution of this equation since it may be found in most text books on automatic control or electrical engineering.

The solution in which we are interested is:

$$T_R = \frac{T_0}{\sqrt{1 + \omega^2 (RC)^2}} \sin(\omega t - \phi) \quad \ldots \quad (389)$$

The amplitude has suffered an attenuation and is now

$$\frac{T_0}{\sqrt{1 + \omega^2 (RC)^2}} \quad \ldots\ldots\ldots\ldots \quad (390)$$

There is a lag and it is given by:

$$\tan \phi = \omega RC \text{ or } \phi = \tan^{-1} \omega RC \quad \ldots \quad (391)$$

From these expressions it can be seen that both frequency and the value of the time constant can influence the response to a significant extent.

Example

The thermal capacitance of an element is given by the product of its mass M and its specific heat *S*. The values are preferably in c.g.s. units. In the example of a mercury-in-steel bulb, the capacitance comprises two parts: that due to the bulb itself and that due to the mercury filling inside the bulb.

Bulb o/d = 1·4cm
Bulb i/d = 1·0cm
Bulb Length = 10·0cm
Vol. V_1 = 7·54cc
Density bulb ρ_1 = 7·90g/cc
material (steel)
Specific heat of S_1 = 0·107cal/s
bulb material (steel)
Thermal capacity of bulb
$C_B = V_1 \times \rho_1 \times S_1 = 7{\cdot}54 \times 7{\cdot}90 \times 0{\cdot}107 = 6{\cdot}37$ g. cal/°C
Vol. bulb liquid V_2 = 7·85cc
(mercury)
Density bulb liquid ρ_2 = 13·56g/cc
(mercury)
Specific heat of bulb S_2 = 0·033cal/g
liquid (mercury)
Thermal capacity of filling liquid
$C_F = V_2 \times \rho_2 \times S_2 = 7{\cdot}85 \times 13{\cdot}56 \times 0{\cdot}033 =$ 3·51g.cal/°C.
Thermal capacity C_1 of bulb and filling =
6·37 + 3·51 = 9·88g.cal/°C.

The calculation of the thermal resistance due to the boundary layer is not so simple as several factors are involved. For a given fluid, the speed of the fluid past the bulb, the Reynolds number, the Prandtl number and the Nusselt number are all involved. Making several assumptions regarding constancy of

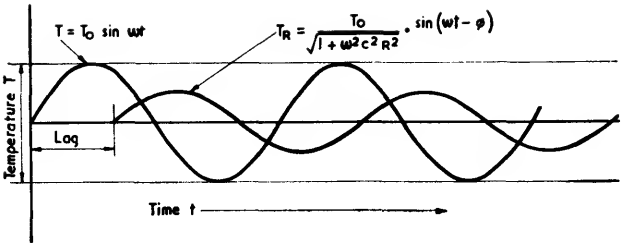


Fig. 242. *Response to a sinusoidal change*

certain factors, the thermal resistance R_1 can be shown to be:

$$R_1 \alpha (d\, v)^{-0.6} \qquad \ldots\ldots\ldots\ldots \quad (392)$$

where d $=$ *o/d* bulb *cms*

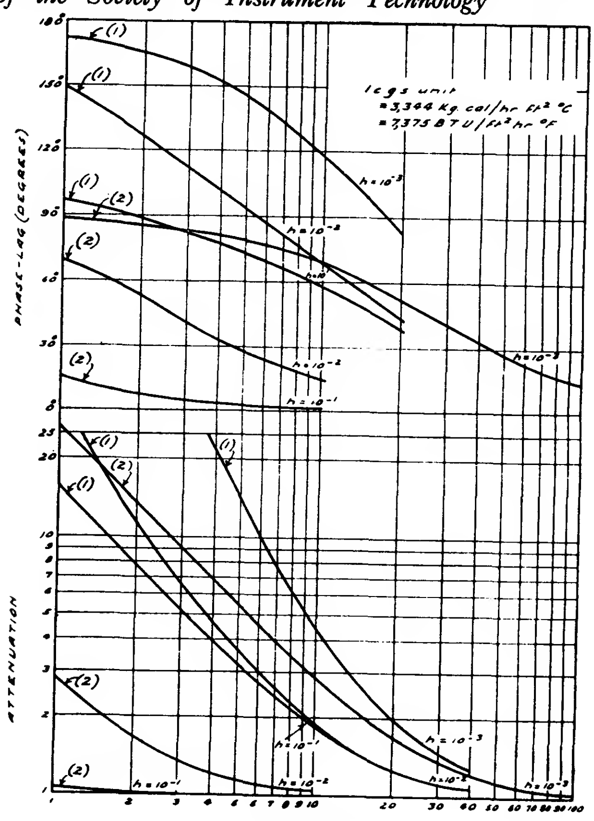
v $=$ speed of fluid past bulb *cm/sec*

Let us take a calculated value for R_1 of 0·70sec/g. cal/°C at a fluid speed of 2ft/sec or 120ft/min.

Then the time contant $=$
$R_1C_1 = 0{\cdot}70 \times 9{\cdot}88 = 6{\cdot}93$sec

The subject has been fully investigated by several research workers notably by Fishwick[1] and Aikman[2] in this country. From the latter reference *Fig.* 243 is produced. The curves are calculated from formulae of the type in equations (389) and (391). The curves emphasize the nature of the phase lag and attenuation for two different types of resistance thermometers in protecting sheaths. *h* on the graphs is a convection film heat transfer coefficient from fluid to sheath.

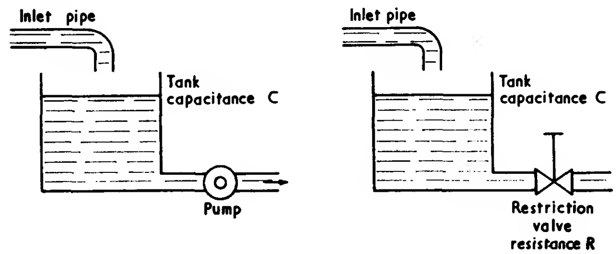
Fig. 243. *Calculated frequency response for two resistance thermometers in sheaths. Reproduced by permission of the Society of Instrument Technology*



First Order Lags: Plant Or Process

Figs. 244(a) and (b) illustrate two very simple types of process. In *Fig.* 244(a) the tank is filled by the inlet pipe and liquid is drawn off by the positive displacement pump in the outflow pipe. In *Fig* 244(b) on the other hand, the rate of outflow depends entirely on the difference between the pressure exerted by the head of liquid in the tank and the pressure existing on the downstream side of the restriction valve.

Let us see what happens in both examples when there is a sudden or step increase in the rate of inflow. Initially the rates of inflow and outflow are considered equal and the levels constant.



(a) (b)

Fig. 244(a) and (b). *Simple first order processes with pump and restriction in outflow line*

In *Fig.* 244(a) the rate of outflow created by the pump remains unaltered and the level commences to rise and continues to rise at a constant rate. The rate is proportional to the increase in inflow rate. Now the tank has a fluid capacity of *C* and its value is equivalent to the area of the tank at the level of the liquid. In a tank of uniform cross section this remains constant at all levels. If *Q* is the increase in inflow rate and *h* the head of liquid at any time *t*,

$$\frac{dh}{dt} \alpha \frac{Q}{C} \qquad \ldots\ldots\ldots\ldots \quad (393)$$

With such a system there cannot be any new equilibrium value.

In *Fig.* 244(b) the initial effect is for the level to rise with the new inflow rate. But this increases the head of liquid in the tank. Assuming that the pressure on the down stream side of the valve remains constant, the new head value initiates a bigger outflow rate. A position of equilibrium is eventually reached where the head is sufficient to produce an increased outflow rate equal to the increased inflow rate. There is, in fact, inherent regulation in such a system whereas that of *Fig.* 244(a) possesses none. By inherent regulation we mean the property of a process to reach equilibrium after a disturbance (such as the increase in inflow rate in *Figs.* 244(a) and (b) without the aid of a controller.

Fig. 244(b) possesses a capacity *C* and a fluid resistance *R*. An equation similar to (382) can be derived:

$$RC\frac{dh}{dt} + h = h_1 \qquad \ldots\ldots\ldots\ldots \quad (394)$$

where h_1 $=$ the new equilibrium value of the head.

The solution is

$$h = h_1\,(1 - e^{-\frac{t}{RC}}) \qquad \ldots\ldots\ldots \quad (395)$$

This bears a close similarity to (383).

The increase in level is of the same form as *Fig.* 240, and is of the same form i.e. an exponential lag.

For decreases in inflow rate the level falls at a constant rate for *Fig.* 244(a). For *Fig.* 244(b) there is a

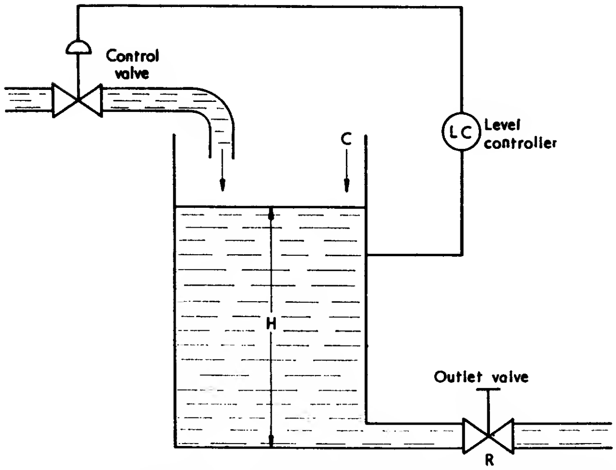


Fig. 245. *Simple first order process with level controlling the inflow rate*

new equilibrium head less than the original one, where the fall in outflow rate equals the fall in inflow rate. The decrease in level is given by the general equation:

$$h = h_1 e^{-\frac{t}{RC}} \qquad \ldots\ldots\ldots\ldots (396)$$

The opposite action could occur with the inflow rate remaining constant and the outflow rate changing due to a varied demand. But it is conceivable that both could change together, and the question of introducing automatic control arises. An elementary control scheme is outlined in *Fig.* 245, where the rate of inflow is controlled by the value of the liquid level. Even for this relatively simple scheme the control equations are involved. Eckman[3] has analyzed the controllability of *Fig.* 245 for the four main controller actions: two step, proportional, proportional plus integral and proportional plus derivative. Here, it will be only feasible to refer to the action of two step control with overlap. *Fig.* 210 of Chapter 13 indicates the general behaviour of two step control without overlap. *Fig.* 246 indicates the action with overlap present. For a single capacitance process similar to *Fig.* 244(a), with pump operated outflow, Eckman shows the amplitude A of oscillation to be:

$$A = G + \frac{ML}{2C} \qquad \ldots\ldots\ldots\ldots (397)$$

where A = the amplitude
G = the overlap
M = maximum value of the inflow
L = the value of the dead time which is considered to be present
C = the capacity of the tank

The period of oscillation is given by:

$$T = \left(L + \frac{2GC}{M}\right) \frac{M^2}{U(M - U)} \qquad \ldots (398)$$

Here U = the rate of outflow

If the frequency of oscillation is to be reduced with overlap, or what is equivalent, the period made large, M must be small and C large. This is observable in equation (398). *Fig.* 247 shows the nature of the oscillation.

Example

Consider a tank of area 4ft² with a maximum outflow rate of 2ft/sec. The dead time is 2sec. Find the amplitude and period of oscillation for two step controller action.

$$\text{Amplitude } A = G + \frac{ML}{2C}$$

G = 2in = 1/6ft.
L = 2sec
C = Area of tank = 4ft²
M = 3ft/sec (As a rough working rule the maximum inflow rate is taken as 50% more than the maximum outflow rate)

$$A = \frac{1}{6} + \frac{3 \times 2}{2 \times 4}$$

$$A = \frac{1}{6} + \frac{3}{4} \text{ ft.} = 11\text{in.}$$

$$\text{Period } T = \left(L + \frac{2GC}{M}\right)\left(\frac{M^2}{U(M - U)}\right) \text{sec}$$

U is 2ft/sec and L, G, C and M are the same as above.

$$T = \left(2 + \frac{2 \times 2 \times 4}{3 \times 12}\right)\left(\frac{3^2}{2(3 - 2)}\right)$$

$$T = \left(2 + \frac{4}{9}\right)\frac{9}{2}$$

$$T = 11 \text{ sec.}$$

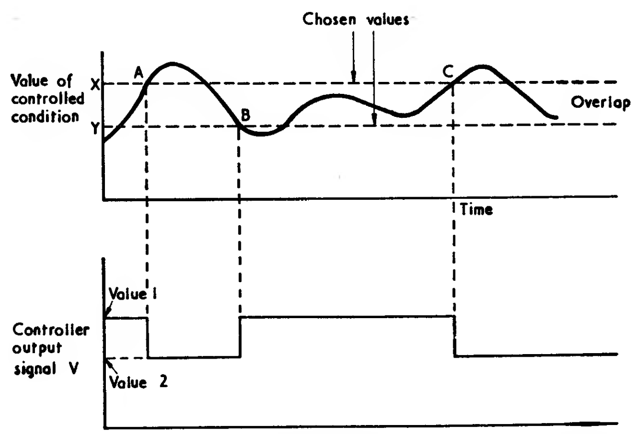
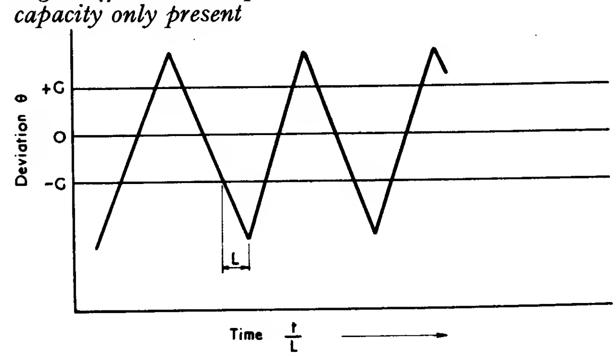


Fig. 246. *Two step controller action with overlap*

For the alternative process where the outflow passes through a restriction valve, the oscillations take the form of segments of exponential shape. This is to be expected from a system possessing a time constant RC.

Fig. 247. *Two step controller action with overlap capacity only present*



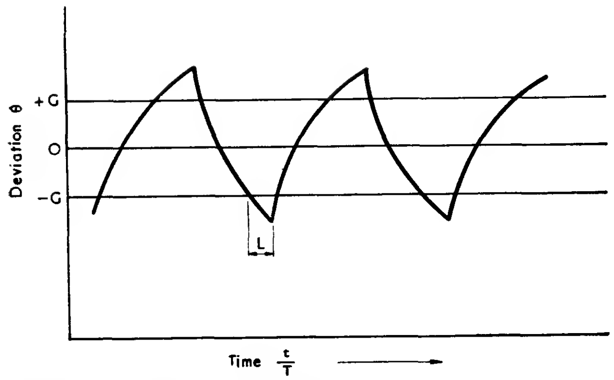


Fig. 248. *Two step controller action with overlap time constant present*

With level rising, Eckman[3] deduces the following equation:

$$\theta = (\theta_m + \theta_s)\,(1 - e^{-\frac{t}{RC}}) - \theta_s \quad . . \quad (399)$$

With level falling,

$$\theta = K\,(e^{-\frac{t}{RC}}) + \theta_a \quad . . . . . . \quad (400)$$

where θ = the deviation
$\theta_m = (RM + U - \theta_s)$
R = outflow restriction fluid resistance
M = maximum value of inflow
U = value of outflow at any time
θ_s = set value
K = a constant
C = tank fluid capacitance
$\theta_a = (U - \theta_s)$

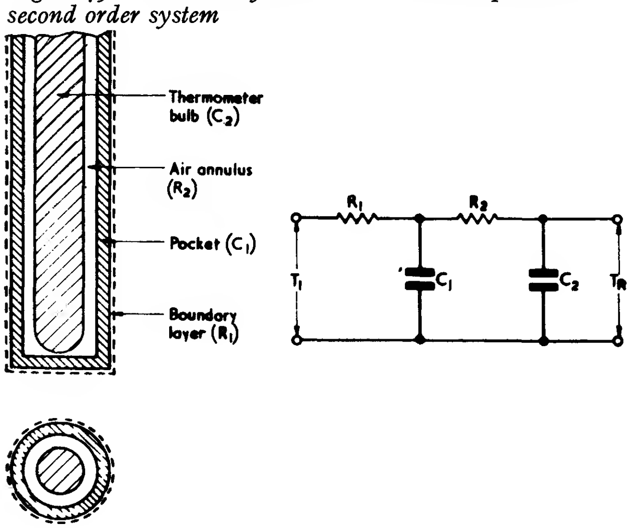
From *Fig.* 248, the dependence of the period of oscillation on the time constant $T = RC$ and the dead time L can be seen.

Before leaving first order systems one may perhaps draw attention to two aspects of proportional control. The offset θ_x can be expressed as

$$\theta_x = \frac{-U}{RK_1 + 1} \quad . . . . \quad (401)$$

From this we may observe that as the proportional action factor K_1 is increased, the offset is reduced. From equation (267) in Chapter 13, this is equivalent to reducing the proportional band, which confirms the deductions made in that chapter.

Fig. 249. *A mercury-in-steel bulb with pocket as a second order system*



The stabilization time T_s can be expressed as:

$$T_s = \frac{4\,RC}{RK_1 + 1} \quad . . . . \quad (402)$$

The broad definition here of stabilization time is the time for the value of the controlled condition to reach 98% of its new value. Again it can be seen that increasing K_1 reduces stabilization time.

Second Order Lags: Detecting Element

In *Fig.* 249, a mercury in steel bulb is shown with a protecting pocket. We have now a two stage system. There is the thermal resistance R_1 formed by the boundary layer on the outer surface of the pocket and a thermal capacitance C_1 formed by the pocket. Inside the pocket is a second thermal resistance R_2 formed by the air annulus between the inner surface of the pocket and the outer surface of the bulb. Finally, there is the thermal capacitance C_2 formed by the bulb and filling. Two individual time constants $R_1C_1(L_1)$ and $R_2C_2(L_2)$ are involved.

From the electrical analogy at the right hand side of *Fig.* 249, there is interaction between the two stages. This is occasioned by the fact that the temperature drop across the first stage R_1C_1 depends on the rate of heat flow through the second stage R_2C_2. One can deduce equivalent independent stages. Assigning the symbols L'_1 and L'_2 to these, their values may be determined from the following relations:

$$L'_1 + L'_2 = R_1C_1 + R_2C_2 + R_1C_2 \quad . . \quad (403)$$

$$L'_1L'_2 = R_1C_1\,R_2C_2 \quad . . . . \quad (404)$$

The relation between the temperature of the medium and the temperature as measured by the mercury in steel bulb at any instant for a step change has been calculated by ref. 4. It is given by:

$$T_R = T_1\left(1 + \frac{L'_1\,e^{-\frac{t}{L'_1}} - L'_2\,e^{-\frac{t}{L'_2}}}{L'_2 - L'_1}\right) \quad . . . \quad (405)$$

The operative factor here is the value of R_1C_2. If this is small compared with the values of R_1C_1 and R_2C_2 one can take the values of L_1 and L_2 with little error for evaluating the time constants and the response mode. But of the units involved, the value of R_1 can probably be the most influential, and this, again, depends on the annular width of the air pocket. The reduction of this to a minimum by careful design is obviously a fundamental requirement. Apart from this, however, attempts have been made to fill the space between the inner wall of the pocket and the bulb with a liquid, the improved thermal conductivity reducing the thermal resistance. In the case of resistance thermometers, in one design the element has been wound on a flat former, and heat conducting springs attached to the outside of the element so formed make good thermal contact with the inner surface of the pocket. A thermocouple can be particularly bad if, as is often the case, the couple is suspended inside the pocket with a relatively large air gap between it and the pocket. It is possible to ensure that the junction of the couple is in metallic contact with the inner surface of the pocket, but one should note that this may lead to earthing troubles. It is probably true to say that there is no universal panacea for this problem particularly with regard to the very large range of temperatures likely to be encountered in modern processes.

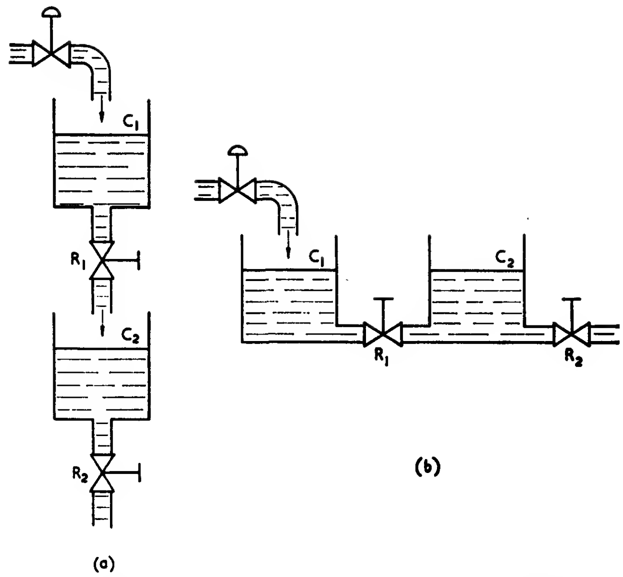


Fig. 250(a) and (b). *Simple two stage processes:*
(a). *No connection between stages*
(b). *Connection between stages*

Second Order Lags: Process or Plant

Figs. 250(a) and (b) represent very simple two stage processes. In *Fig.* 250(a) there is no physical connection between the two liquid level plants. In *Fig.* 250(b), on the other hand, there is a physical connection via the restriction valve R_1. The response of the system in *Fig.* 250(a) to a step change supply is given by:

$$\theta_2 = k\,\theta_1\left[1 - \frac{L_1L_2}{L_2 - L_1}\left(\frac{1}{L_1}e^{-\frac{t}{L_2}} - \frac{1}{L_2}e^{-\frac{t}{L_1}}\right)\right] \quad . (406)$$

where $L_1 = R_1C_1$
$L_2 = R_2C_2$
$\theta_2 =$ the output signal value
$\theta_1 =$ the input signal value

To illustrate further the effect of time constants in such a system it is instructive to consider a sinusoidal input signal

$$\theta = \theta_a \sin \omega t \text{ or } \theta = \theta_a \sin \frac{2\pi t}{T}$$

The amplitude A of the output signal is given by

$$A = \theta_a\left[\left(1 - \frac{4\pi^2}{T^2}L_1L_2\right)^2 + \frac{4\pi^2}{T^2}(L_1 + L_2)^2\right]^{-1} \quad . . (407)$$

The lag is

$$\phi = \tan^{-1}\left[\frac{\frac{2\pi}{T}(L_1 + L_2)}{1 - \frac{4\pi^2}{T^2}L_1L_2}\right] \quad . . (408)$$

In the example of *Fig.* 250(b) similar expressions may be obtained by substituting L'_1 for L_1 and L'_2 for L_2, L'_1 and L'_2 having the same values as in expressions (403) and (404).

Whilst liquid level systems have been taken for explanation purposes, the same reasoning can be applied to two stage time constant plants involving other physical conditions such as temperature. In the example of a two stage thermal process, the exponential lag of the process itself may well be augmented by the exponential lag of the detecting element.

Transmission Lags

We have two main forms of transmission lines in the process control field: electrical and pneumatic. In the case of electrical lines the transmission time for a signal is negligible even for large distances. For pneumatic pipe lines, however, finite times are experienced for the transmission of signals, and these, dependant on pipe internal diameter and length, can cause significant lags.

The subject has been investigated both theoretically and practically by J. E. Samson[5]. Treatment is not easy but the author shows that the pneumatic transmission line can be considered analogous to an electrical line if the following broad assumptions are made:

(a) Pressure disturbance values are small.
(b) There is no steady flow of air in the line.
(c) There is uniform distribution of the fluid resistance, capacitance and inductance parameters throughout the line.
(d) The load is lumped.
(e) The small disturbances that do occur do not vary the mean pressure level in the line.

In this work a pneumatic sinusoidal generator was used and at the receiving end various pneumatic valve motors were coupled to the line. The paper gives a large number of curves for phase displacement and attenuation or gain over lengths of line up to 1000ft.

Another author, M. Bradner[6], has experimented with step and constant rate input signals to the line. His results are expressed in terms of lags and lengths of line for various internal diameters. To illustrate the transmission lags likely to be experienced in practice we may quote some typical figures.

A pipe with internal diameter 0·305in. has a lag of 1sec for a length of 350ft, but this rises to 6sec for 1000ft and at 2000ft is no less than 20sec. The effect of decreasing the diameter is significant. For a pipe 0·188in internal diameter the lag is 0·8 for 100ft length, 4sec for 500ft, 13sec for 1000ft and reaches 40sec for 2000ft. Both pipe diameter and length are a matter for consideration in control plant design.

MISCELLANEOUS CONTROLS AND CONTROL FUNCTIONS

Programme Control

This is a control in which the set value is automatically changed from time to time in accordance with a pre-determined programme. In a number of instances a cam whose contour is cut to the time-value relation of the controlled condition is used as the basic control device, the follower adjusting the set value according to the required programme. The cam is normally driven by a motor similar to that which operates the recording chart, and it bears on it calibration markings the same as the chart. A cut chart is often pasted on the cam for this purpose.

Time-cycle Control

Here, one or more correcting units are actuated on a time basis only.

Self-actuated Controller

A self-actuated controller derives the necessary operating power solely from the controlled physical condition. An example is furnished by the gas regulator in *Fig.* 251. A tapping from the downstream side

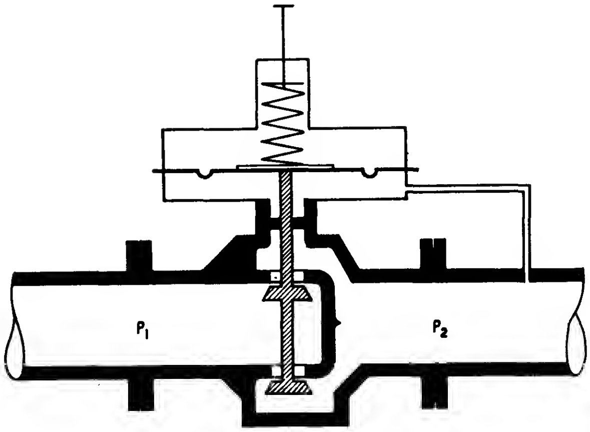


Fig. 251. *Self-actuated controller*

of the valve is taken to the diaphragm chamber. An increase in pressure raises the diaphragm and closes the valve orifice, whilst a decrease in pressure lowers the diaphragm and opens the orifice. The device acts as a controller with a small fixed proportional band to regulate the pressure.

Ratio Control

Ratio control is particularly applied to regulating two flow rates so that their ratio remains constant. *Fig.* 252 shows in simple diagrammatic form how this can be accomplished. *FT* is a flow rate transmitter operating from an orifice, nozzle or Venturi tube in one pipe line. This sends a signal to flow controller *FC* which is operating from an orifice etc in the second pipe line. The signal from *FT* is used to adjust the set value of the controller in such a manner that the desired ratio between the flow rates in the two pipe lines is maintained. As the set value is altered, the controller output signal is varied to provide the necessary new flow rate in the upper pipe line.

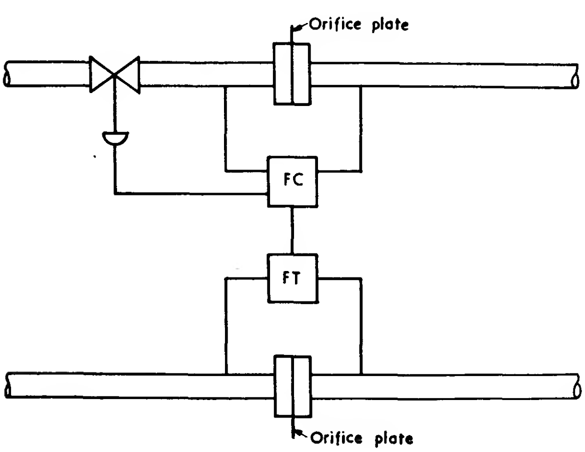
Cascade Control

In a cascade control system one controller alters the set value of one or more other controllers.

A typical example is offered by *Fig.* 253.

In *Fig.* 253 the stream feeding into the column comprises propane, isobutane, normal pentane, isopentane and natural gasoline. The sampling system

Fig. 252. *Flow ratio control*



is taken from the 9th tray of the column, and propane, normal pentane, isobutane, isopentane are the products of interest for analysis purposes. Controlling action, however, is effected by one component only—isopentane—whose peak area or height provides the signal for corrective action.

Two recorder controllers are involved in a double cascade operation. One is the temperature recorder controller whose measuring element is at the 15th tray and the other is the flow recorder controller which controls the steam flow in to the column heater. The cascade action can be seen from *Fig.* 253. A signal from the chromatograph is used to adjust the set value on the temperature recorder controller which in turn supplies a signal to alter the set value on the flow recorder-controller.

As a matter of interest, it is claimed that there was increased recovery of important components such as propane and butane and, coupled with reduced consumption of steam and cooling water, it was considered that the gas-liquid chromatograph paid for itself every six weeks on this particular installation.

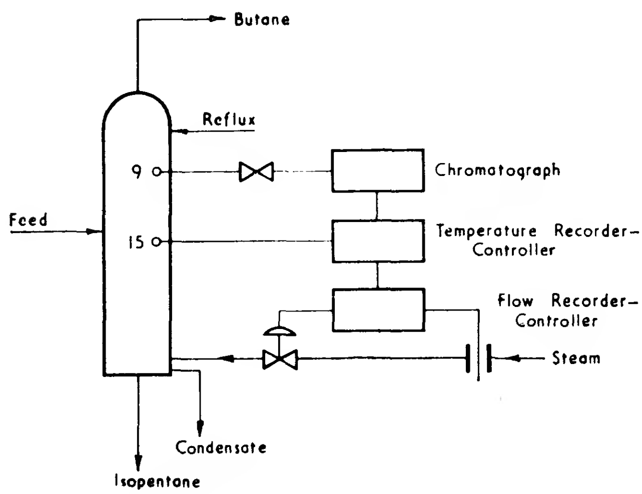


Fig. 253. *Cascade control*

The 9th tray was chosen since there was a higher concentration of components there and the instrument need not be too sensitive. It would have been preferable to analyze the overhead product since the 9th tray products may not always be correlated to this.

Dead Zone

The dead zone is a zone in which a change of value of input signal takes place without initiating any perceptible change in output signal. It can be due to friction between moving parts and to lost motion of linkages, etc. It must not rise to more than a small fraction of the scale range, e.g. it is desirable to restrict the value to less than 0·1% as an arbitrary value.

Dead Time

The dead time interval occurs between the reception of a change in signal and the initiation of a perceptible response. The control action will be delayed until the variable has reached the edge of the dead zone, if the system was previously balanced. *Fig.* 253 shows the effect of dead zone and dead time.

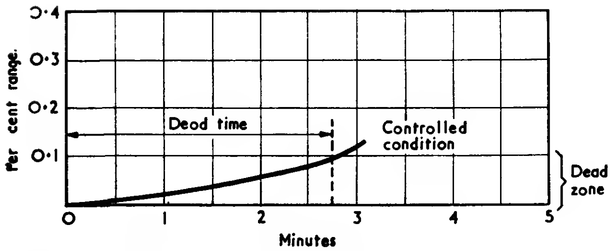


Fig. 254. *Dead zone and dead time*

Accuracy

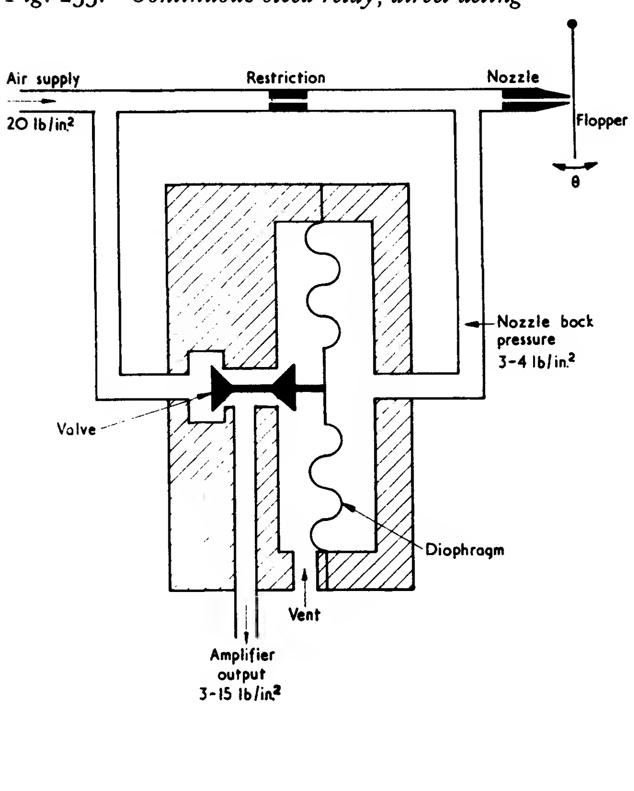
The basic meaning usually associated with accuracy is the nearness with which the measured or controlled condition is actually measured or controlled to the true value. The definition of accuracy has probably provided more argument than any other, but the foregoing will serve pending any official definition. Most manufacturers quote accuracy in one of three ways:

1. As a percentage of the instrument span, at any reading.
2. As a percentage of the maximum scale reading, at any reading.
3. As a percentage of the actual scale reading.

As an example, consider an instrument with a span of 400°F. to 1200°F. and with measurement at 900°F. Let us see what an accuracy of ± 1% means in the three cases.

1. *Accuracy a percentage of the instrument span*
   Instrument span 1200°F. — 400°F. = 800°F.
   ± 1% of 800°F. = ± 8°F. = error limits at any reading, e.g. 900°F.
2. *Accuracy a percentage of the maximum scale reading.*
   Maximum scale reading = 1200°F.
   ±1% of 1 200°F. = ±12°F. = error limits at any reading, e.g. 900°F.

Fig. 255. *Continuous bleed relay, direct acting*



3. *Accuracy a percentage of the actual scale reading*
   Scale reading = 900°F.
   ± 1% of 900°F. = ± 9°F.

It can be seen that the three forms of specifying accuracy lead to different limits of error.

Reproduceability

An important function of an instrument is its ability to reproduce its readings, i.e. the calibration should not change. A calibration alteration is not easy to detect because of its gradual occurrence, and only efficient maintenance and checking will prevent it. It can be due to several causes such as thermocouple and resistance thermometer contamination or metallurgical structure change, to a permanent set in a component such as a spring or a link, or to a stress in some instrumental member. Wear and erosion of orifice plates can also cause calibration changes.

This, of course, must influence the accuracy of measurement or control, but accuracy in the first place is concerned with a system before any calibration changes have occurred.

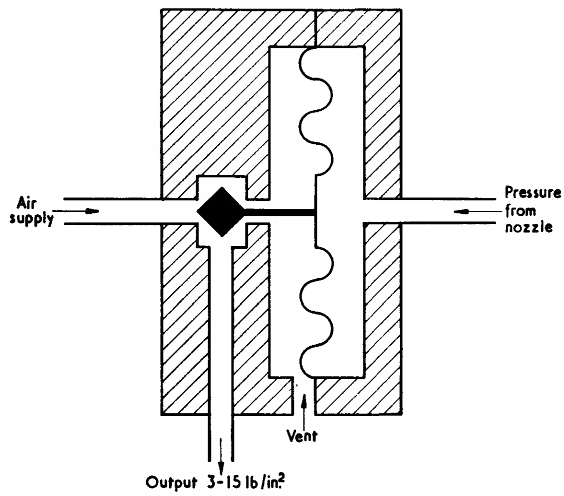


Fig. 256. *Continuous bleed relay, reverse acting*

PNEUMATIC RELAYS

With many nozzle and flapper units a pilot valve or relay is often associated. This reduces any lag which the feed-back bellows may have introduced, and is also necessary where a large volume of air is involved in the connecting piping to the control valve and in the diaphragm chamber of the valve itself. If the nozzle alone carries out the operation, all the air supply must come from it, and the inflation and deflation of the various volumes may take some time, introducing an undesirable lag. A relay valve is useful, therefore, in improving the response, and is in the nature of an amplifier of volume as well as pressure. The valves fall into two general classes: continuous bleed and non-bleed.

Continuous Bleed Type Relay

In *Fig.* 255 is a typical pattern of continuous bleed relay. It will be seen that the nozzle back pressure is fed into the diaphragm chamber and used to position the internal valve between its seatings. The pressure at the output is determined by the amount of air

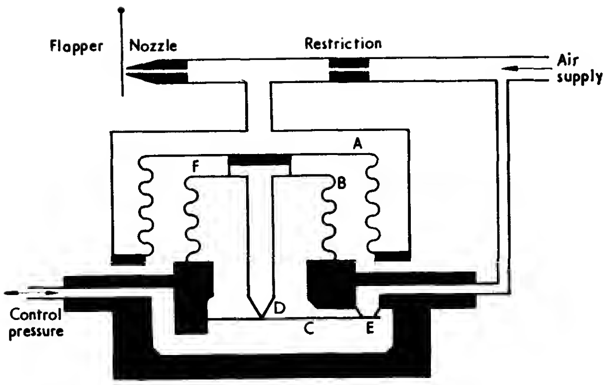


Fig. 257. *Non-bleed relay*

allowed to bleed to atmosphere via the vent. As a result there is a continuous consumption of air (typically 0·5cu ft min.).

The relay shown in *Fig.* 255 is of the "direct" acting type, i.e. an increase in nozzle back pressure results in an increase in output pressure.

Fig. 256 illustrates a "reverse" acting pattern and the diagram shows that the output pressure decreases as the nozzle back pressure increases.

Non-Bleed Type Relay

This title is only approximately correct. *Fig.* 257 indicates a typical design. The nozzle pressure is applied to the exterior of the large outer bellows *A*. The control line pressure is exerted on the interior of the small bellows *B*, and when the forces due to the two are equal, a balanced condition exists. The relay flapper *C* then covers both the exhaust nozzle *D* and the supply port *E*. Imagine the primary nozzle pressure to increase. The larger bellows is deflected downwards carrying the inner smaller one with it. *D* forces the flapper *C* away from the mouth of the supply port, but itself remains closed during this operation. Air is admitted to the control valve line and the interior of *B*. The force supplied by *B* increases and the bellows assembly is now moved upwards, proceeding until *C* arrives back in its original position, and the supply port is closed. On a decrease in pressure in the primary nozzle, the outer bellows moves upwards taking *B* with it. The exhaust nozzle now operates, since its mouth is uncovered by travelling away from *C*. Air bleeds away to atmosphere from the interior of *B* and the control valve line via exhaust port *F*. Pressure is reduced, and the bellows assembly begins to move downwards, continuing until nozzle *D* meets flapper *C*, and the exhaust passage is closed. Observe that only when a deviation occurs is there air consumption. The gain of such a relay is about 5, allowing, as in the other type, of a lower pressure in the primary nozzle.

A dead spot may occur in this pattern if a spring is used to provide a positive closing force for *C*. Any increase in nozzle pressure must overcome the spring pressure. The disadvantage can be reduced by making the spring tension a minimum, and using as large a bellows area as possible.

Transmitters and Distance Transmission

The nozzle and flapper position balance or the force balance unit can be used as a basis for a transmitter for remote signalling. They may also be used with transducers or converters to produce electrical signals at the transmitting end. Chapter 16 describes some common transducers.

The question will arise as to the choice of transmitting media. So many factors enter into the consideration that hard-and-fast rules cannot be laid down. The following remarks are for general guidance.

(1) In a plant liable to contain explosive or inflammatory vapours air has some advantages from the point of safety. With electrical or electronic instruments, unless these are certified flameproof, there is a certain risk. The flameproofing of instruments generally adds a fair amount to the price.

(2) Air transmission appears to have a limit of 1000-2000 feet. Above this figure, electrical transmission becomes necessary, and this can be carried out where large distances are involved (e.g., a few miles) over telephone wires or power lines, provided certain precautions are observed.

(3) External interference does not arise with air transmission. With electrical transmission, on the other hand, unless the system is deliberately designed so that any interfering signal is nullified, shielding of the lines may become necessary.

(4) If special lines have to be installed for electrical transmission, the cost of these as compared with a run of copper tubing should be borne in mind (particularly if the electrical transmission involves three or more wires). On the other hand, air-operated instruments require a compressor of fairly large capacity. If this is already on site to supply a number of other instruments in the plant, no extra cost is involved. A compressor especially for the distance transmission instruments may introduce a cost factor to their disadvantage.

(5) The time lags involved from the point of actual transmission time bias the media heavily in favour of electrical signalling.

(6) If installing an electrical transmission system, the effect of variation in supply voltage or frequency must be well considered.

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Chapter 16

TRANSDUCERS AND ELECTRONIC CONTROL SYSTEMS

INTRODUCTION

To operate an electronic controller one must have electrical input signals. Some of the detecting and measuring elements such as thermocouples, resistance thermometers and pH electrodes produce electrical signals directly, but others such as the non-electrical thermometers described in Chapter 8 and the flowmeters of Chapter 4 initiate non-electrical signals, and to use these with electronic controllers needs some form of converter or transducer. A transducer is simply a device which changes a signal in one form to a corresponding signal in another form. Strictly speaking, the nozzle and flapper unit is a transducer since it translates a displacement into a corresponding pressure signal. The two-phase servo-motor mentioned at the end of Chapter 9 could be classed under this heading since it converts an electrical signal into a displacement of the motor shaft. For our purposes in this chapter, however, we are concerned with transducers for converting non-electrical signals into electrical ones. The transducers most commonly used involve inductance, capacity or resistance. There are many types in each one of these classes and it will only be possible to call attention to one typical example in each category.

INDUCTIVE TRANSDUCERS

Differential Transformer

The differential transformer has found wide application as a means for converting a displacement, corresponding to the value of a controlled condition, to an electrical signal.

It comprises a transformer with an adjustable core. There are two secondary windings electrically equal and, indeed, wound adjacent to each other in the manner of *Fig.* 258 which shows the electrical arrangement of windings. When the core is symmetrically placed with the electrical centre of the transformer, the e.m.f.s E_1 and E_2 in the secondary windings are equal. As the core is moved away from the centre, however, one secondary e.m.f. becomes greater than the other. Referring to *Fig.* 258, as the core moves upwards from the centre, E_2 becomes bigger than E_1. On the other hand, as the core moves downwards from the centre, E_1 becomes larger than E_2. If, therefore, the displacement of the core is made proportional to the value of a controlled condition, the ratio or difference in the two e.m.f.s provides an electrical signal also, proportional to the value. *Fig.* 259, illustrates a simple measuring circuit in which the two secondary windings form two arms of a bridge, the other two being provided by resistances R R. If this bridge is balanced for zero by the potentiometer, when the core moves from the centre position the out-of-balance current or e.m.f. can be used as a means of measurement directly or as an input signal to an electrical controller.

CAPACITIVE TRANSDUCERS

In many designs of capacitive transducers, a diaphragm forms one plate of a capacitor, and the deflection of the diaphragm, in response to a pressure acting on it, varies the capacitance between it and a fixed plate. From *Fig.* 260, we see that, even if the deflection is small, the effect is not that of a simple capacitor where one free plate moves in a perpendicular direction to the other. Neubert[1] has shown that the relation between pressure and capacitance change is given by equation (409) for a clamped diaphragm.

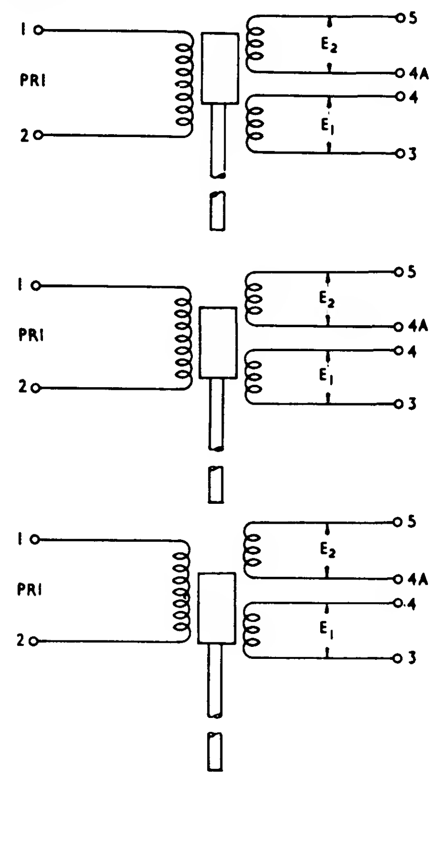
$$\frac{\delta C}{C} = \frac{0{\cdot}0625\,(1-\nu^2)\,a^4 p}{E d t^3} \qquad \dots\dots\dots (409)$$

where C = the original capacitance.
δC = the change in capacitance due to pressure p.
δ = Poisson's ratio.
a = radius of circular diaphragm.
t = thickness of diaphragm.
E = Young's modulus of diaphragm material.

This, estimates Neubert[1], is only about $\frac{1}{3}$ of the deflection of an equivalent free plate or piston movement.

Differential pressure determination is not only important in the pressure measurement field itself, but

Fig. 258. *Electrical arrangement of differential transformer windings*



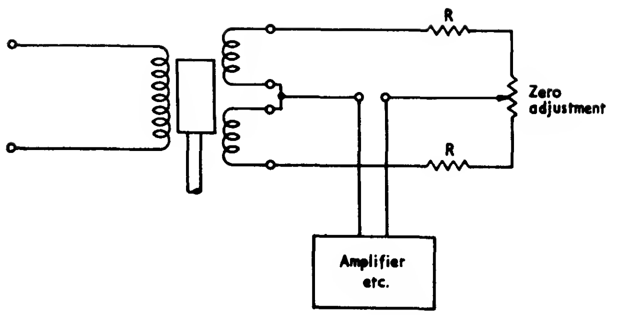


Fig. 259. *Simple measuring circuit for differential transformer*

forms the basis of measurement in flow rate and liquid level installations. The provision of a differential pressure transducer in association with electrical control systems is, therefore, an essential requirement.

The transducer of *Fig.* 261 is an interesting exercise in that it is a force balance device and combines a capacitive transducer with an inductive one.

The measuring element as shown is a diaphragm for differential pressure determination but it may comprise a special U-shaped Bourdon tube for gauge pressure measurement. It is coupled to a sealing

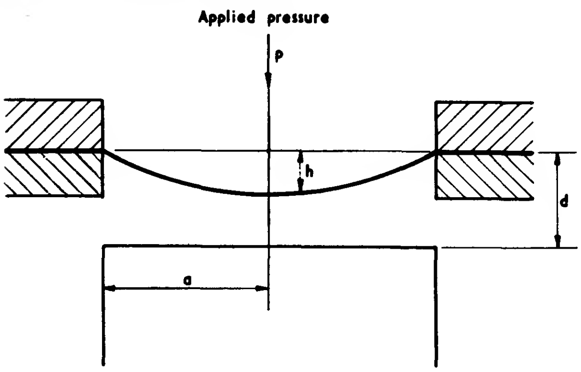


Fig. 260. *Diaphragm pattern capacitive transducer*

diaphragm and from the exterior of the diaphragm a connection is made to the upper or primary member of a three-beam lever system. The sealing diaphragm is at the pivot point of the primary lever and has negligible influence on the force set up by the measuring diaphragm. This arrangement also ensures that the effects of static pressure variation are reduced to a minimum. Friction in the lever system itself is minimized by the introduction of hardened and polished bearing surfaces.

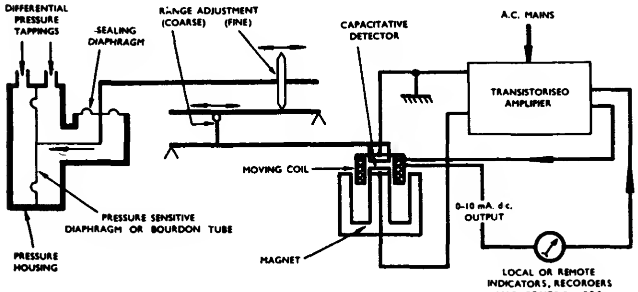


Fig. 261. *Force balance inductive-capacitive transducer.* (*Reproduced by permission of Bailey Meters & Controls Ltd.*)

On the right-hand end of the lower or tertiary lever member is a coil. This moves in the field of a permanent magnet. The inside surface of the coil former acts as one plate of a capacitor, the other plate being formed by the top surface of the central pole of the magnet.

On the application of a differential pressure to the diaphragm a force is produced by the diaphragm proportional to the differential pressure. This force tends to deflect the diaphragm in the direction shown in *Fig.* 261. The deflection is transmitted by the lever system to the coil element. The movement of the upper plate of the capacitor element, i.e. the surface of the coil former, adjusts the value of the capacitance. The element forms one arm of a capacitance bridge and any alteration in the value upsets the bridge balance. This causes a change in the output current from the amplifier unit in the range 0–10mA d.c. The current is passed through the moving coil. It varies, in proportion, the magnetic force acting on the coil. The magnetic force acts in the opposite direction to that due to the differential pressure, and a position of equilibrium is reached where a true force balance is established. The output current from the amplifier is then a direct measure of the pressure. As shown in *Fig.* 261 this current may be used to operate appropriate indicators and recorders or may be supplied as the input signal to an electronic controller.

A later design of the same firm employs a triple bellows system filled with silicone liquid in the pressure chamber. The moving coil device is retained as an element in the force balance unit, but the system is simplified in that the capacitor arrangement is eliminated.

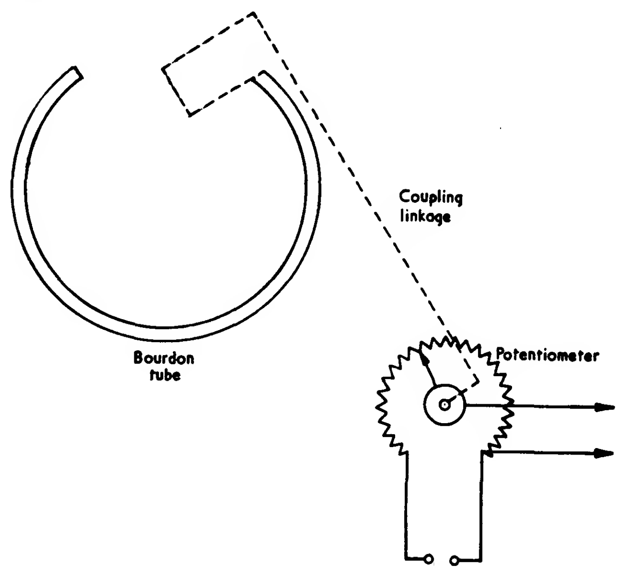


Fig. 262. *Potentiometer pattern resistive transducer*

RESISTIVE TRANSDUCERS

A very simple form of resistive transducer is formed by coupling a potentiometer requiring a very small operating torque to a prime mover such as a Bourdon tube. Intermediate lever or other movement magnification is necessary. Potentiometers with operating torques as low as ¼ gram cm are available and these make this form of transducer possible. *Fig.* 262 indicates the principle of the arrangement.

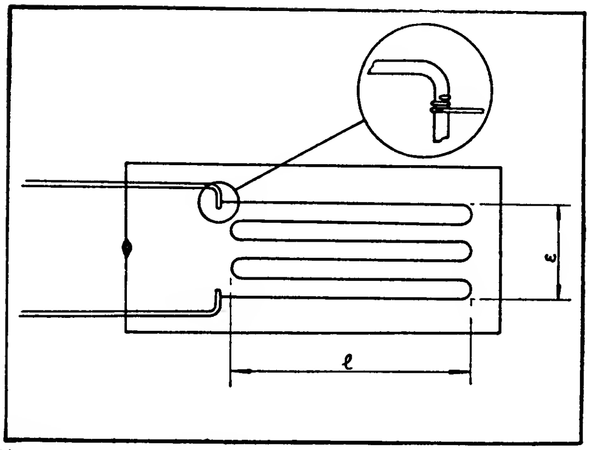


Fig. 263. *Grid form of strain gauge*

Another form of resistive transducer is the strain gauge and this is in widespread use as a means for measuring stress, force or weight.

Strain Gauges

Wire Wound Type

Consider a wire of resistance R, length L, cross-sectional area A and specific resistance ρ. Then,

$$R = \frac{\rho L}{A} \quad \ldots\ldots\ldots\ldots\ldots \quad (410)$$

Let us examine the effect of straining the wire, i.e. increasing its length from L to $(L + \triangle L)$. Then,

$$\frac{\frac{\triangle R}{R}}{\frac{\triangle L}{L}} = 1 + 2\sigma + \frac{\frac{\triangle \rho}{\rho}}{\frac{\triangle L}{L}} \quad \ldots\ldots\ldots \quad (411)$$

The left-hand side is variously termed the *sensitivity factor, the strain sensitivity factor* or *the gauge factor K*. If no change is envisaged in ρ, then the right-hand side of equation (411) becomes $1 + 2\sigma$. If the maximum value of σ is taken as 0·5, the value of K should not exceed 2, and, generally, since σ for most metals is approximately 0·3, it should be less than 2. This is not borne out wholly in practice where K may vary from 1·7 to 2·3.

If the strain is caused by a stress S, the resulting change in resistance from the unstrained condition affords a method of evaluating S.

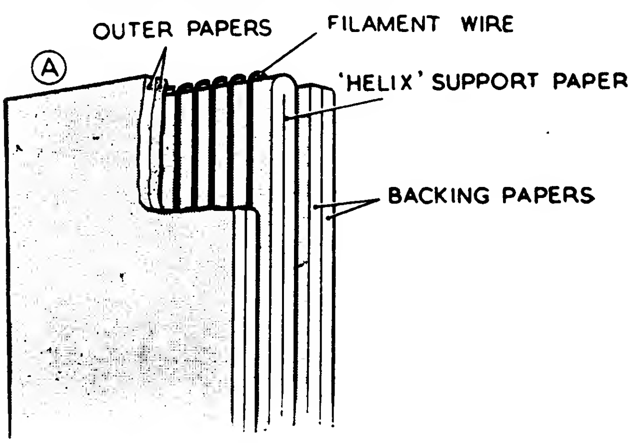


Fig. 264. *Helical wound pattern strain gauge*

Wire

The wire for a strain gauge ideally should possess high strain sensitivity, high specific resistance and a low temperature coefficient.

No wire used must, of course, be strained beyond its elastic limit, and this sets a maximum value for the strain of the order of 0·1 per cent. If 2 is taken as an average value of the gauge factor, $\frac{\triangle R}{R}$ is about 0·2 per cent for the average gauge. With R as 100Ω, this means that $\triangle R$ is only 0·2Ω.

The construction of wire wound gauges takes two general forms: the grid and the helical. A grid wound gauge is shown in *Fig.* 263. It consists of a flat grid winding of wire of about 0·001in. diameter with a backing of material suitable to withstand the temperature and other conditions under which it is likely to operate. Very thin paper, bakelized paper or bakelite itself have been used. The winding is protected by other layers of paper. The ends of the wire are soldered or spot welded to connecting leads.

The helical pattern is wound round a former in the manner of *Fig.* 264 and is protected by layers of suitable material as for the grid pattern.

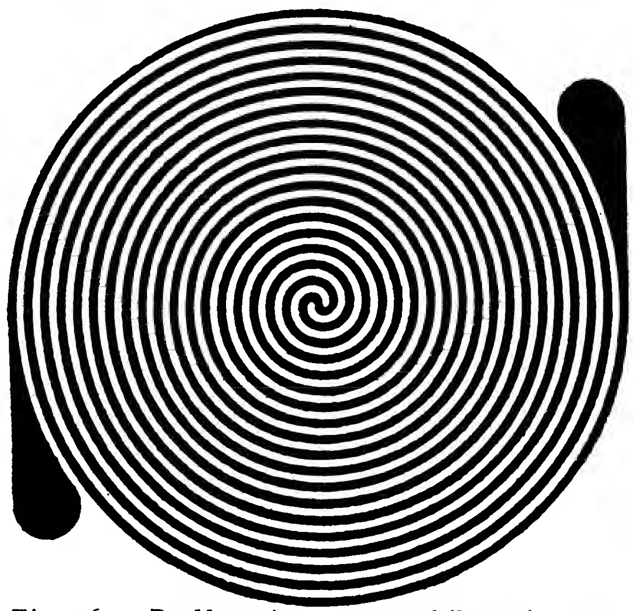


Fig. 265. *Double spiral pattern foil strain gauge*

Foil Type

A totally different design gauge is the foil pattern strain gauge. The basic principle is that of a conducting or resistance path produced by printed circuit techniques on a suitable backing material. A typical example is shown in *Fig.* 265 which is the spiral type. With the printed circuit method of manufacture, special shapes can easily be produced, and the whole gauge can be rendered extremely flexible. The main advantages claimed for the foil type are:

1. The transmission of strain from the specimen to the foil is performed to a high degree of efficiency because of the construction.
2. The ratio of element contact area to volume is very high.
3. It is easily waterproofed.
4. The soldering tags and the gauge proper are of one continuous structure.
5. Special patterns are easily produced.

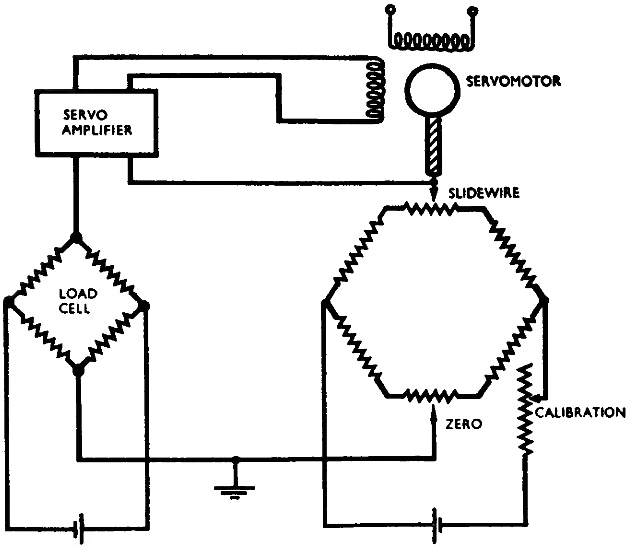


Fig. 266. *A typical bridge arrangement for load cells*

Load Cells

One of the most useful industrial applications of strain gauges as transducers is in the load cell.

In a typical pattern of load cell, four strain gauges are used. They are formed from fine cupro-nickel wire and are bonded in pairs, round the surface of the central cylindrical portion of a pillar type cell. An exclusive bonding cement is used to attach the strain gauges to the cells in a permanent fashion, free from creep or other undesirable effects which would affect accuracy and reproducibility.

The basic circuit for load cell operation is shown in *Fig.* 266. The four strain gauges are used to form a resistance bridge. Two of these, in opposite arms, are used for measuring, and the other two, also in opposite arms, are used for temperature compensation. It may be useful to note at this stage that the cells are enclosed in extremely strong cases and packed with water-repellent material and hermetically sealed. This renders them proof against humidity effects. The weight acts on the cylindrical pillar and produces proportional stresses. These are communicated to the strain gauges. Assuming that the bridge is balanced, stress occurring in either of the measuring strain gauges will vary their resistance and upset the balance of the bridge. This unbalanced output is fed into the servo amplifier. The output of the amplifier is connected to one winding of a servo-motor, the other winding of which is supplied with a reference voltage. On receipt of an unbalanced signal, the motor shaft rotates and, since it is linked to a potentiometer slider, it moves this slider in such a direction as to rebalance the potentiometer circuit.

Also coupled to the motor shaft is the pointer of an indicator or the pen of a recorder and these are re-set to the new values. A feature of this form of instrumentation is that it lends itself to digital indication. To effect this, it is necessary to couple to the motor shaft a digitizer, which converts the rotation into a digit train feeding into a digital type indicator, a card-punching machine, a type-punching machine or an electric typewriter or any of these in combination.

BASIC ELECTRONIC CIRCUITS

Fig. 267 indicates a d.c. amplifier with an input network Z_i and a feedback network Z_f. The amplifier has a gain of —A, i.e. there is a reversal of sign between input and output. It is shown as a four terminal arrangement wherein all voltages are referred to the common earth line. In the simple circuits which follow, the earth line will be omitted but it must be understood that any voltages shown in these figures are with respect to earth.

One next makes the assumption that the current taken by the input stage of the amplifier is zero.

This is equivalent to assuming that the input impedance is infinite. A further assumption normally made is that the output impedance is zero.

If θ_i is the input voltage to the circuit and θ is the actual voltage existing at the input to the amplifier, i_1 the current flowing through Z_i is given by

$$i_1 = \frac{\theta_i - \theta}{Z_i} \qquad \ldots\ldots\ldots\ldots \quad (412)$$

Similarly, the current i_2 flowing through the feedback network is

$$i_2 = \frac{\theta - \theta_o}{Z_f} \qquad \ldots\ldots\ldots\ldots \quad (413)$$

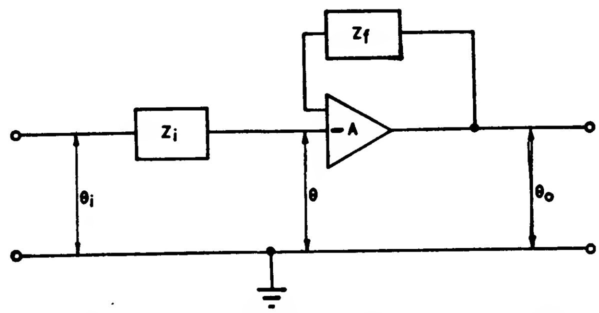


Fig. 267. *D.C. amplifier with feedback network*

But with the above assumptions,

$$i_1 = i_2 \qquad \ldots\ldots\ldots\ldots \quad (414)$$

$$\frac{\theta_i - \theta}{Z_i} = \frac{\theta - \theta_o}{Z_f} \qquad \ldots\ldots\ldots\ldots \quad (415)$$

The amplifier has a gain of $-A$ so that

$$\theta_o = -A\,\theta \qquad \ldots\ldots\ldots\ldots \quad (416)$$

Substituting for θ in (415) and rearranging,

$$\frac{\theta_o}{\theta_i} = -\frac{Z_f}{Z_i}\left[\frac{1}{1 + \frac{1}{A}\left(1 + \frac{Z_f}{Z_i}\right)}\right] \qquad \ldots\ldots\ldots \quad (417)$$

If the gain A is very large, equation (417) reduces to

$$\frac{\theta_o}{\theta_i} = -\frac{Z_f}{Z_i} \qquad \ldots\ldots\ldots\ldots \quad (418)$$

This is the transfer function of the circuit.

Now let us examine *Fig.* 268 shown in the single line arrangement. In the feedback line is a capacitor C and in the input line a resistor R. These are equivalent to Z_f and Z_i respectively in *Fig.* 267. Making the same assumption as for *Fig.* 267, that i_1 equals i_2,

$$\frac{\theta_i}{R} = -C\,\frac{d\theta_o}{dt} \qquad \ldots\ldots\ldots\ldots \quad (419)$$

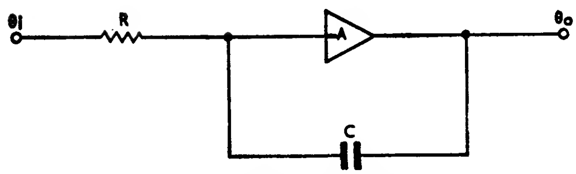


Fig. 268. *Analogue integral action circuit*

$$\frac{\theta_i dt}{R} = -C\,d\theta_o \qquad \ldots\ldots\ldots\ldots (420)$$

Integrating both sides,

$$\theta_o = -\frac{1}{CR}\int \theta_i dt \qquad \ldots\ldots\ldots (421)$$

This implies that we can obtain an integral action with the circuit.

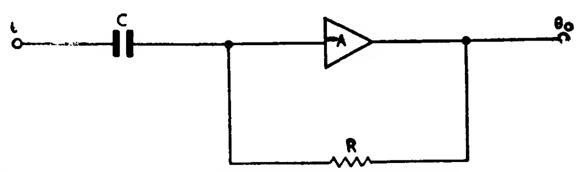


Fig. 269. *Analogue derivative action circuit*

Next consider *Fig.* 269. This has a capacitor input and a resistor in the feedback line. C is now equivalent to Z_i and R to Z_f. Again assuming that the current i_1 is equal to i_2,

$$C\frac{d\theta_i}{dt} = -\frac{\theta_o}{R} \qquad \ldots\ldots\ldots\ldots\ldots (422)$$

From which

$$\theta_o = -RC\frac{d\theta_i}{dt} \qquad \ldots\ldots\ldots\ldots (423)$$

This gives a derivative relation between output and input voltages.

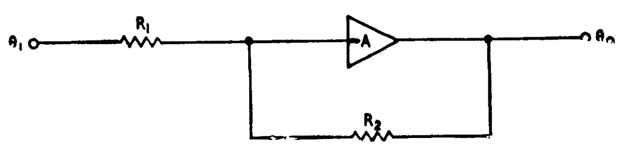


Fig. 270· *Analogue proportional action circuit*

A third circuit is shown in *Fig.* 270. Here both input and feedback elements are resistors. Without much difficulty it can be seen that

$$\theta_o = -\frac{\theta_i R_2}{R_1} \qquad \ldots\ldots\ldots\ldots (424)$$

If θ_o could be regarded as the full range output signal of an electronic controlling element, by adjusting the value of R_1 a range of input values could be obtained for this same value of θ_o. This suggests, from the definition of proportional band in Chapter 13, that we have in *Fig.* 270 the basis of a proportional controller with proportional band adjustment. In a similar fashion, the circuits of *Figs.* 268 and 269 could form the basis of electronic integral and derivative controller actions respectively. From the circuits shown in the three figures one should be able to devise two term or three term controllers corresponding to the pneumatic examples of *Figs.* 216 to 227 in Chapter 14.

Figs. to 268 are 270 strictly speaking analogue computer circuits, but their application in process controllers will be apparent when actual electronic designs are considered.

Before dealing with these, however, it will be most convenient to introduce the differential operator.

Use of the Calculus Operator

So far in using differentiation and integration the equations have been expressed in the full terminology. There is, however, a kind of mathematical shorthand which may be used, rendering equations less formidable looking and the derivation of expressions easier to follow.

For $\frac{d}{dt}$ is written D, so that $\frac{d\theta}{dt}$ becomes $D\,\theta$. $\frac{d^2\theta}{dt^2}$ is written as $D^2\theta$ and $\frac{d^n\theta}{dt^n}$ generally as $D^n\theta$.

If I represents the integration operator,

$$I = \frac{1}{D}$$

From this, $\int\theta\,dt$ can be reduced to $\frac{\theta}{D}$.

A few examples will clarify the procedure.

Equation (421) can be written,

$$\theta_o = -\frac{1}{CR}\frac{\theta_i}{D} \qquad \ldots\ldots\ldots\ldots (425)$$

Equation (423) becomes,

$$\theta_o = -RC\,D\theta_i \qquad \ldots\ldots\ldots\ldots (426)$$

Again, for a controller unit producing proportional plus integral action

$$V = -K_1\theta - K_2\int\theta dt \qquad \ldots\ldots (427)$$

or $$V = -K_1\left(\theta + \frac{1}{t_i}\int\theta dt\right) \qquad \ldots\ldots (428)$$

with the differential operator D,

$$V = -K_1\left(\theta + \frac{\theta}{t_i D}\right) \qquad \ldots\ldots\ldots (429)$$

Now we can manipulate the operators in a similar fashion to algebraic factors so that (429) can be written

$$V = -\theta K_1\left(1 + \frac{1}{t_i D}\right) \qquad \ldots\ldots\ldots (430)$$

or in terms of a transfer function

$$\frac{V}{\theta} = -K_1\left(1 + \frac{1}{t_i D}\right) \qquad \ldots\ldots\ldots (431)$$

In a similar manner, for a proportional plus dervative controller,

$$\frac{V}{\theta} = -K_1\left(1 + t_D D\right) \qquad \ldots\ldots\ldots (432)$$

The letters p and s are sometimes used instead of D for the operator.

Electronic Proportional Plus Integral Controller Units

Next let us consider further circuits for two and three term control, keeping to the analogue designs for the time being. *Fig.* 271 represents an arrangement for producing integral controller action.

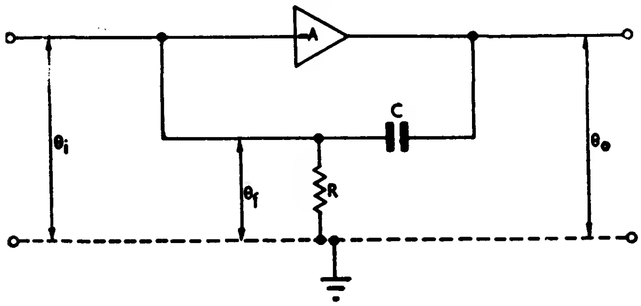


Fig. 271. *Integral controller action circuit*

If this circuit is examined it will be found to be basically the same as *Fig.* 194 of Chapter 13 with a feed forward and a feedback channel. If the gain of the unit G_1, which corresponds to the amplifier in *Fig.* 271, is very high, the transfer function is given by

$$\frac{\theta_o}{\theta_i} = -\frac{1}{9_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (433)$$

where

G_2 = the transfer function of the feedback network.

In *Fig.* 271,

$$G_2 = \frac{\theta_f}{\theta_o} = \frac{Ri}{Ri + \frac{1}{C}\int idt} \qquad \ldots\ldots\ldots (434)$$

where i = the current flowing through R and C.

Writing $\frac{i}{D}$ for $\int idt$

$$G_2 = \frac{Ri}{Ri + \frac{i}{CD}} \qquad \ldots\ldots\ldots\ldots (435)$$

$$G_2 = \frac{R}{R + \frac{1}{CD}} \qquad \ldots\ldots\ldots\ldots (436)$$

$$G_2 = \frac{1}{1 + \frac{1}{RCD}} \qquad \ldots\ldots\ldots\ldots (437)$$

But $\frac{\theta_o}{\theta_i} = -\frac{1}{9_2}$

Hence

$$\frac{\theta_o}{\theta_i} = -\left(1 + \frac{1}{RCD}\right) \qquad \ldots\ldots\ldots\ldots (438)$$

This is of the same basic form as previous equations for integral controller action. Replacing RC by t_1,

$$\frac{\theta_o}{\theta_i} = -\left(1 + \frac{1}{t_1 D}\right) \qquad \ldots\ldots\ldots\ldots (439)$$

Reverting to the single lead arrangement, figs. 272 and 273 show proportional plus integral control circuits. *Fig.* 272 is an advanced negative feedback method.

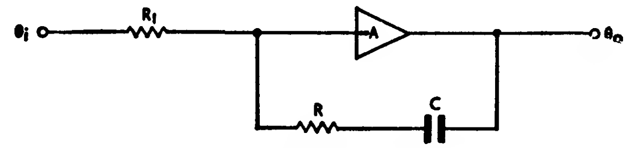


Fig. 272. *Integral controller action circuit with advanced negative feedback*

The transfer function for this figure can be shown to be

$$\frac{\theta_o}{\theta_i} = -K_a\left(1 + \frac{1}{t_1 D}\right) \qquad \ldots\ldots\ldots (440)$$

where

$$K_a = \frac{R}{R_1}$$

$$t_1 = RC$$

It can be seen that equation (439) is of the same form as (436), the basic one defining proportional plus integral controller action.

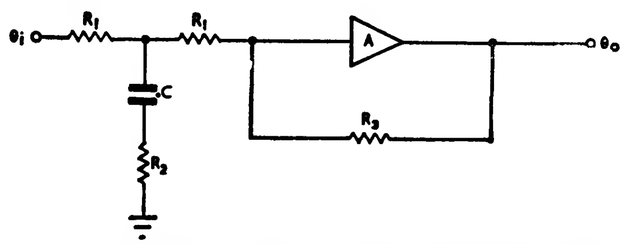


Fig. 273. *Integral controller action circuit with delayed input*

In the delayed input method of *Fig.* 273 the transfer function is

$$\frac{\theta_o}{\theta_i} = -K_B\left(\frac{k\,t_1 D + 1}{t_1 D + 1}\right) \qquad \ldots\ldots\ldots (441)$$

where

$$K_B = \frac{R_3}{2R_1}$$

$$k = \frac{2R_2}{(2R_2 + R_1)}$$

$$t_1 = \frac{C\,(2R_2 + R_1)}{2}$$

Electronic Proportional Plus Derivative Controller Units

A derivative control circuit is given in *Fig.* 274. Like *Fig.* 271 for integral controller action it is used fairly frequently, often in association with *Fig.* 271.

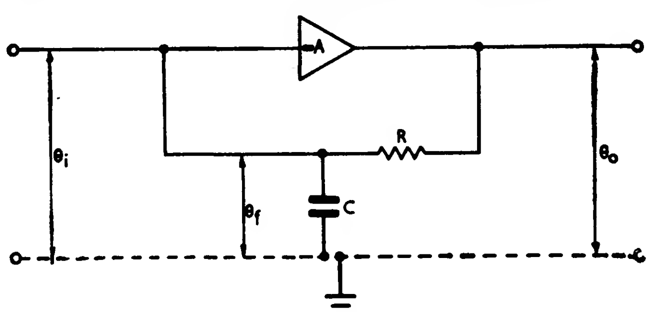


Fig. 274. *Derivative controller action circuit*

Following similar reasoning to that for the integral example,

$$G_2 = \frac{\theta_f}{\theta_o} = \frac{\frac{1}{C}\int idt}{Ri + \frac{1}{C}\int idt} \qquad \ldots\ldots\ldots\ldots (442)$$

Writing $\frac{i}{D}$ for $\int idt$

$$G_2 = \frac{\frac{i}{CD}}{Ri + \frac{i}{CD}} \qquad \ldots\ldots\ldots\ldots (443)$$

$$G_2 = \frac{1}{RCD + 1} \qquad \ldots\ldots\ldots\ldots (444)$$

From which,

$$\frac{\theta_o}{\theta_i} = -\left(RCD + 1\right) \qquad \ldots\ldots\ldots\ldots (445)$$

Replacing RC by t_D

$$\frac{\theta_o}{\theta_i} = -\left(t_D D + 1\right) \qquad \ldots\ldots\ldots\ldots (446)$$

Two arrangements are possible: delayed negative feedback and advanced input.

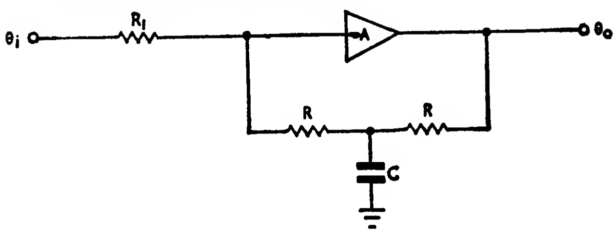


Fig. 275. *Derivative controller action circuit with delayed negative feedback*

Fig. 275 shows the basic circuit for delayed negative feedback. The transfer function can be derived as

$$\frac{\theta_o}{\theta_i} = -K_C\,(t_D\,D + 1) \qquad \ldots\ldots\ldots (447)$$

where

$$K_C = \frac{2R}{R_1}$$

$$t_D = \frac{RC}{2}$$

Observe that the delayed negative feedback principle has been described before when dealing with pneumatic proportional plus derivative controller action in Chapter 14. In *Fig.* 225 for example there is a similar derivative resistance or restriction feeding into the negative feedback capacitor formed by the bellows.

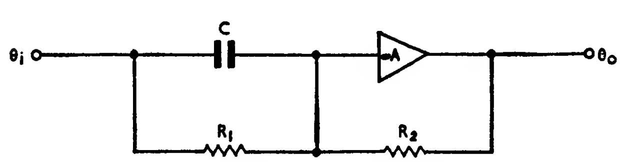


Fig. 276. *Derivative controller action circuit with advanced input*

The advanced input method of *Fig.* 276 is not so much used with electronic controllers. The transfer function can be shown to be

$$\frac{\theta_o}{\theta_i} = -K_D\,(t_D\,D + 1) \qquad \ldots\ldots\ldots (448)$$

where

$$K_D = \frac{R_2}{R_1}$$

$$t_D = R_1 C$$

True Integral Action

If a step change is applied to an RC network the curve relating the change in value of the output signal to time is an exponential one. This applies both to pneumatic and electronic examples. In *Figs.* 222 and 223 of Chapter 14, if p represents a step change, $\frac{dp_1}{dt}$ is continuously changing as p_1 increases to the final value p. Only initially will $\frac{dp_1}{dt}$ be proportional to p whereas the requirement is for $\frac{dp_1}{dt}$ to be proportional to p during the whole of the operating time. This calls, in effect, for a constant pressure drop across the integral restriction. Feedback action coupled with a high gain nozzle-flapper amplifier can produce an approximation to this requirement.

For electronic integral action consider *Fig.* 268. With an amplifier gain of $-A$, for a step change θ_i the relation between θ_o and θ_i can be written,

$$\theta_o = -\theta_i A\,(1 - e^{-\frac{t}{RC(1+A)}}) \qquad \ldots (455)$$

Expanding the term inside the brackets and retaining just the first two terms of the series,

$$\theta_o = -\frac{\theta_i t}{RC}\left(1 - \frac{t}{2RCA}\right) \qquad \ldots\ldots\ldots (456)$$

The true integral action is represented by $\frac{\theta_i t}{RC}$ and the error or departure x from this value is given by

$$x = \frac{t}{2RCA} \qquad \ldots\ldots\ldots\ldots\ldots (457)$$

Assigning some figures to equation (457), some idea of the minimum gain required by the amplifier may be found. Let $x = 1\%$, $RC = 10$sec and $t = 500$sec, then

$$\frac{1}{100} = \frac{500}{2 \times 10 \times A}$$

From this, A must be at least 2500.

Different requirements for RC and t, however, can alter this value considerably, and there are other methods of ensuring true integral action in electronic controllers as we shall see when the various types are considered.

Integral Limiter or Desaturator (Reset Windup)

It is thought desirable to include a brief note on one unfavourable aspect of integral controller action. Under some conditions, particularly during the start-up period, the amplifier of the controller unit can

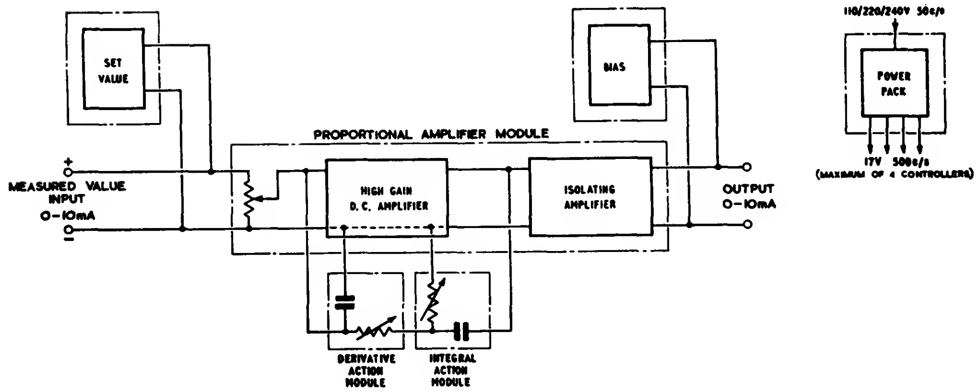


Fig. 277. Three term modular electronic controller. (Reproduced by permission of Bailey Meters & Controls Ltd.)

saturate. This applies to both pneumatic and electronic versions, and results in the controller unit producing its maximum output signal, despite the changes in the value of the controlled condition, until the set value is reached. The correcting unit has, of course, moved to the fully open position and has remained there, obeying the value of the controller output signal. It can be seen without much difficulty that conditions for overshoot are produced, the controller output signal not changing its sign until the set value has been reached. In the case of batch production where start-ups may be on a regular basis this is clearly undesirable since the value of the overshoot may be such as to damage the product. Some form of limiting operation on the integral controller action is necessary. Depending on process characteristics the introduction of derivative controller action may reduce the overshoot value, but integral limiting devices are included in many proportional plus integral action controllers, and references 2 and 3 give typical examples of these. Other examples will be found in the descriptions of electronic controllers which follow.

ELECTRONIC CONTROLLERS

It is not intended to present an exposition of every electronic controller available in this country, but rather, by choosing one or two examples, to illustrate how the basic circuits so far discussed are applied in commercial models of electronic controllers.

Fig. 277 shows the build-up of the Bailetronic* modular controller of Bailey Meters & Controls Ltd. The fundamental unit here is the proportional amplifier with isolated input and output terminals. The amplifier is a high gain chopper type d.c. design, the chopper frequency being 500c/s. The latter frequency, together with the isolation scheme, gives a large amount of protection against unwanted induced a.c. signals. Integral and derivative controller actions are included by adding separate modules and it can be seen that both actions are produced by networks of the types shown in *Figs.* 271 and 274 in the feedback path of the amplifier. The reference line here is the negative line and not earth as indicated in the figures quoted. The measured value is compared with the set value and the deviation between the two is applied to the proportional band adjusting potentiometer shown at the input to the d.c. amplifier.

Signal input is 0-10mA and the output signal standard range is 0-10mA. The proportional band may be adjusted between 2 and 1000% or in terms of gain from 0·1 to 50. Integral action times can be adjusted in a series of twelve steps between 3sec and 10min. Derivative action time is adjustable between 0·5 and 120sec.

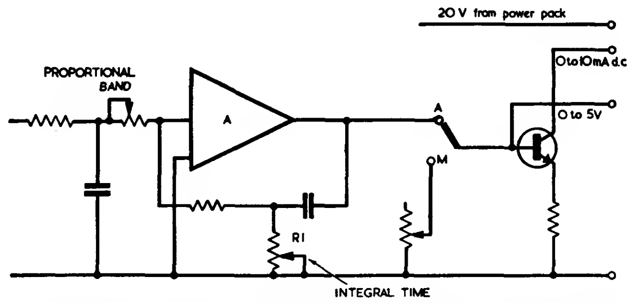
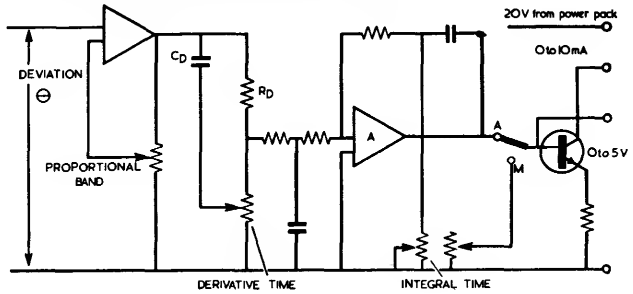


Fig. 278. Two term electronic controller. (Reproduced by permission of George Kent Ltd.)

A simplified diagram of a two term electronic controller of George Kent Ltd. is that of *Fig.* 278. Proportional band adjustment is by the input potentiometer and integral action time is produced by a network of the type of *Fig.* 271. Input signal is 0-10mA and output signal is 0-10mA. The proportional band is continuously variable between 5 and 500%. Integral action time is adjustable in ten steps from 0·5 to 25sec.

In the three term version of the same firm (*Fig.* 279), it will be observed that the proportional band adjustment and the derivative action time circuit are made part of the feedback channel of one amplifier and the integral action time network is in the feedback channel of a second amplifier. Proportional band adjustment covers the same range as in the two term design, but the integral action time is now variable from 0·5 to 20min and derivative action time from 0 to 5min.

Fig. 279. Three term electronic controller. (Reproduced by permission of George Kent Ltd.)



* Registered Trade Name

In the Conzel* controller of Negretti & Zambra Ltd., shown in *Fig.* 280, the integral action time network is shown as "Reset" and the derivative action network, designated "Rate", is included in the feedback channel of the amplifier. Observe the integral limiter connected as a feedback channel. It ensures that the controller output signal value is altered as soon as the signal from the process itself changes direction. This prevents the saturation effect described in the previous section.

As with the other designs, the controller is an operational amplifier with variable networks for producing proportional, integral and derivative responses.

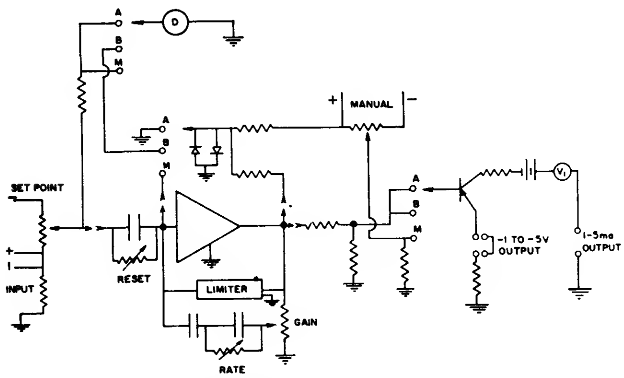


Fig. 280. *Three term electronic controller.* (*Reproduced by permission of Negretti & Zambra Ltd.*)

The amplifier is a diode modulator stabilized type. Ultra-low leakage surface passivated diodes are used to modulate the d.c. current error signal flowing through the integral resistor. The resultant current resolution allows for long integral action time constants at low proportional gains.

The modulated current is transformer coupled to a high gain transistor amplifier whose output is changed back into d.c. at a greatly amplified signal level.

The signal is then mixed with the higher frequency components of the input signal and further amplified by a direct-coupled transistor amplifier to the final output signal level. The high frequency response of the d.c. section gives wide frequency response to the proportional part of the input signal.

The ratio of output voltage to input current is typically $10^{14}\Omega$. For example, a 1·0 micromicroampere input current would produce 100 volts on the output if the amplifier were operated under open loop conditions and could give this much voltage swing. The high gain is necessary to maintain sufficient feedback round the amplifier for all combinations of proportional band, integral and derivative settings and have a stable and noise insensitive controller.

Proportional band adjustment is from 1 to 500%, derivative time from 0·15min to 20min or alternatively from 0·003 to 4min.

In *Fig.* 281 is shown a simplified diagram of the Evershed & Vignoles A.C.313 Mk. 4 electronic three term controller.

The three control terms are generated at different stages of the circuitry, and are free from interaction. They can provide a wide range of proportional, integral and derivative adjustments.

The measured value signal is applied to transistor (3). The output of (3) is fed to the special "*R-W*" coupler (2).

A large d.c. voltage, proportional to the relatively small current through transistor (3), is generated. This signal *Vm* (a voltage in the range of 0-130 volts) is applied across the resistor (5) in opposition to the "set value" voltage *Vd* generated in the "set value" circuit embodying an indicator and the stabilizer valve (4). As a result, a voltage proportional to the magnitude and the direction of the deviation between *Vm* and *Vd* is applied to the grid of valve (7).

In parallel with valve (7), a current *Is* (derived from the d.c. supply circuit) is applied and its magnitude and direction is arranged to make *Is* and *Ip* equal and opposed to each other when no error signal is applied to the grid of the valve (7), i.e. no voltage drop occurs across the circuit constituted by the resistor (8) and the regulating unit (18) when *Vm* equals *Vd*.

Any difference between *Vm* and *Vd* will modify the current through valve (7), thereby creating a voltage drop across the resistor (8). By varying the moving contact of resistor (8), the magnitude of the current change and, hence, of the voltage drop relative to a given change in the error signal *Vm-Vd* can be adjusted and a proportional band adjustment is created. To accommodate a wide range proportional band, the value of the resistor (5) can be changed by a switch.

Any voltage drop across resistor (8) is applied as an input signal to the integrating circuit consisting of the variable resistor (9) (integral time adjustment), and the integral capacitor (10). Three different values for the latter can be selected by a switch, thereby providing an integral range adjustment. The voltage across the capacitor (10) which is connected between the grid and cathode of valve (16) thus represents the integral of the error.

The disadvantages of a resistance-capacity integral network (exponential charge, leakage at low inputs) are overcome by valve (16) operating in conjunction with the "*R-W*" coupler (15), the output of which is arranged to be equal to the integrating voltage across the capacitor (10), and is applied in opposition to the integrating voltage. The net voltage across the integral circuit is thus always zero. In this manner long time constants, linear integration which "stays put" when the error returns to zero, are achieved.

To provide derivative controller action, the "*R-W*" coupler (6) is arranged to apply a relatively large voltage (which is proportional to the current through the valve (7) and thus to the error) to capacitor (11), resistor (12) and capacitor (14). In steady conditions capacitor (11) is charged to the input voltage *Vp*. A change in the error will alter *Vp* and a voltage, which, due to the characteristics of the "*R-W*" coupler, is proportional to the rate of change of the error, will appear across resistor (12). This voltage is applied to capacitor (10), thereby modifying the current through valve (16) and thus providing the derivative component in the controller's output signal. The variable resistor (12) provides the derivative action time adjustment. (In practice (R12) is a two-gang potentiometer utilizing (R13) in conjunction with (C14) as a shunt across (R17), effective during transient conditions to enhance the performance of the arrangement.)

* **Registered Trade Name**

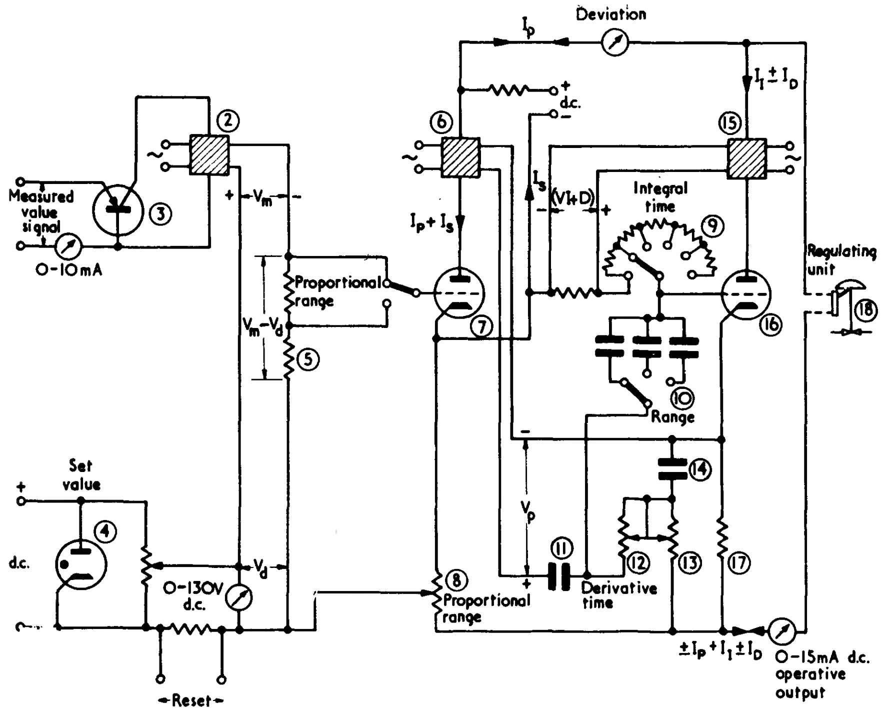


Fig. 281. Three term electronic controller. (Reproduced by permission of Evershed & Vignoles Ltd.)

Proportional band range is from 5-325%, integral action time has three ranges: 3-20sec, 0·3 to 3min and 3-50min. Derivative action time is continuously variable from 0-3min.

Correcting Units for Electronic Controllers

The electrical nature of the output signal from an electronic controller has posed a problem with relation to the correcting unit. It seems logical for the motor element of the correcting unit to be an electrical one. Until recently, however, satisfactory electrical motors which would respond to, say, a three term electrical control signal were not available. The control loop has included a converter or transducer to change the electrical signal into a corresponding pneumatic one so that the normal pneumatic control valve could be used. The converters or transducers perform the reverse functions to those described at the beginning of the chapter. It is not proposed to describe any of these as they contain many of the same elements but operated in the reverse manner. But attention must be drawn to a development existing at the moment of writing for utilizing electrical output signals from an electronic controller directly.

Steromotor Actuator

The Steromotor is in effect an a.c. induction motor. The stator coils produce a rotating magnetic field. The rotor is a permanent magnet and is free to rotate round the stator bore and it follows the rotating magnetic field. There is no metallic contact between rotor and stator bore, however, since the rotor is equipped with two resilient tyres which run in annular tracks formed in the epoxy resin compound in which the motor is potted. The motion of the rotor is epicyclic, and this motion is transmitted to the output shaft through a flexible linkage. There are two versions: a synchronous one and an asynchronous one. The latter has smooth tyres and tracks which permit a negligible amount of slip under continuous running or stepping conditions.

When used in association with a control valve, the output shaft drives a simple screw jack directly coupled to the spindle of the valve. The position of the spindle is measured by a pick-off device and the signal from this is compared with the value of the signal from the electronic controller. Any unbalance between the two signals is detected and utilized to energize the Steromotor which continues to operate until the balance is restored.

No mechanical braking or electrical braking is required as the drive shaft of the Steromotor stops immediately the energizing supply is cut-off.

If N is the speed of the motor in rev/min, ω the velocity of the rotating field (proportional to the supply frequency), D the diameter of the stator bore and d the rotor diameter,

$$N \propto \frac{\omega (D - d)}{d} \qquad \qquad (458)$$

Motor speeds at the present range from 2 to 200rev/min for the synchronous versions and 20rev/min, 30rev/min and 60rev/min for the asynchronous types.

References and Literature for Further Reading

1. Neubert, H. K. P. *Instrument Transducers.* Oxford University Press. 1963.
2. Eckman, D. P. *Automatic Process Control.* Chap. 4. John Wiley, 1958.
3. Young, A. J. *An Introduction to Process Control System Design.* Longmans, 1955.

APPENDIX 1

B.S.1042, Part 1, covers the following types of orifices (in addition to nozzles and Venturi tubes).

1. Square edged with corner tappings.
2. Square edged with D and D/2 tappings.
3. Square edged with flange tappings.
4. Conical entrance.
5. Quarter circle (quadrant-edge).

1. Square Edged Orifice, Corner Tappings

This type of orifice may be used for measuring flow rate in

a) Pipes of internal diameter not less than 1 inch.

b) Between two large chambers divided by a partition containing the orifice.

c) At the outlet or inlet of a pipe with internal diameter not less than 1 inch discharging into or receiving from a large chamber.

The smallest allowable orifice diameter is 0·25 inch. The general form is shown in *Figs.* 61 (page 30) and 63 (page 31) and involves a square edge on the upstream side and a bevel on the downstream side. If the plate is sufficiently thin, the bevel may be omitted.

The orifice may be used with liquids, gases and vapours with the proviso that it is not suitable for metering viscous liquids. It is, in addition, not suitable for critical flow operation.

With compressible fluids, the pressure difference across the orifice in inches of water must not exceed 5·5 times the absolute upstream pressure in lbf/in^2.

In the specification, data is presented for Reynolds numbers of 10 000 upwards for pipes 1 in to 2 in diameter and 20 000 upwards for pipes of 2 in diameter

2. Square Edged Orifice, D and D/2 Tappings

The requirements are generally similar to those for orifices with corner tappings, but the orifice is not applicable to measuring flow between two chambers.

3. Square Edged Orifice, Flange Tappings.

The general requirements are similar to those for orifices with corner tappings, but the minimum internal pipe diameter is now 2 in, and the minimum orifice diameter 0·2 in. Data is presented for Reynolds numbers from 100 000 to 1 000 000 for pipes 2 to 3 in diameter and 100 000 to 10 000 000 for pipes of 4 to 14 in diameter.

It cannot be used for measuring flow between two chambers.

4. Conical-Entrance Orifice Plate

In effect, this is a reversal of the three previous designs in that it involves a bevelled edge on the upstream side and a square edge on the downstream side.

This orifice is suitable for viscous liquids such as oil.

Data is given for area ratios less than 0·1 and Reynolds numbers from 250 to 200 000.

The minimum pipe diameter is 1 in and the minimum orifice diameter is 0·25 in.

5. Quarter-Circle Orifice Plate

This orifice has a rounded edge on the upstream side and a square edge on the downstream side.

It is particularly suitable for viscous liquids such as oil, but it is not suitable for measuring flow between two chambers.

The minimum pipe diameter is 1 in and the minimum orifice diameter is 0·6 in.

Data is supplied for Reynolds numbers up to 100 000.

Factors Influencing Choice of Device

These are some of the main factors mentioned in B.S.1042 Part 1:—

1. Viscous fluids. Square edged orifices and nozzles are generally unsuitable. Conical entrance orifices, quarter-circle orifices and venturi tubes are suitable, but not unreservedly so.

2. Lower flow limit. Smallest for orifices with corner or *D* and *D*/2 tappings.

3. Upper flow limit. This is limited in the case of conical entrance and quarter circle orifices to the Reynolds numbers specified above. For other devices, there is, generally speaking, no upper limit to the flow rate.

4. No device specified in the Standard is suitable for pulsating flow conditions.

5. A minimum length of straight pipeline is required upstream of the devices. This varies with the different types.

6. The internal surface condition of the pipe preceding the orifice influences the minimum pipe diameter.

7. There is no upper limit of pipe size.

8. Flow between two large chambers. Orifice plates with flange or *D* and *D*/2 tappings quarter-circle plates and Venturi tubes are unsuitable.

9. Only a nozzle is suitable for flow measurement at pressure differences above the critical.

10. Cost. Orifices have considerable advantages on this score.

11. Net pressure loss. Venturi nozzles and Venturi tubes have appreciably less net pressure loss than orifices or nozzles.

12. Venturi tubes have a length between two and six pipe diameters.

13. Accuracy. This is influenced by circumstances of installation. (Section 4 and Appendix D of B.S.1042 Part 1 discuss this in some detail).

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